

# A SYSTEM DYNAMICS MODEL FOR IMPROVING PRIMARY EDUCATION ENROLLMENT IN A DEVELOPING COUNTRY

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## Abstract

The system dynamics approach is a holistic way of solving problems in real-time scenarios. This is a powerful methodology and computer simulation modeling technique for framing, analyzing, and discussing complex issues and problems. System dynamics modeling is often the background of a systemic thinking approach and has become a management and organizational development paradigm. This paper proposes a system dynamics approach for studying the importance of infrastructure facilities on the quality of primary education system in a developing nation. The model is built using the Cross Impact Analysis (CIA) method of relating entities and attributes relevant to the primary education system in any given community. The CIA model enables us to predict the effects of infrastructural facilities on the community's access of primary education. This may support policy makers to take more effective actions in campaigns that attempt to improve literacy.

**Keywords:** developing countries, system modeling, cross impact analysis, simulation, system dynamics, primary education

## 1. Introduction

The first stage of compulsory education is primary or elementary education. In most countries, it is compulsory for children to receive primary education, though in many jurisdictions it is permissible for parents to provide it. The transition to secondary school or high school is somewhat arbitrary, but it generally occurs at about eleven or twelve years of age. Some educational systems have separate middle schools with the transition to the final stage of education taking place at around the age of fourteen.

The major goals of primary education are achieving basic literacy and numeracy amongst all pupils, as well as establishing foundations in science, geography, history and other social sciences. The relative priority of various areas, and the methods used to teach them, are areas of considerable political debate. Some of the expected benefits from primary education are the reduction of infant mortality rate, population growth rate, crude birth and death rate, and so on.

Because of the importance of primary education, there are several models proposed to study the factors influencing the primary school enrollment and progression. These are logistic regression models (Admassu 2008), poisson regression models (Admassu 2008), system models (Altamirano and van Daalen 2004, Karadeli et al. 2001, Pdamallu 2001, Terlou et al. 1991), behavioral models (Benson 1995, Hanushek et al. 2008)

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constructed for the context of different countries. Several factors which influence the school enrollment and drop outs are identified in various studies. Some of the vital factors at the macro level are social, economic and logistics factors (Benson 1995), and at the micro level there are parental education, household wealth/income, distance to school, financial assistance to students and quality of school (Admassu 2008, Benson 1995, Rena 2007). An early system dynamics model to investigate the low efficiency of primary education in Latin America is introduced by Terlouw et al. (1991). This model investigates the progression through primary school and includes causal chains leading to progression, dropout and repetition of students. Karadeli et al. (2001) develop a model to analyze the future quality of the Turkish educational system based on the budget of the Ministry of National education. In this model, quality of education and progression of students is influenced by the student to teacher ratio and student to class ratio. Altmirano and van Daalen (2004) propose a system dynamics model to analyze the educational system of Nicaragua and helps in identifying and analyzing the consequences of policies that are aimed at improving the coverage of the different educational programs, reducing illiteracy and increasing the average number of schooling years of the population. This study shows that implementing literacy programs and introducing a program in which families in extreme poverty receive a subsidy has an effect on school coverage as well as on the number of illiterate people. More recently, Hanushek et al. (2008) shows that school quality and grade completion by students are directly linked. The World Bank has published several reports on achieving universal primary education (Bruns et al. 2003, Serge 2009). In particular, Serge (2009) focuses on the infrastructure challenge in Sub-Saharan Africa and the constraints to scale up at an affordable cost.

The model proposed in this study aims at identifying the importance of infrastructural facilities on school enrollment and progression beside factors such as quality of teaching and income level. This point is also investigated by Akar (2008) who reports about the infrastructural problems at Turkish schools and their negative impact on students. Here, we present the details about the model constructed for this purpose, the selection of attributes and entities and the simulation results that identify the variables that impact the quality of primary education. The simulation is conducted by using the Gujarat primary education data in India (Pedamallu 2001).

## **2. The model**

The model proposed here is developed by using the cross impact analysis method (CIA). The CIA method is one of the most popular systems thinking approach developed for identifying the relationships among the variables defining the systems (Gordon and Hayward 1968, Julius 2002, Weimer-Jehle 2006). This method first was developed by Theodore Gordon and Olaf Helmer in 1966 in an attempt to answer a question whether perceptions of how future events may interact with each other can be used in forecasting. As it is well known, most events and trends are interdependent in some ways. The CIA method provides an analytical approach to the probabilities of an element in a forecast set, and it helps to assess probabilities in view of judgments about potential interactions between those elements. (We refer to Lane (1999) and Mohapatra et al. (1994) for more detailed information on system dynamics modeling.) CIA has been used to model and simulate several real-time problems (for example: Pedamallu et al. 2009, Hayashi et al. 2006). Here, we briefly describe the steps of the CIA method through a block diagram given in Figure 1.

### **2.1. Definition of the system**

Systems defined based on entities, which interact with each other and produce some outputs that are either designed or natural. A system receives inputs and converts them through a process and produces outputs. All the outputs of a system need not be desirable. In the present context, the system represents the primary education system.

#### *a. Environment*

Every system functions in an Environment, which provides inputs to the system and receives outputs from the system. In our context, the Environment is the society.

#### *b. Structure*

All systems have a Structure. The 'body' of a system's structure is represented by the entities of the system and their interrelationships or linkages or connections. The entities in our system are defined as follows.

1. student,
2. teacher,
3. parents,
4. educational officials,
5. infrastructure and

6. local community.

c. *Linkages*

The linkages among entities may be physical (e.g., facilitates), electro-magnetic (e.g., electrical, electronic and communications systems, and so on), and information-based (e.g., influence, and so on). It is important to try and understand, what linkages exist in the system's structure, which entities are linked with each other, and the implications of these linkages on the behavior of the entities in particular. The entity relationship diagram of the system is illustrated in Figure 2. Exchange of matter, information and/or spirit between two entities causes a change in the state of both entities. This is reflected as system behavior.

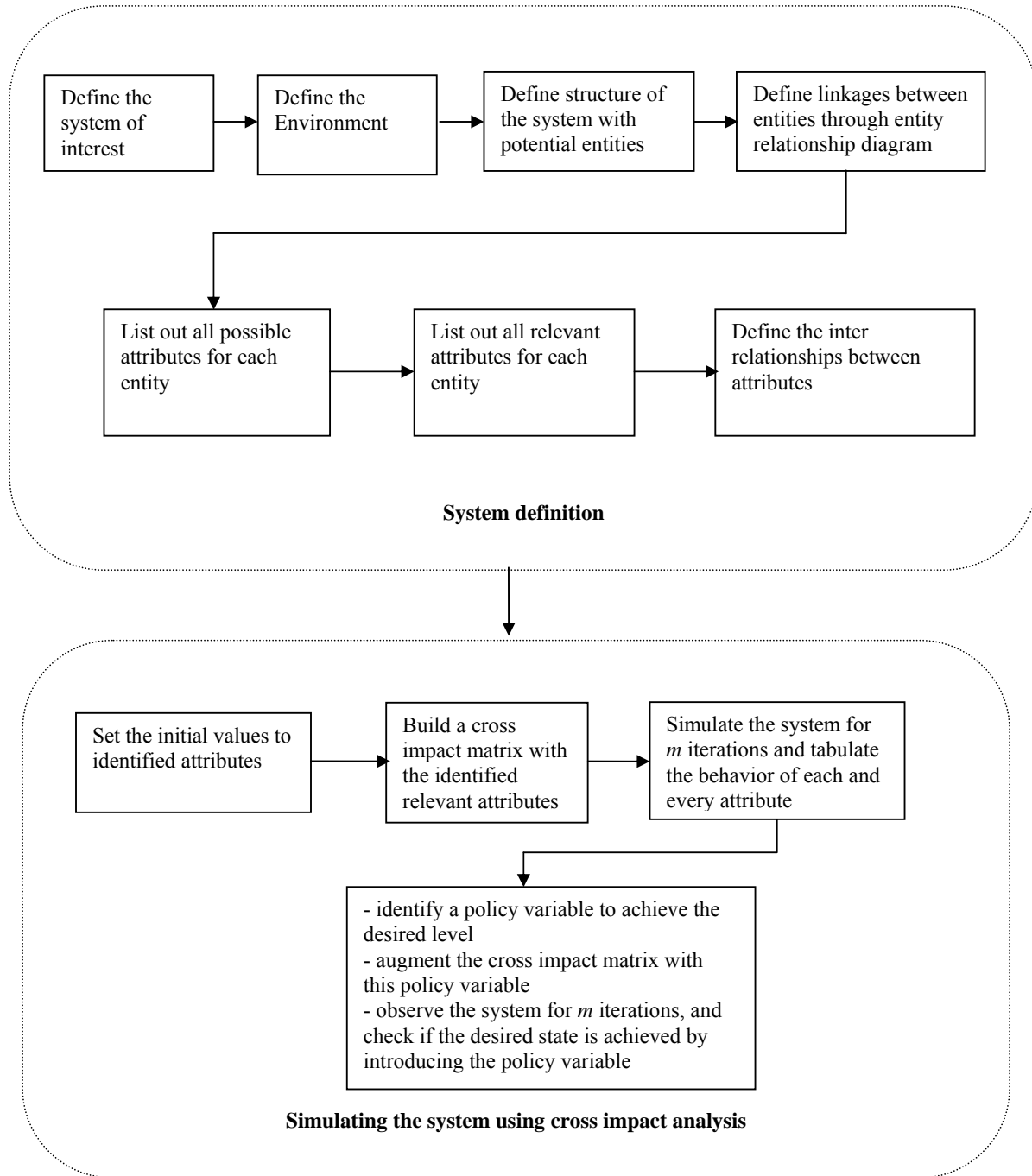


Fig. 1: Block diagram for the steps of the CIA method.

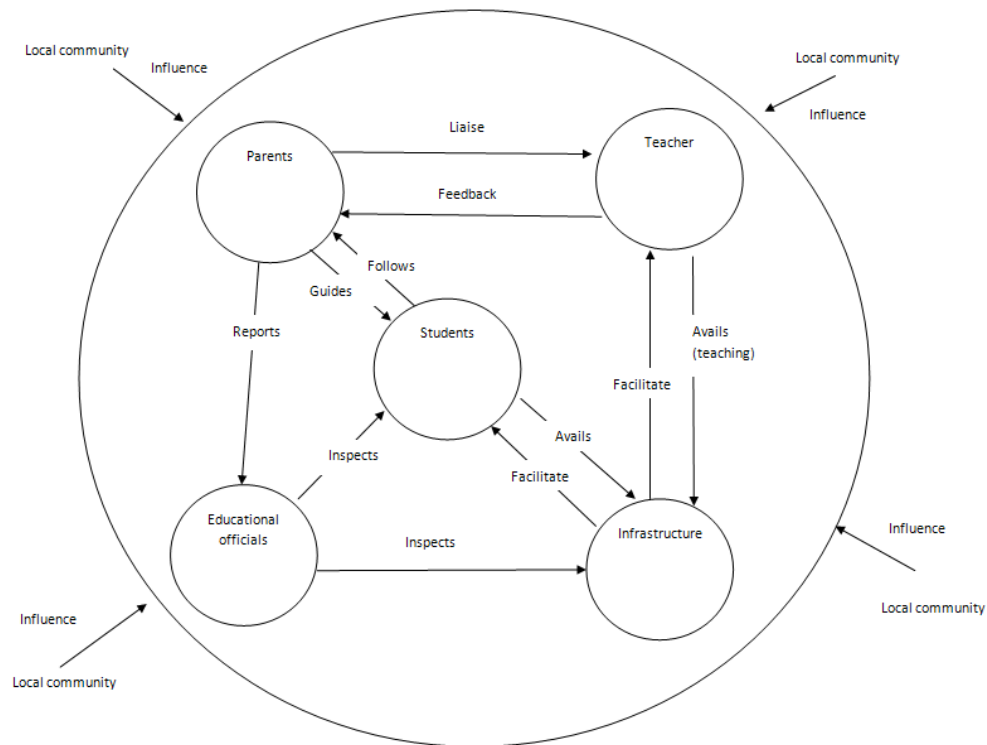


Fig. 2: Entity relationship diagram for the primary education system.

## 2.2. System entities and relationships equations

The dynamic change of the system state is referred to as *system behavior*. The state of a system is an instantaneous snapshot of levels (or, amounts) of the relevant attributes (or, characteristics) possessed by the entities that constitute the system. In all systems, every entity possesses many attributes, but only a few attributes are ‘relevant’ with respect to the problem at hand. Some attributes are of immediate or short-term relevance while others may be of relevance in the long run. The choice of relevant attributes has to be made carefully, keeping in mind both the short-term and long-term consequences of solutions (decisions). All attributes can be associated with given levels that may indicate quantitative or qualitative possession. The set of attributes identified for the model are given below.

### **Entity 1:** *Student:*

- 1.1 Level of Enrollment (*loe*).
- 1.2 Level of boys dropouts in a school (*lbd*).
- 1.3 Level of girls dropouts in a school (*lgd*).
- 1.4 Level of repeaters in a school (*lr*).

### **Entity 2:** *Teacher:*

- 2.1 Level of perceived quality of teaching by the Students (*lts*).
- 2.2 Level of perceived quality of teaching by the Parents (*ltp*).

### **Entity 3:** *Parents:*

- 3.1 Educational level of parents (*elp*).
- 3.2 Income level of parents (*ilp*).
- 3.3 Level of expectations from school by the parents (*leps*).

**Entity 4: Educational officials:**

4.1 Level of perceived quality of teaching by the District educational officer (DEO) (*ltd*).

**Entity 5: Infrastructure:**

5.1 Level of Space and ventilation available in a Classroom (*lsv*).

5.2 Level of cleanliness and other facilities such as board, mats, table/chair, educational aids (maps, toys, charts, etc.) (*lc*).

5.3 Level of sanitation facilities for general purpose (for both boys and girls) (*ls\_g*).

5.4 Level of separate sanitation facilities for girls (*ls\_s*).

5.5 Level of drinking water facility available (*ldw*).

5.6 Level of availability of Playground area and other equipment for children used in playing (*lpa*).

5.7 Level of bad organisation in the classrooms (*lbo*):

a. Number of cases in which more than one class is conducted in a single instructional classroom.

b. Number of cases in which more than 40 people are accommodating in a single instructional classroom.

**Entity 6: Local community:**

6.1 Level of participation of local community (*llc*).

6.2 Level of awareness of local community about educational benefits (*lale*).

When entities interact through their attributes, the levels of the attributes might change, i.e., the system behaves in certain directions. Some changes in attribute levels may be desirable while others may not be so. Each attribute influences several others, thus creating a web of complex interactions which eventually determine system behavior. In other terms, attributes are variables that vary from time to time. They can vary in the system in an unsupervised way. However, variables can be controlled directly or indirectly, and partially by introducing new intervention policies. The interrelationships among variables should be analyzed carefully before introducing new policies.

The following **conjectures** are valid in the systems approach (the following is motivated by Julius (2002)).

**a. Modeling and forecasting the behavior of complex systems are necessary if we are to exert some degree of control over them.**

**b. Properties of variables and interactions in large scale system variables are bounded such that:**

*i. System variables are bounded.* It is now widely recognized that any variable of human significance cannot increase indefinitely. There must be distinct limits. In an appropriate set of units these can always be set to a value between one and zero:

$$0 \leq x_i(t) \leq 1, \text{ for all } i = 1, 2, \dots, N \text{ and all } t \geq 0, \text{ where } x_i(t) \text{ is the level of variable } i \text{ in period } t.$$

*ii. A variable increases or decreases according to whether the net impact of the other variables is positive or negative.*

To preserve boundedness,  $x_i(t + \Delta t)$  is calculated by the transformation

$$x_i(t + \Delta t) = x_i(t)^{P_i},$$

where the exponent  $P_i(t)$  is given by

$$P_i(t) = \frac{1 + \Delta t | \text{sum of negative impacts on } x_i |}{1 + \Delta t | \text{sum of positive impacts on } x_i |}.$$

*iii. A variables' response to a given impact decreases to zero as that variable approaches its upper or lower bound. It is generally found that bounded growth and decay processes exhibit this sigmoidal character.*

*iv. All other things being kept fixed (constant), a variable (attribute) will produce a greater impact on the system as it grows larger (*ceteris paribus*).*

*v. Complex interactions are described by a looped network of binary interactions (this is the basis of the *cross impact analysis*).*

### 3. Simulating the system using cross impact analysis

There are four steps to follow while implementing the cross impact analysis in our case. First, we conduct the simulation by considering the primary education system without human intervention. Then, we run the same analysis after implementing some selected policy variables such as infrastructure improvement and observe the change in system dynamics.

We now describe how we construct the model in the following four steps.

**Step 1.** Set the initial values for attributes. The initial values are obtained from published sources and surveys conducted. Here, we use the survey data reported in Pedamallu (2001). Table 1 illustrates the initial values for various attributes identified in this study.

Table 1. Initial values for attributes

Attribute	Initial value
Level of Enrollment ( <i>loe</i> )	0.71
Level of Space and ventilation available in a Classroom ( <i>lsv</i> )	0.5
Level of cleanliness and other facilities such as board, mats, table/chair, educational aids (maps, toys, charts, etc.) ( <i>lc</i> )	0.5
Educational level of parents ( <i>elp</i> )	0.35
Income level of parents ( <i>ilp</i> )	0.35
Level of expectations from school by the parents ( <i>leps</i> )	0.6
Level of perceived quality of teaching by the Students ( <i>lts</i> )	0.45
Level of perceived quality of teaching by the Parents ( <i>ltp</i> )	0.35
Level of perceived quality of teaching by the District educational officer (DEO) ( <i>ltd</i> )	0.35
Level of sanitation facilities for general purpose (for both boys and girls) ( <i>ls_g</i> )	0.39
Level of separate sanitation facilities for girls ( <i>ls_s</i> )	0.28
Level of availability of Playground area and other equipment for children used in playing ( <i>lpa</i> )	0.3
Level of participation of local community ( <i>llc</i> )	0.25
Level of awareness of local community about educational benefits ( <i>lale</i> )	0.25
Level of repeaters in a school ( <i>lr</i> )	0.05
Level of boys dropouts in a school ( <i>lbd</i> )	0.2
Level of girls dropouts in a school ( <i>lgd</i> )	0.29
Level of bad organisation in the classrooms ( <i>lbo</i> )	0.69
Level of drinking water facility available ( <i>ldw</i> )	0.34

**Step 2.** Build a cross impact matrix with the identified relevant attributes. Summing the effects of column attributes on rows shows the effect of each attribute in the matrix. The parameters  $\alpha_{ij}$  can be determined by creating a pairwise correlation matrix after collecting the data, and these can be adjusted by subjective assessment. In Table 2, qualitative impacts are quantified subjectively. The impact of infrastructural facilities on primary school enrollments and progression become visible by running the simulation model. A cross-impact matrix for the attributes listed above is illustrated in Table 3.

**Step 3.** Simulate the system for a number of 50 iterations ( $m$  iterations) and tabulate the behavior of each and every attribute in each every iteration. Plot the results on a worksheet.

We apply Step 3 and illustrate, in Figure 3, the simulation of the system for 50 iterations without any policy related variables. It is observed that there is sharp increase in enrollment rate at the beginning phase of the simulation (i.e., for the first 12 iterations). However, there is a steady decrease in the enrollment rate after a certain period of time. The trend is observed in the number of dropouts and repeaters. In order to observe the

effect of infrastructure attributes, we include them as policy variables in our next step. The policy variable that is selected involves additional investment in the infrastructure related attributes and elements which we call it as “policy variable”.

**Step 4.** Identify a policy variable to achieve the desired level or state and augment the cross impact matrix with this policy variable with the qualitative assessment of pairwise attribute interactions. Re-simulate the model.

Table 2. Impact rates of variables (attributes).

Representation of Impact	Value	Description
++++	0.8	Very strong positive effect
+++	0.6	Strong positive effect
++	0.4	Moderate positive effect
+	0.2	Mild positive effect
0	0	Neutral
-	-0.2	Mild negative effect
--	-0.4	Moderate negative effect
---	-0.6	Strong negative effect
----	-0.8	Very strong negative effect

Table 3. Cross impact matrix for primary education system.

	loe	lsv	lc	eip	ilp	leps	lts	ltp	lid	ls_g	ls_s	lpa	llc	lale	lr	lbd	lgd	lbo	ldw	
loe	0	+++	+++	+++	+++	+++	++	++	0	+++	+++	+++	0	0	0	0	0	0	0	+++
lsv	---	0	0	0	0	0	0	0	0	0	0	0	+	+	0	0	0	0	-	0
lc	---	0	0	0	0	0	0	0	+++	0	0	0	+	+	0	0	0	0	-	0
eip	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ilp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
leps	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lts	0	++	++	0	0	0	0	0	+	0	0	+	0	0	0	0	0	0	---	0
ltp	0	0	0	0	0	0	0	0	+	0	0	+	0	0	0	0	0	0	---	0
lid	0	0	0	0	0	0	+	+	0	0	0	0	0	0	0	0	0	0	---	0
ls_g	---	0	0	0	0	0	0	0	+++	0	0	0	0	+	0	0	0	0	0	0
ls_s	---	0	0	0	0	0	0	0	+++	0	0	0	+	+	0	0	0	0	0	0
lpa	---	0	0	0	0	0	0	0	0	0	0	0	+	+	0	0	0	0	0	0
llc	0	0	0	0	0	0	0	0	0	0	0	0	+	+	0	0	0	0	0	0
lale	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lr	+	0	0	0	0	0	---	---	0	0	0	0	+	0	0	0	0	0	+++	0
lbd	0	---	---	---	---	---	-	-	0	---	0	---	0	-	++++	0	0	0	++++	-
lgd	0	---	---	---	---	---	-	-	0	---	---	---	0	-	++++	0	0	0	++++	-
lbo	++++	0	0	0	0	0	0	0	---	0	0	0	0	0	0	0	0	0	0	0
ldw	---	0	0	0	0	0	0	0	0	0	0	0	+	+	0	0	0	0	0	0

In this re-simulation run, we select an improvement in infrastructural facilities as the policy variable. In Table 4, we include the relationship of the policy variable to other attributes. We observe the system for 50 iterations, and check if the desired state is achieved by introducing the policy variable. We then compare the results obtained in the two simulation runs. The detailed rates of change in all variables during the two simulation runs taken before and after adding the policy variable are indicated in the Appendix.

Figure 4 illustrates the results of the simulated system after adding the identified policy variable in Step 4. Here, it is observed that the policy variable is effective on improving the enrollment and dropout and repeater rates.

### 5. Conclusion

A cross-impact model is developed here to study the influence of infrastructure facilities on primary education enrollment and progression. The cross-impact matrix illustrates the influence of one variable over the others and it also has a provision to identify the impact variables (i.e., policy variables). Here, we construct a model based on primary education data obtained in a survey conducted in Gujarat, India. Simulation results show that infrastructure improvement would indeed increase the enrollment rate in primary education.

Table 4. Cross impact matrix for primary education system after adding policy variable.

	loe	lsv	lc	elp	ilp	leps	lts	ltp	ltd	ls_g	ls_s	lpa	llc	lale	lr	lbd	lgd	lbo	ldw	policy
loe	0	+++	+++	+++	+++	+++	++	++	0	+++	+++	+++	0	0	0	0	0	0	0	0
lsv	---	0	0	0	0	0	0	0	+++	0	0	0	+	+	0	0	0	-	0	+++
lc	---	0	0	0	0	0	0	0	+++	0	0	0	+	+	0	0	0	-	0	+++
elp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ilp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
leps	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lts	0	++	++	0	0	0	0	0	+	0	0	+	0	0	0	0	0	---	0	0
ltp	0	0	0	0	0	0	0	0	+	0	0	+	0	0	0	0	0	---	0	0
ltd	0	0	0	0	0	0	+	+	0	0	0	0	0	0	0	0	0	---	0	0
ls_g	---	0	0	0	0	0	0	0	+++	0	0	0	0	+	0	0	0	0	0	+++
ls_s	---	0	0	0	0	0	0	0	+++	0	0	0	+	+	0	0	0	0	0	+++
lpa	---	0	0	0	0	0	0	0	0	0	0	0	+	+	0	0	0	0	0	+++
llc	0	0	0	0	0	0	0	0	0	0	0	0	+	+	0	0	0	0	0	0
lale	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lr	+	0	0	0	0	0	---	---	0	0	0	0	+	0	0	0	0	+++	0	0
lbd	0	---	---	---	---	---	-	-	0	---	---	---	0	-	++++	0	0	++++	-	0
lgd	0	---	---	---	---	---	-	-	0	---	---	---	0	-	++++	0	0	++++	-	0
lbo	++++	0	0	0	0	0	0	0	---	0	0	0	0	0	0	0	0	0	0	---
ldw	---	0	0	0	0	0	0	0	0	0	0	0	+	+	0	0	0	0	0	+++

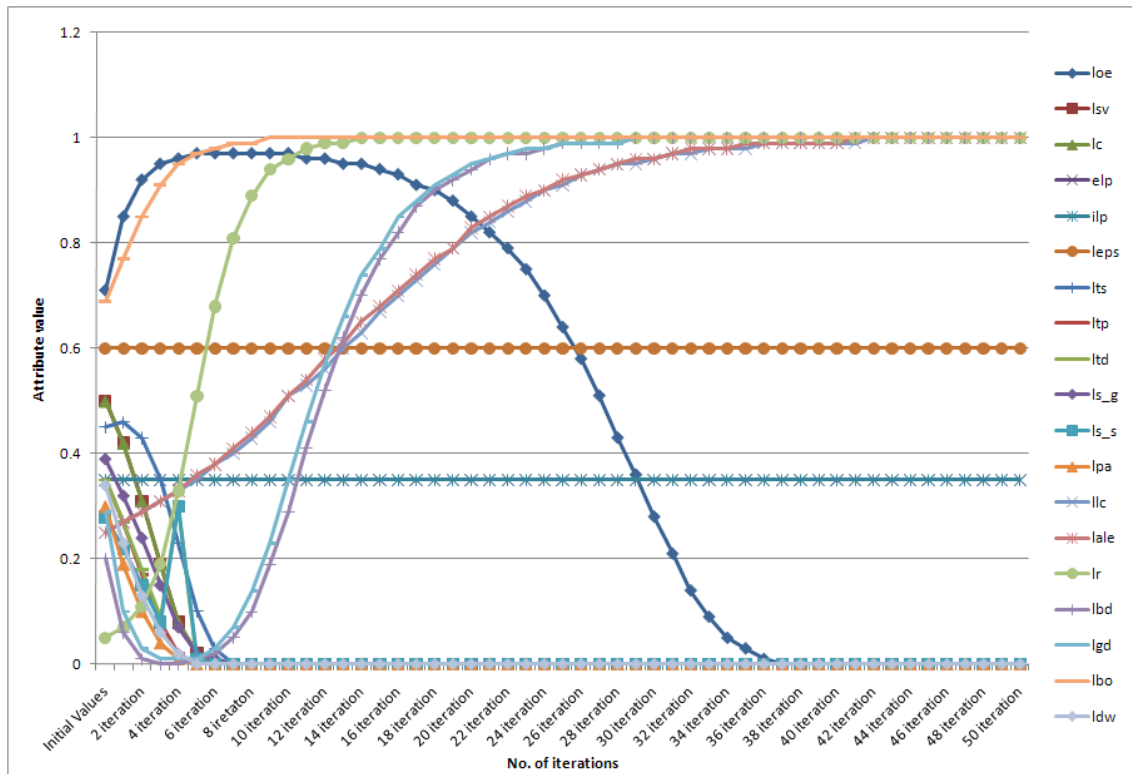


Fig. 3: Behavior of primary educational system before adding the policy variable.



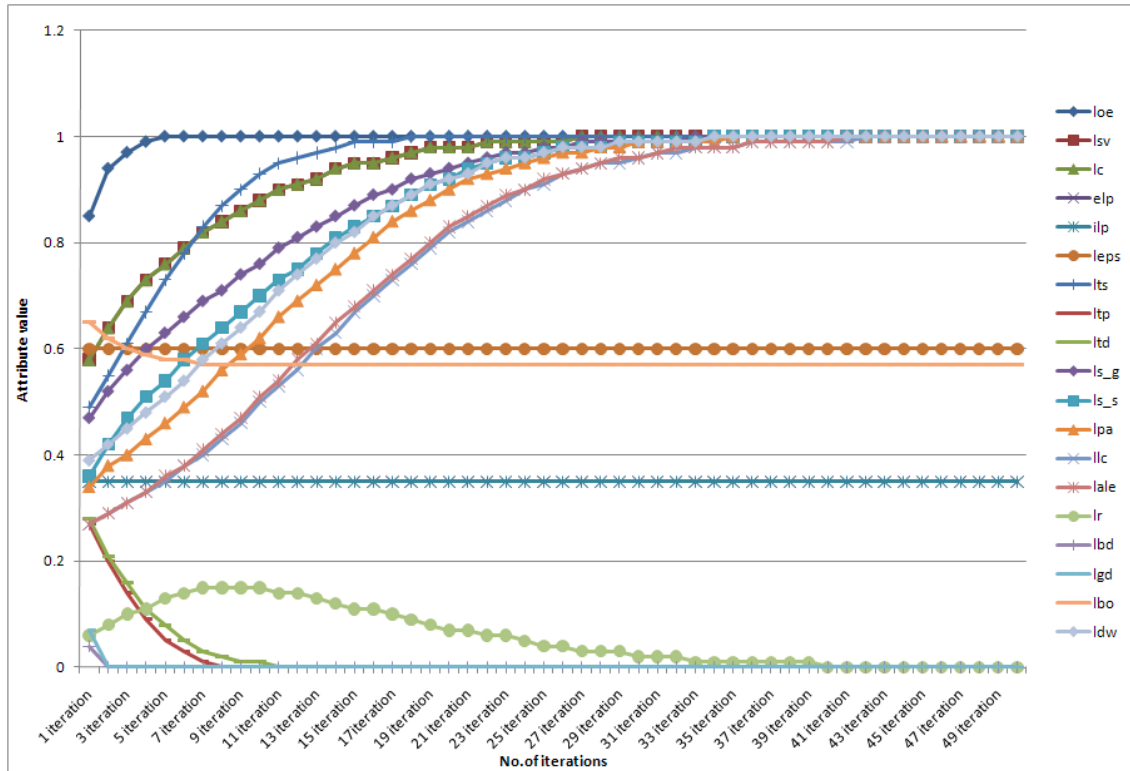


Fig. 4: Behavior of primary educational system after adding the policy variable.

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## Appendix

### 1. Simulation results for attributes before adding the policy variable

Attributes	Initial Values	1 iteration	2 iteration	3 iteration	4 iteration	5 iteration	6 iteration	7 iteration	8 iteration	9 iteration	10 iteration	11 iteration	12 iteration	13 iteration	14 iteration
loe	0.71	0.85	0.92	0.95	0.96	0.97	0.97	0.97	0.97	0.97	0.97	0.96	0.96	0.95	0.95
lsv	0.5	0.42	0.31	0.19	0.08	0.02	0	0	0	0	0	0	0	0	0
lc	0.5	0.42	0.31	0.19	0.08	0.02	0	0	0	0	0	0	0	0	0
elp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
ilp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
leps	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
lts	0.45	0.46	0.43	0.35	0.23	0.1	0.03	0	0	0	0	0	0	0	0
ltp	0.35	0.27	0.17	0.08	0.02	0	0	0	0	0	0	0	0	0	0
ltd	0.35	0.27	0.18	0.09	0.3	0	0	0	0	0	0	0	0	0	0
ls_g	0.39	0.32	0.24	0.15	0.07	0.02	0.01	0	0	0	0	0	0	0	0
ls_s	0.28	0.22	0.15	0.08	0.3	0.01	0	0	0	0	0	0	0	0	0
lpa	0.3	0.19	0.1	0.04	0.01	0	0	0	0	0	0	0	0	0	0
llc	0.25	0.27	0.29	0.31	0.33	0.35	0.38	0.4	0.43	0.46	0.51	0.53	0.56	0.6	0.63
lale	0.25	0.27	0.29	0.31	0.33	0.36	0.38	0.41	0.44	0.47	0.51	0.54	0.58	0.61	0.65
lr	0.05	0.07	0.11	0.19	0.33	0.51	0.68	0.81	0.89	0.94	0.96	0.98	0.99	0.99	1
lbd	0.2	0.06	0.01	0	0	0.01	0.02	0.05	0.1	0.19	0.29	0.41	0.52	0.62	0.7
lgd	0.29	0.1	0.03	0.01	0.01	0.01	0.03	0.07	0.14	0.23	0.35	0.46	0.57	0.66	0.74
lbo	0.69	0.77	0.85	0.91	0.95	0.97	0.98	0.99	0.99	1	1	1	1	1	1
ldw	0.34	0.23	0.13	0.06	0.02	0	0	0	0	0	0	0	0	0	0

Attributes	15 iteration	16 iteration	17 iteration	18 iteration	19 iteration	20 iteration	21 iteration	22 iteration	23 iteration	24 iteration	25 iteration	26 iteration	27 iteration	28 iteration	29 iteration
loe	0.94	0.93	0.91	0.9	0.88	0.85	0.82	0.79	0.75	0.7	0.64	0.58	0.51	0.43	0.36
lsv	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lc	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
elp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
ilp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
leps	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
lts	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ltp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ltd	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ls_g	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ls_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lpa	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
llc	0.67	0.7	0.73	0.76	0.79	0.82	0.84	0.86	0.88	0.9	0.91	0.93	0.94	0.95	0.95
lale	0.68	0.71	0.74	0.77	0.79	0.83	0.85	0.87	0.89	0.9	0.92	0.93	0.94	0.95	0.96
lr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
lbd	0.77	0.82	0.87	0.9	0.92	0.94	0.96	0.97	0.97	0.98	0.99	0.99	0.99	0.99	1
lgd	0.79	0.85	0.88	0.91	0.93	0.95	0.96	0.97	0.98	0.98	0.99	0.99	0.99	0.99	1
lbo	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ldw	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Attributes	30 iteration	31 iteration	32 iteration	33 iteration	34 iteration	35 iteration	36 iteration	37 iteration	38 iteration	39 iteration	40 iteration	41 iteration	42 iteration	43 iteration	44 iteration
loe	0.28	0.21	0.14	0.09	0.05	0.03	0.01	0	0	0	0	0	0	0	0
lsv	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lc	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
elp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
ilp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
leps	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
lts	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ltp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ltd	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ls_g	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ls_s	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lpa	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
llc	0.96	0.97	0.97	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	1	1	1
lale	0.96	0.97	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1	1	1
lr	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
lbd	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
lgd	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
lbo	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ldw	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Attributes	45 iteration	46 iteration	47 iteration	48 iteration	49 iteration	50 iteration
loe	0	0	0	0	0	0
lsv	0	0	0	0	0	0
lc	0	0	0	0	0	0
elp	0.35	0.35	0.35	0.35	0.35	0.35
ilp	0.35	0.35	0.35	0.35	0.35	0.35
leps	0.6	0.6	0.6	0.6	0.6	0.6
lts	0	0	0	0	0	0
ltp	0	0	0	0	0	0
ltd	0	0	0	0	0	0
ls_g	0	0	0	0	0	0
ls_s	0	0	0	0	0	0
lpa	0	0	0	0	0	0
llc	1	1	1	1	1	1
lale	1	1	1	1	1	1
lr	1	1	1	1	1	1
lbd	1	1	1	1	1	1
lgd	1	1	1	1	1	1
lbo	1	1	1	1	1	1
ldw	0	0	0	0	0	0

## 2. Simulation results for attributes after adding the policy variable

Attributes	1 iteration	2 iteration	3 iteration	4 iteration	5 iteration	6 iteration	7 iteration	8 iteration	9 iteration	10 iteration	11 iteration	12 iteration	13 iteration	14 iteration
loe	0.85	0.94	0.97	0.99	1	1	1	1	1	1	1	1	1	1
lsv	0.58	0.64	0.69	0.73	0.76	0.79	0.82	0.84	0.86	0.88	0.9	0.91	0.92	0.94
lc	0.58	0.64	0.69	0.73	0.76	0.79	0.82	0.84	0.86	0.88	0.9	0.91	0.92	0.94
elp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
ilp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
leps	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
lts	0.49	0.55	0.61	0.67	0.73	0.78	0.83	0.87	0.9	0.93	0.95	0.96	0.97	0.98
ltp	0.27	0.2	0.14	0.09	0.05	0.03	0.01	0	0	0	0	0	0	0
ltd	0.28	0.21	0.16	0.11	0.08	0.05	0.03	0.02	0.01	0.01	0	0	0	0
ls_g	0.47	0.52	0.56	0.6	0.63	0.66	0.69	0.71	0.74	0.76	0.79	0.81	0.83	0.85
ls_s	0.36	0.42	0.47	0.51	0.54	0.58	0.61	0.64	0.67	0.7	0.73	0.75	0.78	0.81
lpa	0.34	0.38	0.4	0.43	0.46	0.49	0.52	0.56	0.59	0.62	0.66	0.69	0.72	0.75
lpc	0.27	0.29	0.31	0.33	0.35	0.38	0.4	0.43	0.46	0.5	0.53	0.56	0.6	0.63
lale	0.27	0.29	0.31	0.33	0.36	0.38	0.41	0.44	0.47	0.51	0.54	0.58	0.61	0.65
lr	0.06	0.08	0.1	0.11	0.13	0.14	0.15	0.15	0.15	0.15	0.14	0.14	0.13	0.12
lbd	0.04	0	0	0	0	0	0	0	0	0	0	0	0	0
ljd	0.07	0	0	0	0	0	0	0	0	0	0	0	0	0
lbo	0.65	0.62	0.6	0.59	0.58	0.58	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57
ldw	0.39	0.42	0.45	0.48	0.51	0.54	0.58	0.61	0.64	0.67	0.71	0.74	0.77	0.8

Attributes	15 iteration	16 iteration	17 iteration	18 iteration	19 iteration	20 iteration	21 iteration	22 iteration	23 iteration	24 iteration	25 iteration	26 iteration	27 iteration	28 iteration	29 iteration
loe	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
lsv	0.95	0.95	0.96	0.97	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	1	1
lc	0.95	0.95	0.96	0.97	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	1	1
elp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
ilp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
leps	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
lts	0.99	0.99	0.99	1	1	1	1	1	1	1	1	1	1	1	1
ltp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ltd	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ls_g	0.87	0.89	0.9	0.92	0.93	0.94	0.95	0.96	0.97	0.97	0.98	0.98	0.99	0.99	0.99
ls_e	0.83	0.86	0.87	0.89	0.91	0.92	0.94	0.95	0.96	0.96	0.97	0.98	0.98	0.98	0.99
lpa	0.78	0.81	0.84	0.86	0.88	0.9	0.92	0.93	0.94	0.95	0.96	0.97	0.97	0.98	0.98
lpc	0.67	0.7	0.73	0.76	0.79	0.82	0.84	0.86	0.88	0.9	0.91	0.93	0.94	0.95	0.96
lale	0.68	0.71	0.74	0.77	0.8	0.83	0.85	0.87	0.89	0.9	0.92	0.93	0.94	0.95	0.96
lr	0.11	0.11	0.1	0.09	0.08	0.07	0.07	0.06	0.06	0.05	0.04	0.04	0.03	0.03	0.03
lbd	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ljd	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lbo	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57
ldw	0.82	0.85	0.87	0.89	0.91	0.92	0.93	0.95	0.96	0.96	0.97	0.98	0.98	0.98	0.99

Attributes	30 iteration	31 iteration	32 iteration	33 iteration	34 iteration	35 iteration	36 iteration	37 iteration	38 iteration	39 iteration	40 iteration	41 iteration	42 iteration	43 iteration	44 iteration
loe	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
lsv	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
lc	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
elp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
ilp	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
leps	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
lts	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
ltp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ltd	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ls_g	0.99	0.99	0.99	1	1	1	1	1	1	1	1	1	1	1	1
ls_s	0.99	0.99	0.99	0.99	1	1	1	1	1	1	1	1	1	1	1
lpa	0.99	0.99	0.99	0.99	1	1	1	1	1	1	1	1	1	1	1
lpc	0.96	0.97	0.97	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	1	1	1
lale	0.96	0.97	0.98	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	1	1	1	1
lr	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0
lbd	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ljd	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lbo	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57
ldw	0.99	0.99	0.99	0.99	1	1	1	1	1	1	1	1	1	1	1

Attributes	45 iteration	46 iteration	47 iteration	48 iteration	49 iteration	50 iteration
loe	1	1	1	1	1	1
lsv	1	1	1	1	1	1
lc	1	1	1	1	1	1
elp	0.35	0.35	0.35	0.35	0.35	0.35
ilp	0.35	0.35	0.35	0.35	0.35	0.35
leps	0.6	0.6	0.6	0.6	0.6	0.6
lts	1	1	1	1	1	1
ltp	0	0	0	0	0	0
ltd	0	0	0	0	0	0
ls_g	1	1	1	1	1	1
ls_s	1	1	1	1	1	1
lpa	1	1	1	1	1	1
lpc	1	1	1	1	1	1
lale	1	1	1	1	1	1
lr	0	0	0	0	0	0
lbd	0	0	0	0	0	0
ljd	0	0	0	0	0	0
lbo	0.57	0.57	0.57	0.57	0.57	0.57
ldw	1	1	1	1	1	1