MODEL-GENERALIZATION OF BUILDING OUTLINES BASED ON SCALE-SPACES AND SCALE-SPACE EVENTS

Helmut Mayer

Chair for Photogrammetry and Remote Sensing
Technische Universität München, D-80290 Munich, Germany

E-mail: helmut@photo.verm.tu-muenchen.de URL: http://www.photo.verm.tu-muenchen.de

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ABSTRACT

Generalization is not only mandatory to make maps more readable, or to make the visualization of larger areas computationally feasible. It is also needed in the form of model-generalization, to transform data into the right level of abstraction for a particular analysis task. In this paper the formally well-defined theory of scale-spaces is introduced as a basis for model-generalization. More specifically, an approach is proposed which simplifies building outlines employing vector-based morphology and discrete curvature space. So-called scale-space events, for which the link to the semantics of objects has been shown, play a major role for the simplification. The proposed approach renders it possible to preserve and even enforce right angles, and makes the link to iterative cartographic displacement easy. Results show the validity of the approach.

1 INTRODUCTION

Generalization is not only mandatory to make maps more readable, or to make the visualization of larger areas computationally feasible, but it is also needed in the form of model-generalization, to transform data into the right level of abstraction for a particular analysis task, e.g., in a geographic information system (GIS). This paper deals with the model-generalization of building outlines based on the formally well-defined theory of scale-spaces, and particularly on so-called "scale-space events", for which the link to the semantics of objects has been shown in (Mayer, 1996).

Mathematical morphology and the so-called "discrete curvature space" are the main scale-spaces used in this paper. They are employed on vector data of the building outline. Vector data does not only render it possible to preserve right angles, but it also gives means to enforce them. In this paper the scale-spaces are incrementally applied, i.e., the vector outline of the building is shifted in small steps. An important reason for an incremental processing is the occurrence of scale-space events. They occur, when parts of the building like a porch are eliminated, when a building is split into parts, when an inner courtyard arises, or when two buildings merge. The events are classified into internal and external events. To handle the events, simple geometrical rules can be defined.

Sections 2 and 3 give an overview over the state of the art of model-generalization and scale-space theory, respectively. Section 4 introduces the link of scale-space events in an image or a geometry based representation to the abstraction in a symbolical representation. Section 5 presents the main body of the paper, namely the definition of the internal and external events as well as of the discrete curvature space. For the different scale-spaces and particularly for their combination, results which have been compared with another approach show the validity of the proposed approach. In Section 6 conclusions are given.

2 MODEL-GENERALIZATION

Model-generalization is part of the so-called "object-generalization" and aims at a simplification of the geometry, topology, and semantics of objects, but not of their graphical representation. This results in a modeling of geographical information on different levels of abstraction. Ideally, this renders it feasible that data are analyzed on the level of abstraction, where their variance is maximum, or where the spatial processes are understood best (Müller et al., 1995b). While model-generalization is sufficient for subsequent analysis, e.g., in a GIS, for output on media in most cases a cartographic generalization is needed. It deals with problems arising when objects are to be visualized based on signatures of a certain size, or originating from constraints, like, e.g., minimum distances. As the basic problem is model-generalization, it should be developed with highest priority (Grünreich, 1995b). Basically, the research on both types of generalization has a strong tendency to use object-oriented structures, i.e., declarative knowledge, and rules, i.e., procedural knowledge (Grünreich, 1995a, Müller et al., 1995a).

The state of the art of automated generalization, as presented in (Mackaness et al., 1997), comprises also commercial systems, but they are far from solving the problems. A very important point for research in this area is seen to be the acquisition and encoding of expert knowledge. To model the semantics, the purpose of the generalization and especially a close interaction with geometric, i.e., spatial structure, are of major importance. The modeling of these structures can be done raster- or vector-based, making different properties of space explicit. Apart from expert knowledge, semantics, and geometry, quality control is seen to play a critical role. It can not only be used to measure the quality of the product, indicating good strategies for generalization, but it can also give information needed to control the different steps of the generalization. During generalization often conflicts, mostly of geometric nature, arise. For constraint satisfaction cartographic displacement techniques are used.

The main topic of this paper, **automatic generalization of buildings**, has a long tradition. A rather old, but important approach is (Staufenbiel, 1973). Besides more conceptual ideas, it deals with building outlines for which a fixed processing scheme is proposed based on techniques like the intersection of straight lines. The merging of buildings is made dependent on the change of the position of a building and the treatment of the corresponding region. One step further is taken in (Meyer, 1989). It does not longer only focus on the simplification of the building, but the building is classified if possible. Classes are besides the rectangle, e.g., L-, T-, U-, and Z-shaped regions. The classification is based on correlation of the rasterized buildings with a template. If a classification is not possible, a simplification according to (Staufenbiel, 1973) is employed. (Powitz, 1993) extends and combines (Staufenbiel, 1973, Meyer, 1989) with approaches for the generalization of roads and for cartographic displacement. The latter one is done iteratively. Besides this, the main focus lies on quality control, i.e., the determination and analysis of geometric changes of the generalization.

(Li, 1996) deals with **scale-spaces**, particularly with binary morphology (Serra, 1982) on raster data. Opening and closing based on simple structuring elements are employed for the generalization of regions with smooth outlines. How to treat "changes of the type of geometry" when changing scale (scale-space event) is shown in (Schoppmeyer and Heisser, 1995). An example for such a change is when a region becomes a line or a point. The new geometry has to be delineated and a treatment of the neighborhood might be necessary. The fact that objects can appear and disappear when zooming in or out, i.e., when the scale is changed, is analyzed conceptually in (Timpf and Frank, 1997).

Cartographic displacement is not dealt with in this paper, but it is a very important process, which must accompany the simplification of a building. While (Jäger, 1991) proposes a "displacement mountain range", (Bobrich, 1996) recommends a physical approach based on springs. The approach closest to the scale-space based approach proposed in this paper is (Burghardt and Meier, 1997), which uses "snakes" (Kass et al., 1987) and is inherently incremental.

As **important points for further research on generalization**, relevant in the context of this paper, (Mackaness et al., 1997) lists

- the identification and development of "missing" operators for generalization and
- the development of methods for the recognition of structures.

It should be noted, that results of human, i.e., of **manual generalization**, can show large differences, which implies a strong subjectivity. This is apparent for the results of an OEEPE test (Spiess, 1995), which show among other things that generalization is an art. This leads at least to a partial answer to the question of "the right interpretation": The expert knowledge fed into the computer should lead to a result, which complies with an average demand accepted by many people.

3 SCALE-SPACES

Scale-space theory from image analysis deals with the formal definition of the concept "scale" (Koenderink, 1984, Lindeberg, 1994). The basic idea is to generate a multi-scale-representation from a one parameter family of derived signals. Data are systematically simplified and details, i.e., information with high "frequency" are eliminated. The scale parameter $\sigma \in R_+$ is intended to describe the current level of scale. Its semantics is in addition dependent on the definition of the particular scale-space.

If isotropy, i.e., independence of direction, and homogeneity, i.e., independence of location, are combined with causality and a continuous scale parameter, it can be shown (Koenderink, 1984), that the scale-space family necessarily satisfies the so-called "diffusion equation". The convolution with the Gaussian Kernel $g : \mathbf{R^2} \times \mathbf{R}_+ \setminus \{0\} \to \mathbf{R}$

$$g_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

is the solution of the diffusion equation for an infinite domain. This leads to the so-called **linear scale-space**.

Another way to define a scale-space is **mathematical morphology** (Serra, 1982). Examples how to generate a morphological scale-space for gray-scale imagery are given in (van den Boomgard and Smeulders, 1994, Bangham et al., 1996). Erosion and dilation, or rather opening and closing are the basic operations of morphological scale-space:

$$\begin{array}{lrcl} \text{Erosion} & (f \ominus g)(\vec{x}) & = & \inf_{\vec{x}' \in G}(f(\vec{x}+\vec{x}') - g(\vec{x}')) \\ \text{Dilation} & (f \oplus g)(\vec{x}) & = & \sup_{\vec{x}' \in G}(f(\vec{x}-\vec{x}') + g(\vec{x}')) \\ \text{Opening} & (f \circ g)(\vec{x}) & = & ((f \ominus g) \oplus g)(\vec{x}) \\ \text{Closing} & (f \bullet g)(\vec{x}) & = & ((f \oplus g) \ominus g)(\vec{x}) \end{array}$$

The scale-space is generated by filtering the image with – often circle-shaped – so-called "structuring functions" of different size (Köthe, 1996). In the case of binary images, instead of the structuring function $g(\vec{x})$ the so-called "structuring element" (Haralick and Shapiro, 1992) is used.

The so-called **reaction-diffusion space** (Kimia and Siddiqui, 1993, Kimia et al., 1995, Siddiqui and Kimia, 1996) for gray-scale images combines linear and morphological scale-space. It comprises a diffusion as well as a morphological component and is defined based on closed contours. Smoothing is done by embedding the contours into a surface function. For this, e.g., a signed distance transformation is well suited. Another way is to use the gray-scale image as surface function. This renders the smoothing of gray-scale imagery possible. The reaction-diffusion space is applied to the contours, i.e., the curves with the same gray value. The basic definition for the displacement of the contours is:

$$\left\{ \begin{array}{rcl} \frac{\partial C}{\partial \sigma} & = & (\beta_0 - \beta_1 \kappa)\vec{N} \\ C(s,0) & = & C_0(s), \end{array} \right.$$

where C is the vector of the coordinates of the contour, $\vec{N}$ the outer normal, s the parameterization, σ the time (scale parameter), κ the curvature of the contour, β_0, β_1 arbitrary functions, and C_0 the initial contour. β_1 controls the strength of the curvature dependent smoothing (diffusion). Opposed to this, β_0 controls the strength of the morphological smoothing (reaction), where circles are used as structuring functions. The diffusion part which is termed curvature-flow in (Malladi and Sethian, 1996), and **curvature space** in this paper.

The introduced scale-spaces have **different characteristics**. While the linear scale-space more or less continuously smoothes the image function, the morphological scale space eliminates regions with a too small spatial extent. The reaction-diffusion space integrates the former two scale-spaces. Besides this, its diffusion component has – compared to linear scale-space – the interesting property to preserve elongated regions by taking into account the curvature of the contours of the image function.

4 SCALE-SPACE EVENTS AND ABSTRACTION

For this paper scale-spaces are of special interest **in combination with a symbolical description of objects** for which two components have to be considered: On the one hand, features, e.g., edges, can be tracked over different scales as well as different features in different scales can be linked (Hartmann and Drüe, 1984). Different features are, e.g., lines and edges, where a line in coarse scale can be matched with two bounding edges in fine scale. That this can be useful is, e.g., shown in (Kosslyn, 1994) from a psychological point of view: The behavior of features when scale changes is important for the recognition of objects corresponding to the features. On the other hand, objects are combined according to their spatial proximity or due to similar properties, to generate a more compact representation or to render a simpler analysis possible. When using spatial proximity, this is termed object aggregation, or a part hierarchy. When similar properties are employed, it is termed class hierarchy (Molenaar, 1996). Both hierarchies describe the behavior of objects with a spatial extent dependent on how they can be represented and analyzed. In this, an implicit scale is inherent. It becomes immediately explicit, when the object is visualized, e.g., as a map.

For an integration with symbolical information the so-called **shocks** of the reaction part of the reaction-diffusion space (Kimia et al., 1995, Siddiqui and Kimia, 1996) are of particular interest. They correspond to those points of the grass fire algorithm used for medial-axis transformation, where the fire front-lines collide and extinguish each other. There are four different types of shocks (cf. Tab. 1 and Fig. 1). The significance of shocks lies in the fact, that a basic set of operations for the description of objects is available for them. Based on this, a grammar was proposed (Siddiqui and Kimia, 1996), which allows for the elimination of impossible configurations of shocks and for a meaningful organization of the rest of the shocks. Based on the shocks alone, only a description of possibly occluded, but not of deformed objects is possible. This defect can be remedied by a combination with curvature-space. It especially smoothes away shocks produced by noise.

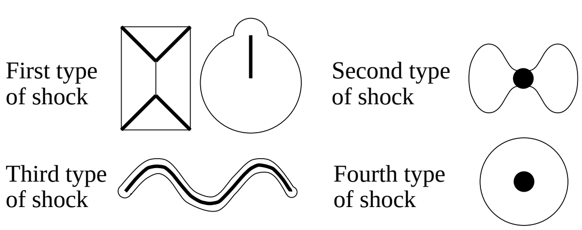


Figure 1: Four types of shocks

The connection of scale-spaces and symbolical information was focused upon in (Mayer, 1996, Mayer and Steger, 1998, Mayer, 1998). There, it was shown that essentially two things happen simultaneously, when an image comprising features representing objects is transformed in scale-space from a finer to a coarser scale:

- The information content of the image is reduced by means of **scale-space events** by eliminating points, edges, lines, and regions: Noise as well as meaningful information is removed thereby.
- The elimination of meaningful information can be considered equivalent to the elimination of parts of objects. What is more, it results in a simplification, i.e., an **abstraction**, and therefore an improved feasibility for object extraction.

With these means the connection of a symbolical representation based on a part hierarchy and a transformation by changing the scale in the image becomes possible. Another finding is that in many cases the geometric representation of an object changes simultaneously with the elimination of parts. While it is a good idea to represent a road in fine scale as a region, it is better to represent it as a line in coarse scale.

Scale-space events can be classified into annihilation, merge, split, and creation. For split and merge Fig. 2 shows an example. “Split” corresponds to the second type of shock (see above). Scale-space events equivalent to the third type of shock can be seen as examples for the change of the type of geometry presented in (Schoppmeyer and Heisser, 1995). Annihilation and creation have a large similarity with the appearing and disappearing of objects in (Timpf and Frank, 1997).

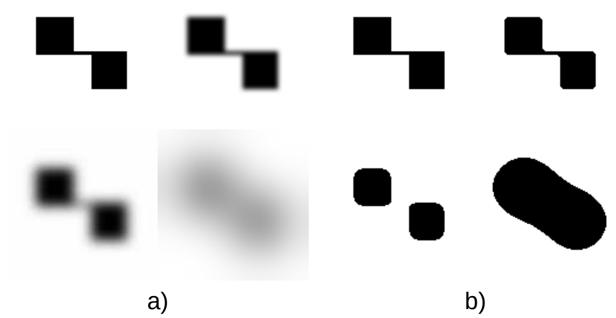


Figure 2: Split and merge while smoothing in linear scale-space — a) Images b) Images after thresholding (from left to right and from top to bottom: Input image, $\sigma = 2$, $\sigma = 5$: Split into two regions, $\sigma = 20$: Merge into one region)

The abstraction capability of scale-spaces and results like, e.g., in Fig. 2 suggest to employ scale-spaces for generalization. That this is basically possible was proven in (Li, 1996) for morphology. At this point it makes sense to ask, how the scale-space might fit into a generalization based on expert knowledge? To answer this question, a basic experience from image analysis can be stressed: Expert knowledge in the form of heuristics is not suited for low-level methods. It is of fundamental importance to employ low-level methods which are mathematically well founded and which have only a few, semantically meaningful parameters. On first sight, this is contradictory to the state of the art in generalization focusing on the use of expert knowledge. But, this only seems to be a conflict: Formal methods are only the well-defined basis on which the expert knowledge is constructed upon.

Type of Shock	Description	Orientation	Curvature
First	Protrusion (Discontinuity in orientation of outline)	non-vanishing $\nabla\phi$	high κ
Second	Neck (spatially separated parts of the outline collide at one point)	isolated vanishing $\nabla\phi$	$\kappa_1\kappa_2 < 0$
Third	Axis of Symmetry (spatially separated points of the outline collide at several points at a time)	non-isolated vanishing $\nabla\phi$	$\kappa_1\kappa_2 = 0$
Fourth	Seed (one object collapses into one point)	isolated vanishing $\nabla\phi$	$\kappa_1\kappa_2 > 0$

Table 1: Four types of shocks (κ_1 and κ_2: Main curvatures of the surface function)

5 MODEL-GENERALIZATION OF BUILDING OUTLINES

Scale-spaces were often employed raster-based and only on smooth contours in image analysis. The rounding of the right angles in Fig. 2 is not suited for the simplification of man-made objects like, e.g., buildings, in terms of a generalization. For this, it is obvious to use a vector-based representation, where non-continuities, particularly right angles, can be represented and enforced easily. The scale-spaces to be utilized should have basic properties like, e.g., causality, which is the case for opening and closing. For vectors the contours are shifted in this case inwards or outwards like for the reaction part of the reaction-diffusion space (cf. left side of Fig. 3).

Again, **scale-space events** occur. In Fig. 3 this is the elimination of the inward pointing notch in the middle of the upper part of the building. The scale-space events are classified into internal and external: *Internal* events refer to topologically local areas and are synonymous with one or more "unnecessary" points or very short segments. They emerge when an incremental displacement like in Fig. 3 is employed. This is the procedure used in the remainder of this paper. The internal events can be further classified into U, Z, and L (cf. Fig. 4), for which simple basic operations can be stated: For U events small inwards or outwards pointing structures are eliminated. Z events are solved by averaging straight lines. For L events two or more short sides are united into one point.

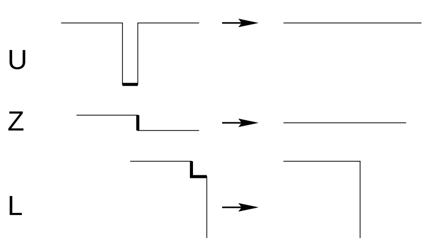


Figure 4: Internal scale-space events

External events emerge when topologically non-local segments of one or more buildings touch or overlap. Typical examples are the split of objects connected by narrow connections in the course of Opening, and the elimination of small entrances when Closing (cf. Fig. 5). Similarly to the latter, the merge of two or more buildings can be treated.

A further scale-space is the **discrete curvature-space**. It is obtained when only segments under a certain minimal length are shifted. This leads to results like in the center of of Fig. 3. The procedure for this is as follows (cf. Fig. 6):

- In the case of one short segment in between two longer segments two things can happen:

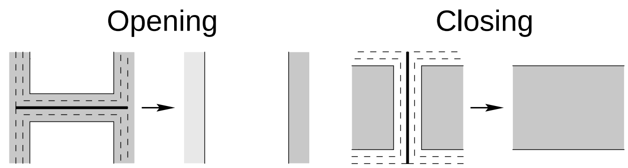


Figure 5: External scale-space events

  - If both angles for the segment are equal, i.e., either 90 or 270, and are forming an inwards or outwards pointing U-shape, the short segment is shifted outwards or inwards, respectively.
  - If both angles are different, i.e., the structure is Z-shaped, the two long segments are shifted in that way that the short segment is getting even shorter.
- For more than one short segment, two segments which are locally L-shaped, are shifted inwards or outwards at a time, depending on the first angle. When the calculation of the shift-values is done in one direction of the building outline, it is guaranteed that also for more than two segments a meaningful result is obtained.

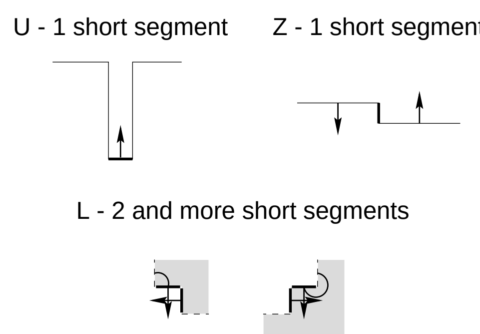


Figure 6: Discrete curvature-space – Procedure

This procedure is similar to curvature-space. If the curvature-radius is defined "regionally" like in Fig. 7 left for the U-shape and Fig. 7 right for the Z-shape, it corresponds to the half length of a segment. The proposed procedure does not shift points more or less depending on the curvature like in curvature-space. Instead, short sides below a certain length, i.e., points with high curvature, are shifted, while other points are not shifted at all. With this, the minimal curvature-radius is eventually enlarged to the half minimal length of the segment. This is the reason why this procedure can be described as *discrete curvature-space*.

Another way to define a scale-space is the **optimization of the compactness** of the object, i.e., the ratio of $perimeter^2$

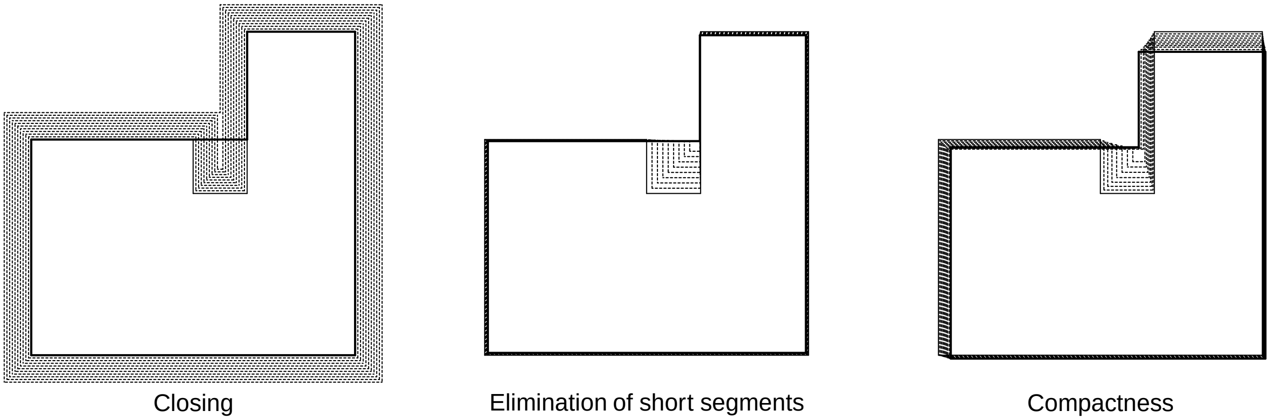


Figure 3: Result of different scale-spaces based on a vector-representation (input = thin line, intermediate steps = dashed line, result = thick line)

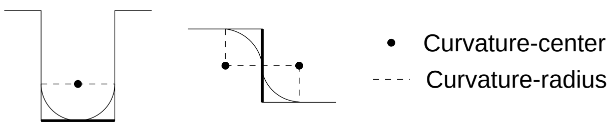


Figure 7: Discrete curvature-space – Curvature radius

/ $regionsize$. This scale-space is similar to linear scale-space. Its optimum is the square for right-angled objects. A result is shown in the right part of Fig. 3.

Based on the incremental processing also a stepwise **rectification** of angles in the interval of [45,135] or an elimination of angles in the intervals]0,45[and]135,180[can be done using the Thales circle, or a straight line, respectively (cf. Fig. 8 for the Thales circle): The points P_{i-1} before and P_{i+1} after the point P_i with the non-right angle are connected ($\overline{P_{i-1}P_{i+1}}$). The center point P_M of $\overline{P_{i-1}P_{i+1}}$ is connected with the point to be shifted and a vector $\overrightarrow{P_M P_i}$ is obtained. P_i is shifted incrementally with a given step-size in the direction $\overrightarrow{P_M P_i}$ in that way, that the distance to P_M approaches $|P_{i-1}P_{i+1}|/2$. If the distance becomes smaller than the step-size, the point is shifted in such a way that $|P_{i-1}P_{i+1}|/2$ is obtained.

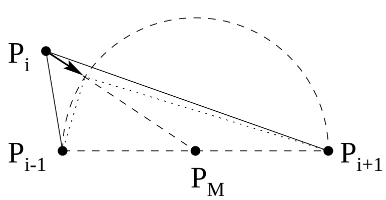


Figure 8: Rectification of angles based on the Thales circle

Results are presented in Fig. 9 and 10 for the examples 10, 12, and 9 from (Staufenbiel, 1973). In Fig. 10 b) it is shown, how non-right angles are incrementally rectified. For these and all the following examples a combination of opening, closing, and discrete curvature-space with always the same parameter values were employed.

Fig. 11 a) presents the external scale-space event that a building with a narrow connection is split into two separate buildings. In Fig. 11 b) an inner courtyard arises from eliminating a small entrance. In both cases the external scale-space events occur in morphological scale-space. In Fig. 11 a) the boundaries of the narrow connection meet during closing, while in Fig. 11 b) the boundaries of the small entrance collide during opening.

Finally, the results have to be **evaluated** in the context of generalization. The results based on morphology and discrete curvature-space are comparable to the results of the approaches presented in Section 2, particularly to that of (Staufenbiel, 1973). Basically, it should be noted that by the selection of the scale-spaces and their parameters the result can be largely varied. What is more, it is very likely that the results comply with the average demand mentioned in Section 2.

A further optimization of the results would be possible by means of techniques for quality control linked with machine learning (Michalski et al., 1984). Because of the huge effort and the experience that meaningful results can be obtained much easier by manual optimization based on visual inspection, machine learning was not utilized.

6 CONCLUSIONS

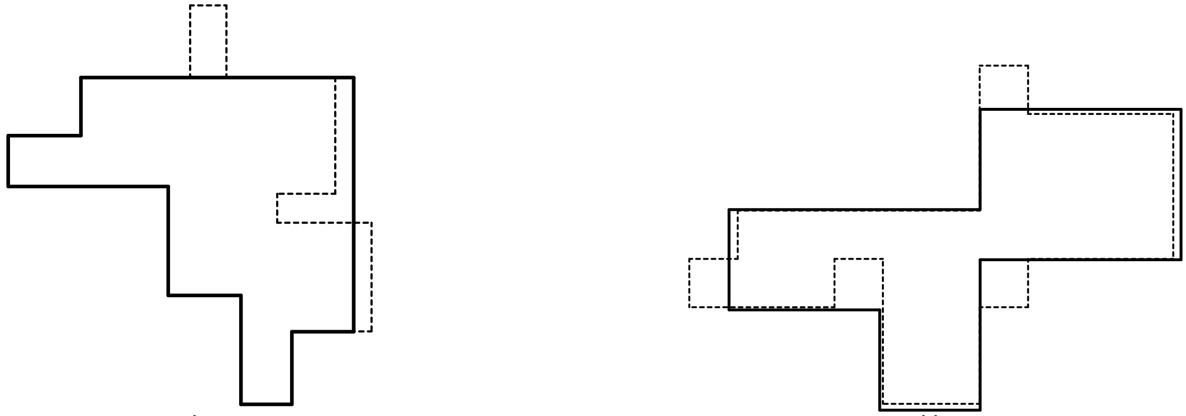
In summary, from the point of view of scale-spaces

- internal and external scale-space events occur.

For generalization,

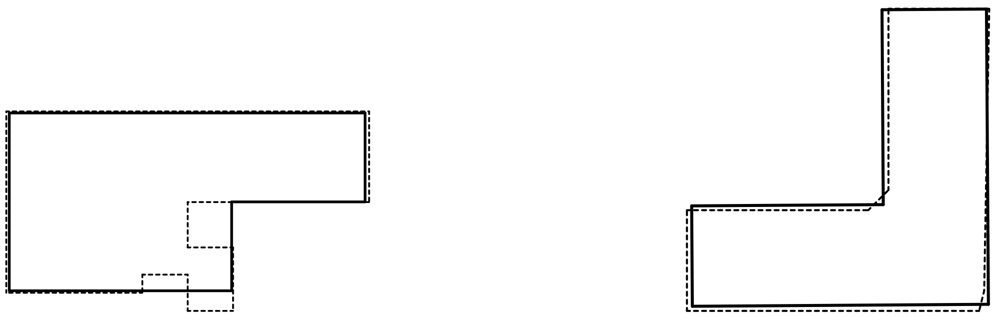
- only relatively few parameters with precisely defined semantics, namely the scale-space and few additional parameters, the step-size, and the number of steps are sufficient.
- By incremental processing
  - the emerging internal and external scale-space events can be handled by simple basic operations,
  - right angles can be enforced, and
  - a link to cartographic displacement is relatively simple.

The last point results from the experiences in (Powitz, 1993), that cartographic displacement should be done in



a) b)

Figure 9: Results of buildings 11 (a)) and 12 (b)) from (Staufenbiel, 1973)



a) b)

Figure 10: a) Result for building 9 from (Staufenbiel, 1973) b) Result for a building with non-right angles

an iterative fashion. Particularly, the proposed approach seems to be well suited for a simple plug-in of techniques for cartographic displacement like the one presented in (Burghardt and Meier, 1997).

What is more, basing generalization onto scale-spaces like it is proposed in this paper contributes to the detection and development of “missing” operators for generalization (Mackaness et al., 1997). Besides, it is an at least implicit step for the development of methods for the recognition of structures: Instead of directly recognizing objects, they are changed into more simple, and therefore more comparable objects.

By and large, the main advantages of the scale-space based model-generalization are that it is based on the sound scale-space theory, and its good interaction with other generalization operations due to the incremental approach. Finally, there is a good chance that these advantages carry over to the 3D case. The final goal to which the project is heading, is the automatic generation of a level of detail (LOD) representation for 3D buildings.

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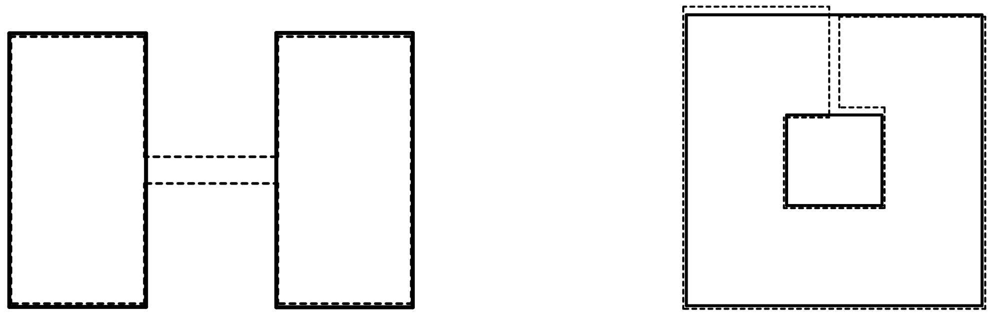

a) b)

Figure 11: a) Narrow connection among buildings ⇒ two buildings b) Small entrance ⇒ Inner courtyard

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