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DIGITAL SIMULATION OF MECHANICAL STEREOPLOTTERS[1]

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ABSTRACT

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Due to design considerations, production tolerances, wear and tear, improrer calibration or maladjustment, aging, external influences, etc., the accuracy and performance state of analog stereo restitution instruments is often not at its optimum. The large number of spatial linkages and optical elements required for a technical reconstruction of the original bundles of rays, gives rise to the accumulation of small systematic errors that are difficult to describe. By making use of the principal flow of information in a stereoplotter, the kinematics of both the spatial linkage mechanisms from the model to the image and the geometric optics between measuring marks and image, however, can be formulated algebraically. The formulation itself can be extended to a computer algorithm which is used as the basis of a digital simulation of the metric state of the stereoplotter.

The paper discusses the principle of the mathematical representation of rotations and translations of spatial linkages and the procedure used in determining the dependent pair variables of a typical stereoplotter mechanism. By a simple ray-tracing method applied to the essential optical elements, the process of photogrammetric static measurements can be simulated. Preliminary test results of the computer simulation program are presented, and its full potential is discussed by artificially introduced instrumental errors. The results are of interest to the efforts of sub-group 2 (Testing of Instruments) of I.S.P.'s Commission II.

INTRODUCTION

Simulation studies can serve mainly three purposes (Evans et al., 1967), viz.: (*1*) the establishment of estimates and guide lines during the preliminary design phase of the development of a system; (*2*) the experimental system design for modifications intended to improve the system; and (*3*) the detailed evaluation of performance of a system under variable conditions.

The purpose of this paper is to show the basic feasibility of a rigorous computer simulation method applied to evaluate the state of performance of a stereo restitution system. The method, therefore, is within the scope of instrument testing. An earlier paper (Dorrer, 1971) discussed the principle of displacement analysis as applied to optical and mechanical linkages in stereoplotters, and tried to draw the attention to computer simulation.

In photogrammetry, computer simulation methods are not entirely new,

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although practical applications have been limited to a few special cases. Two distinctly different utilizations are recognizable:

(*1*) Simulation models for testing and checking the statistical characteristics of certain analytical photogrammetric procedures with the help of computer-generated pseudo random numbers (Doyle, 1966; Wolters, 1968). This technique is known as Monte Carlo simulation (Martin, 1968).

(*2*) Simulation to determine the performance of AS-11B automatic analytical stereoplotter (Whiteside and Lipski, 1968).

It is this second class of problems that is applied here to (non-automatic) analog stereoplotters. Contrary to automatic systems, the dynamic characteristics of conventional optical–mechanical restitution instruments are of secondary importance. The reason is that even in the so-called dynamic or continuous mode of restitutions, i.e., during the compilation of line maps, contour lines, etc., the spatial motions of all mechanical parts still are relatively slow. Of primary concern is the static or stationary mode of restitution, where the output is recorded only when the tracking unit is in a state of rest. This paper, therefore, is restricted to a kinematic analysis of stereoplotters.

The computer simulation approach can be used to find and locate instrumental errors of systematic nature. This is possible because any dislocation or disorientation of optical or mechanical units taking part in the measurement process yields deviations from the ideal mathematical collinearity model. The interference of the large number of such errors, and the interdependency of many of them make simulation of the actual physical system to an interesting tool for a methodical evaluation of the system's capabilities and limitations. The high precision inherent in any stereoplotter can be exploited to its limits, thus improving the static compilation and measuring accuracy.

FLOW OF INFORMATION

Photogrammetric restitution instruments can be interpreted as communication systems, in which information flows from the photographs to the operator and to one or more recording units. A block diagram for a manually controlled stereoplotter is shown in Fig.1. The information contents of two corresponding photographs is carried through a pair of scanning and observation systems by optical signals. Their perception occurs with the operator's eyes, the received information is transmitted by electrical pulses in the sensory nerves to the human brain, where the actual processing follows. The two corresponding, spatially different optical signals are interpreted, correlated to be amalgamated into a single stereoscopic signal, and are compared with a reference signal (floating mark). By experiencing a discrepancy, the brain decides to send electrical signals in the motor nerves to the operator's muscular system in order to create corrective measures. Reaction to the discrepancy consists in carrying kinetic signals to the driving agents; the tracking unit is forced to move and the two projection systems are

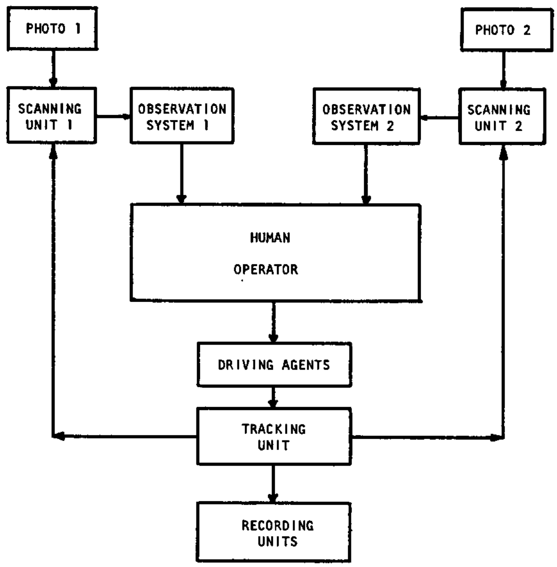


Fig.1. Manual control of a stereoplotter.

displaced together with their scanning units. Thus, the previous photographic signals are changed. From a functional point of view, this communication channel consists of a closed or feed-back loop (Makarovič, 1969).

The described process can be simulated mathematically. Fig.2 shows a descriptive, already computer-oriented flow diagram. The feed-back loop can clearly

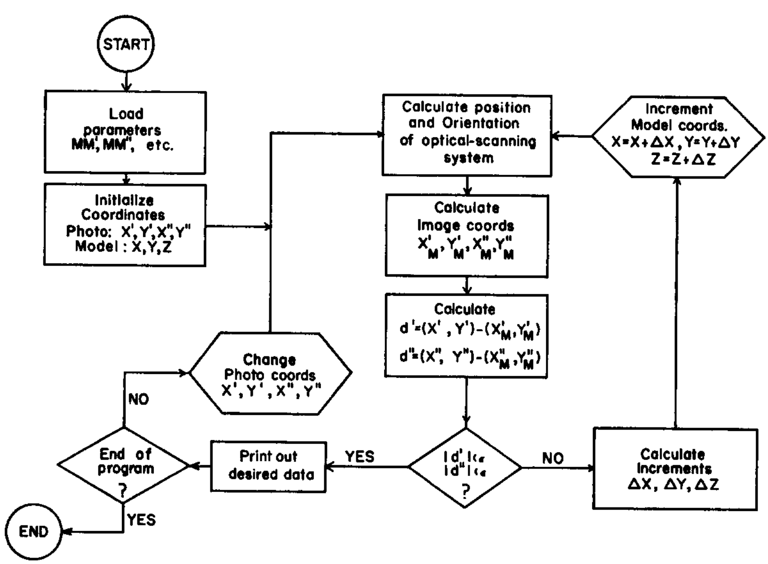


Fig.2. Descriptive flow diagram for the simulation of a stereoplotter in the static mode of restitution.

be recognized. The crucial decision for the feed-back control is made upon comparing the photo coordinates of a pair of correlated photo points with a pair of image coordinates of two measuring marks (*MM'*, *MM''*) projected into the photographs through the optical and scanning systems. If photo points and corresponding projected measuring marks are not coinciding, the model coordinates of the current model point are incremented by an appropriate amount and in a proper direction. The two projection systems are displaced, thus changing the projected image coordinates of the two measuring marks. This process is to be repeated iteratively until the photo-coordinate differences fall within specified limits.

It is also possible to simulate only one projector of a stereoplotter. In this case model coordinates are considered as projection coordinates with one arbitrary coordinate (Z).

DISPLACEMENT ANALYSIS OF SPATIAL MECHANICAL LINKAGES

A stereoplotter may be represented in terms of a kinematic equivalent which is a complex spatial mechanism having several kinematic loops and degrees of freedom. For a computer simulation of such mechanisms an all-algebraic method for the displacement analysis of spatial linkages is essential. This is the main part of the total simulation, although it may not be recognized in Fig.2 (upper central box).

A method based on matrix algebra has been published earlier and has been successfully applied to a variety of mechanical problems (Uicker et al., 1972), The procedure defines all parameters required for a kinematic analysis, and allows formulating spatial problems in terms of matrix equations. The symbolic notation used to represent relations between adjacent links connected by kinematic lower

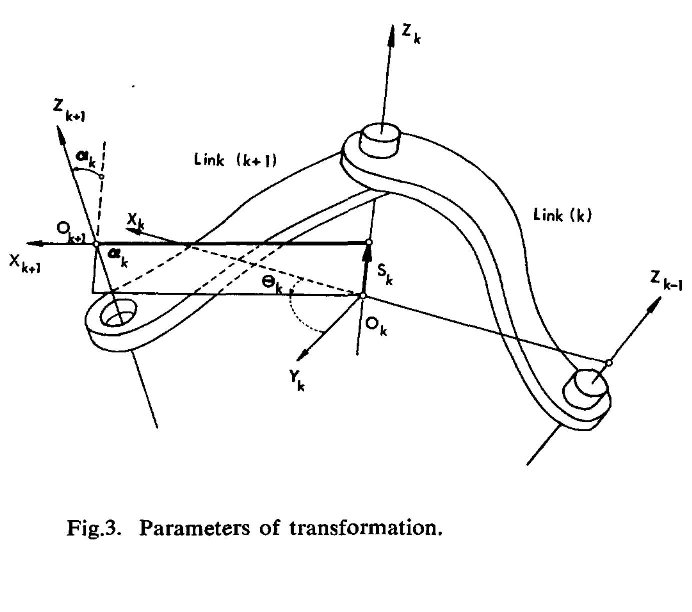


Fig.3. Parameters of transformation.

pairs, is essentially the same as in earlier work and originates from Hartenberg and Denarit (1964). Fig.3 identifies the essential parameters.

The $X_k Y_k Z_k$-axes define a right-handed Cartesian coordinate system rigidly attached to link k of a linkage chain, and the four parameters a_k, α_k, θ_k, S_k define the position of link $(k + 1)$ relative to that of link k. It has been shown that the

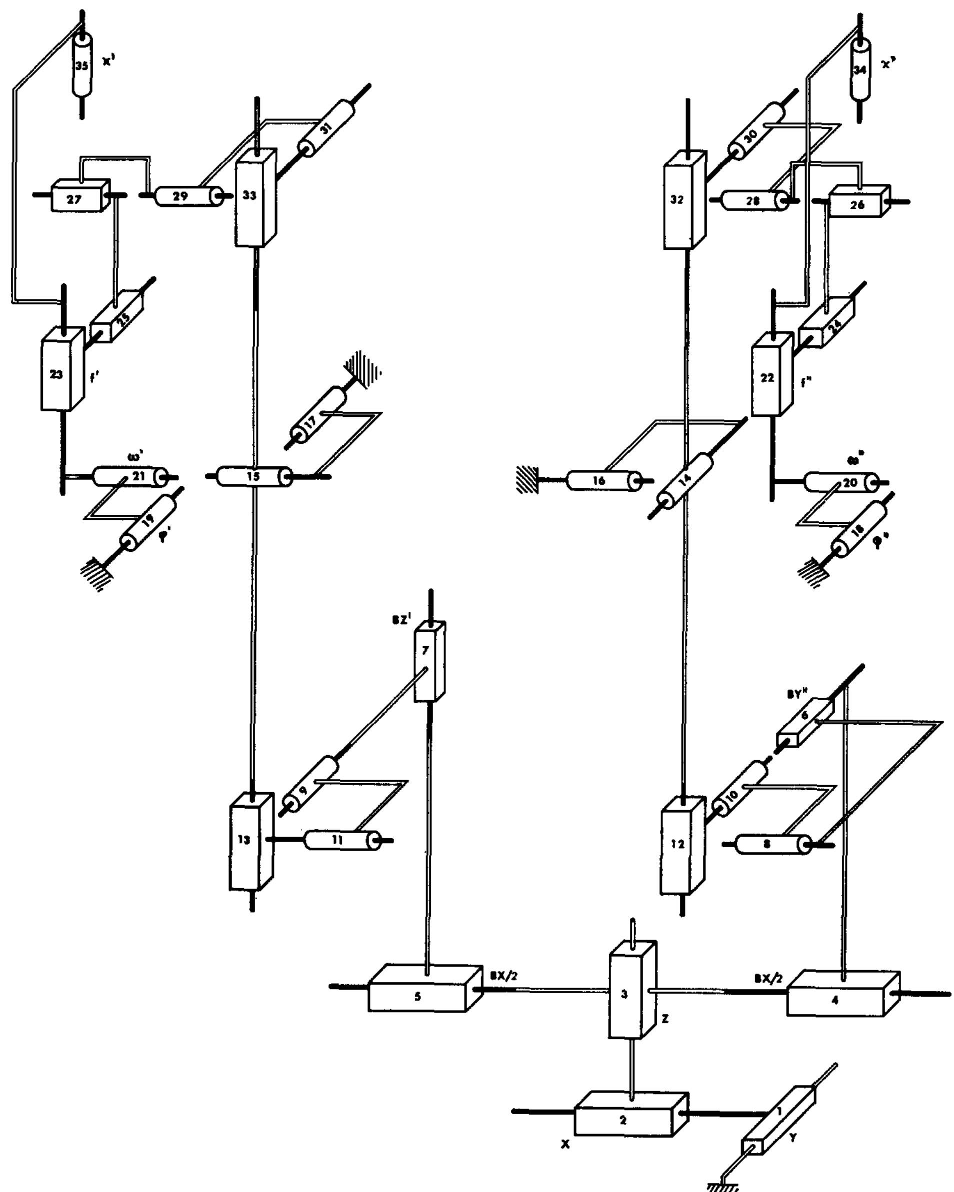


Fig.4. Spatial mechanism of the Wild A-10 stereoplotter with eighteen revolute and seventeen prismatic pairs.

relative position of such two adjacent coordinates systems can be mathematically stated in terms of the following (4×4) transformation:

$$A_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_k \cos\theta_k & \cos\theta_k & -\sin\theta_k \cos\alpha_k & \sin\theta_k \sin\alpha_k \\ a_k \sin\theta_k & \sin\theta_k & \cos\theta_k \cos\alpha_k & -\cos\theta_k \sin\alpha_k \\ S_k & 0 & \sin\alpha_k & \cos\alpha_k \end{bmatrix} \quad (1)$$

Around any closed loop of n links, the $(n + 1)$ coordinate system is the same as coordinate system 1, and this may be expressed as:

$$A_1 A_2 \ldots A_n = \mathrm{I} \quad (2)$$

As a basis for the following derivation, the two-loop spatial mechanism of the left projection system of a Wild A-10 stereoplotter is used. Fig.4 shows the arrangement of mechanical revolutes ("cylinders") and prismatics ("cubes") typical for the A-10. For a revolute, the pair variable is the angle θ; for a prismatic pair, it is the distance S. By denoting with R a revolute matrix A, and with P a prismatic matrix, the two loops associated with the left projector, can be expressed by the following matrix equations:

$$\text{model loop: } P_1 \cdot P_2 \cdot P_3 \cdot P_5 \cdot P_7 \cdot R_9 \cdot R_{11} \cdot P_{13} \cdot R_{15} \cdot R_{17} = \mathrm{I} \quad (3)$$

$$\text{camera loop: } R_{19} \cdot R_{21} \cdot P_{23} \cdot P_{25} \cdot P_{27} \cdot R_{29} \cdot R_{31} \cdot P_{33} \cdot R_{15} \cdot R_{17} = \mathrm{I} \quad (4)$$

The three matrices P_1, P_2, P_3 are defined by the independent variables Y, X, Z as inputs into the system. P_5, P_7, R_{19}, R_{21}, R_{23} are constant matrices for a particular orientation, viz., BX, BZ, φ, ω, f.

Loop 1 (the "model loop") gives rise to a system of equations in five unknown dependent variables (θ_9, θ_{11}, S_{13}, θ_{15}, θ_{17}). With the thus determined pair variables θ_{15} and θ_{17}, the remaining five unknown dependent variables (S_{25}, S_{27}, θ_{29}, θ_{31}, S_{33}) can be found from loop 2 (the "camera loop").

As eq.1 indicates, the expressions are non-linear in the unknown parameters θ_k and S_k'. For a numerical solution, an iterative procedure is therefore required. A detailed derivation of this process is given in Uicker (1965); here, only the principle is shown. Initial estimates θ_i^0, S_k^0 for the values of the unknowns are used to linearize the loop eq.3 and 4:

$$\theta_i = \theta_i^0 + \Delta\theta_i, \qquad S_k = S_k^0 + \Delta S_k. \quad (5)$$

Any matrix R_i or P_k with unknown parameters, after linearization yields:

$$R_i(\theta_i^0 + \Delta\theta_i) = R_i(\theta_i^0) + \left.\frac{\partial R_i}{\partial \theta_i}\right|_{\theta_i^0} . \Delta\theta_i \qquad \text{or}$$

$$P_k(S_k^0 + \Delta S_k) = P_k(S_k^0) + \left.\frac{\partial P_k}{\partial S_k}\right|_{S_k^0} \cdot \Delta S_k$$

Noting that this problem is to be adapted to computer operation, linear operators T_θ, T_S will be introduced to perform the indicated differentiation through the following definitions:

$$\left.\frac{\partial R_i}{\partial \theta_i}\right|_{\theta_i^0} = T_\theta R_i^0, \text{ with } R_i^0 = R_i(\theta_i^0)$$

$$\left.\frac{\partial P_k}{\partial S_k}\right|_{S_k^0} = T_S P_k^0, \text{ with } P_k^0 = P_k(S_k^0)$$

The operator matrices are defined by:

$$T_\theta = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad T_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Condensing,

$$R_i(\theta_i) = (I + T_\theta \Delta\theta_i)\ R_i^0,$$
$$P_k(S_k) = (I + T_S \Delta S_k)\ P_k^0$$

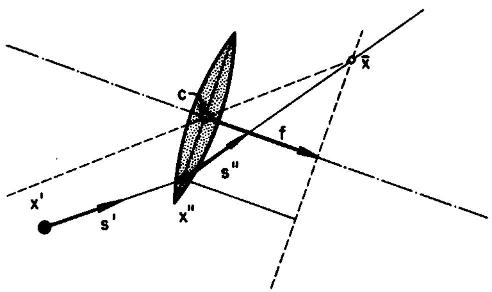
and substituting these forms in eq.3 and 4 yields two sets of sixteen linear equations each in the unknown corrections $\Delta\theta_9$, $\Delta\theta_{11}$, ΔS_{13}, $\Delta\theta_{15}$, $\Delta\theta_{17}$ and ΔS_{25}, ΔS_{27}, $\Delta\theta_{29}$, $\Delta\theta_{31}$, ΔS_{33}. As has been shown by Uicker et al. (1972), even after eliminating the linearly dependent equations of each of the two sets, usually more (viz., nine) equations than unknowns remain. In general, such systems have no exact solution. According to Uicker et al. (1972), however, since the entire method revolves about an iterative approach, no significant error will be introduced if such a system is solved for the closest approximation of a solution to all equations in the least-squares sense.

When the correction terms $\Delta\theta_i$, ΔS_k have been evaluated, they may be added to their corresponding initial estimates to give an improved approximation. The iteration process may be continued in this manner until the correction terms are all smaller than the desired accuracy limits.

In Uicker (1963), for the first time, a digital computer program was written. Since then, the program has been tested on a wide variety of mechanisms, and produced very satisfactory results. The program has recently been applied for the first time in photogrammetry[1], viz., in conjunction with a simulation program of the flow of information in one projector of a stereoplotter (Fig.2), and with a simulation program of the geometrical optical aspects (see the following section).

[1] Thesis of one of the two authors in progress.

	Position Vector	Direction Vector
Input Ray	x'	s' (unit vector)
Lens	c	f
Output Ray	x"	s" (unit vector)

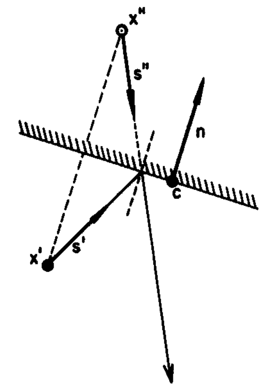


$$[x''-c,\ \bar{x}-x''-f] = \left[I - \frac{s'f^T}{f^Ts'}\right][x'-c,\ c-x'-f]$$

$$s'' = \frac{\bar{x}-x''}{\|\bar{x}-x''\|}$$

Fig.5. Computer simulation of mechanical stereooplottes: mathematical lens model.

	Position Vector	Direction Vector
Input Ray	x'	s' (u.v.)
Mirror	c	n (u.v.)
Output Ray	x"	s" (u.v.)



$$[x''-c,\ s''] = [I - 2nn^T][x'-c,\ s']$$

Fig.6. Computer simulation of mechanical stereoplotters: mathematical mirror model.

SIMULATION OF GEOMETRICAL OPTICS

Only those elements of the optical system that are located between the measuring mark and the photographic image, directly effect the measurement process. For analog instruments based upon the principle of mechanical projection, optical-error sources are less important than for optical projection instruments. This stems from the fact that in the first case only coaxial bundles of rays are used, whereas in the latter case angles up to 90° or more may occur (Schwidefsky, 1967). It is therefore sufficient to simulate only the geometrical optics, without considering physical optical-error sources.

Optical elements such as mirrors, prisms, lenses, beam splitters are assumed to be rigidly attached to the mechanical links. Each optical element is fixed with one particular mechanical link, defined from the instrumental design. Starting from the measuring mark the entire optical train has to be traced mathematically by at least two spatial rays. Mathematically, each optical element represents a transformation of a certain input ray into a corresponding output ray. This is shown in Fig.5 and 6 for a thin lens and a plane mirror. The parameters of a particular optical element as well as the position and direction of an input ray are considered to be known relative to the coordinate system of that mechanical link to which the optical element is attached. In Fig.7, this is link i containing a lens. The output

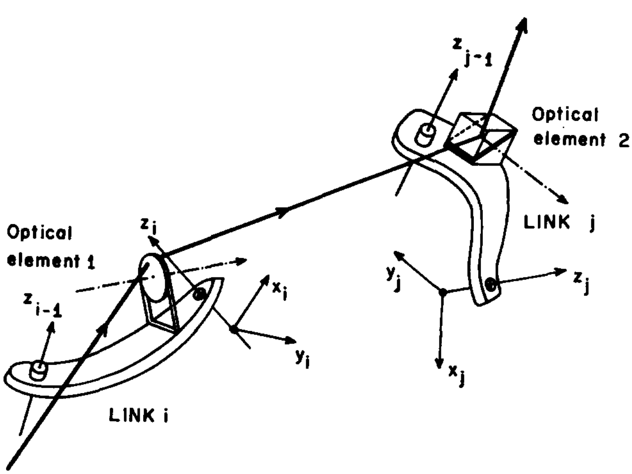


Fig.7. Computer simulation of geometrical optics.

ray of this lens, the parameters of which are still referred to coordinate system (X_i, Y_i, Z_i), is input ray to the neighbouring optical element 2 (prism) within an optical train. This prism is fixed with link j, so are its parameters known relative to system (X_j, Y_j, Z_j). It is therefore necessary to transform the input ray to optical element 2 from system (i) into system (j). This is possible by multiplying the individual (4 × 4) matrices (eq.1) for each link between link i and j along the shortest linkage chain:

$$A_{i,j} = A_i\, A_{i+1}\, A_{i+2} \ldots A_{j-2}\, A_{j-1}.$$

The output ray $[X_i'', S_i'']$ of element 1 can be used as input ray $[X_j', S_j']$ for element 2, if both vectors, X_i'' and S_i'' undergo this (4×4) transformation.

The point of intersection between at least two output rays originating from the last element of the optical train, is considered to be the projected measuring mark after transformation into the image coordinate system (revolutes 34 and 35 in Fig.4). Its image coordinates serve for a comparison with the image coordinates of the original photo point (Fig.2).

APPLICATION OF THE COMPUTER PROGRAM

The original computer program for an iterative displacement analysis of spatial mechanisms was written by Uicker (1963). It was subsequently improved and expanded to cover also the dynamic behaviour of linkages. A number of applications have been published since Uicker (1965), Livermore (1969), Sheth and Uicker (1971). A newer version of the program, written in FORTRAN for the IBM 7090, was made available to us by its originator. It first had to be adapted to the IBM System 360/50 of the University of New Brunswick. Since only displacements were to be considered, all program parts dealing with velocities, accelerations and forces were deleted. The program could therefore be reduced to about 600 FORTRAN-statements, one third of which belongs to the main displacement analysis program. This main routine is supported by five groups of subroutines, which can be described by the following:

(*1*) Control of the displacement analysis of the linkage.

(*2*) Subsequent calculation of the A-matrices, their partial derivatives, their products, and of the design matrix (in the least-squares sense).

(*3*) Solution of the least-squares problem.

(*4*) Correction of the variables and test of the size of error corrections.

(*5*) Auxiliary subroutines.

The main program—amongst other duties—controls input and output. This program version was tested on the spatial mechanism of a Wild A-5 stereoplotter, although on one projection system only.

One of the two mechanical linkage loops, viz., the "model loop", is shown in Fig.8. Together with the "camera loop", the mechanism consists of eleven independent variables $(X, Y, Z, BX, BY, BZ, \omega, \varphi, \Phi, K, f)$ and twelve dependent pair variables (such as α_1, β_1, S, β_2, α_2 in Fig.8).

With constant interior and exterior orientation parameters, the pair variables of each link coud be determined for certain input-model coordinates. Although no optical analysis is used, the program allows an empirical determination of the influence of mechanical deviations from the design dimensions, upon the (mechanical) image coordinates.

This "mechanical" computer program was subsequently extended to simulate

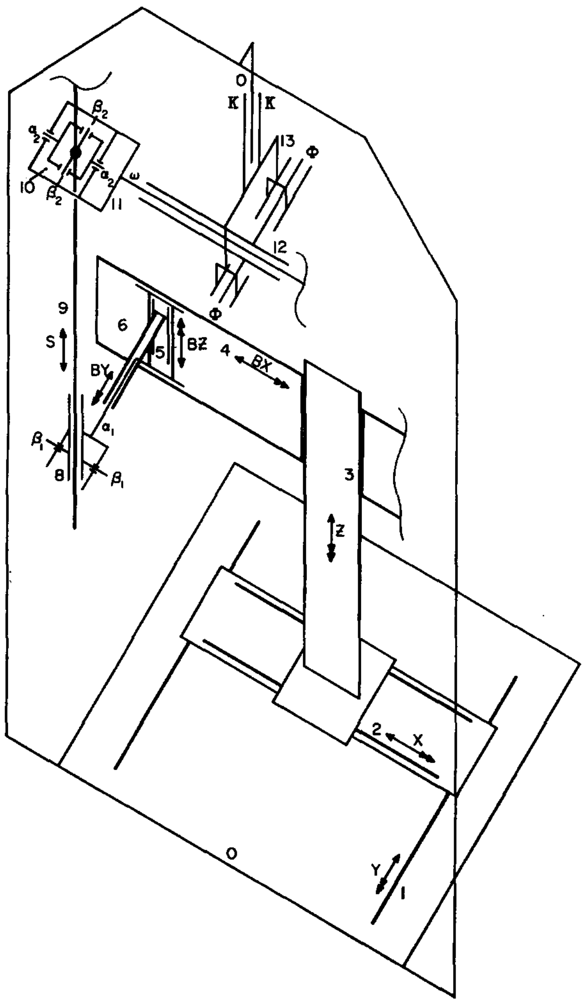


Fig.8. Wild A-5: model loop closure.

the "optics" as well. The authors succeeded in developing several algorithms that describe: (*1*) the transformation of rays through lenses and plane mirrors (Fig.5 and 6); (*2*) the control of all optical elements pertaining to the optical measuring train, and their connection to the mechanical linkage simulation; (*3*) the spatial intersection of two rays; and (*4*) the transformation of the projected measuring mark into the image coordinate system.

The entire computer program presently consists of approximately 800 FORTRAN-instructions, about 200 of which define the main program, the rest is divided into 18 subroutines. No attempts have made sofar to optimize the program, i.e., to improve the convergence of the iterative process, to optimize the least squares solution, and to avoid large and higher dimensional arrays. Together with an extensive hardcopy output of important results, the complete mechanical and optical simulation for one model point requires about 10 sec. The authors, however, believe that this relatively long computing time may still be reduced.

By introducing small errors in the mechanical/optical linkage, a variety of tests have been carried out. As prototype, a Wild A-5 autograph was used. These tests were:

(*1*) Primary and secondary axes of the central gimbal joint are not perpendicular.

(*2*) Primary and secondary axes of the central gimbal joint are not intersecting.

(*3*) Swinging girder is not parallel to the image plane.

(*4*) Rotation axis of the swinging girder is not parallel to the projection camera axis.

(*5*) Swinging girder is not perpendicular to its rotation axis.

(*6*) Omega axis is not perpendicular to phi axis.

(*7*) Omega and phi axes are not intersecting.

The output of the present version of the computer simulation program contains the dimensions of the mechanism, the errors of the mechanism, for 9 (or 25) regularly distributed projection (model) points the spatial coordinates of all optical elements relative to the frame system, a listing of these model coordinates and their corresponding mechanical equivalent and image coordinates, and a rough error vector plot for the image coordinates. Fig.9 shows the output listing for test no.2, where model scale and image scale are identical, all orientation parameters are zero, and the shortest distance between the two joint axes is 1 mm.

CONCLUSIONS

The computer simulation of analog stereo restitution systems of the mechanical projection type has been made possible by use of mechanical displacement and optical transformation analyses. In the stationary mode of measurement, the spatial state of all mechanical links and optical elements taking part in the reconstruction of the bundles of rays, can be mathematically described and put into an algorithm. Preliminary test results indicate the usefulness of the simulation method, to trace deviations against the mathematical collinearity model or against the original design model. It is still too early, however, for a definite answer whether this method can give new insight into the problems of instrument testing (Döhler, 1972) or not. Carefully selected test examples must be chosen, other instrument types must be simulated, and consideration to optimizing the present simulation procedure must be given, before its theoretical as well as practical usefulness is proven. Principally, the method is open for applications.

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SPATIAL POINT ELEVATION   300.000        FOCAL LENGTH   =   150.000
BX    =    0.0            BY =     0.0          BZ    =     0.0
OMEGA=     0.0            PHI=     0.0          KAPPA=      0.0
COMMON PHI=     0.0            COMMON KAPPA=     0.0
```

POINT NO.	MODEL COORD. X	MODEL COORD. Y	MECHANICAL COORD. X	MECHANICAL COORD. Y	IMAGE COORD. X	IMAGE COORD. Y
1 1	100.000	100.000	101.116	100.000	101.116	100.000
1 2	50.000	100.000	50.558	100.000	50.558	100.000
1 3	0.0	100.000	-0.000	100.000	-0.000	100.000
1 4	-50.000	100.000	-50.558	100.000	-50.558	100.000
1 5	-100.000	100.000	-101.116	100.000	-101.116	100.000
2 1	100.000	50.000	101.273	50.000	101.273	50.000
2 2	50.000	50.000	50.636	50.000	50.636	50.000
2 3	0.0	50.000	-0.000	50.000	-0.000	50.000
2 4	-50.000	50.000	-50.637	50.000	-50.636	50.000
2 5	-100.000	50.000	-101.273	50.000	-101.273	50.000
3 1	100.000	0.0	101.342	-0.000	101.342	-0.000
3 2	50.000	0.0	50.671	-0.000	50.671	-0.000
3 3	0.0	0.0	-0.000	-0.000	-0.000	-0.000
3 4	-50.000	0.0	-50.671	-0.000	-50.671	-0.000
3 5	-100.000	0.0	-101.342	-0.000	-101.342	-0.000
4 1	100.000	-50.000	101.273	-50.000	101.273	-50.000
4 2	50.000	-50.000	50.636	-50.000	50.636	-50.000
4 3	0.0	-50.000	-0.000	-50.000	-0.000	-50.000
4 4	-50.000	-50.000	-50.637	-50.000	-50.636	-50.000
4 5	-100.000	-50.000	-101.273	-50.000	-101.273	-50.000
5 1	100.000	-100.000	101.116	-100.000	101.116	-100.000
5 2	50.000	-100.000	50.558	-100.000	50.558	-100.000
5 3	0.0	-100.000	-0.000	-100.000	-0.000	-100.000
5 4	-50.000	-100.000	-50.558	-100.000	-50.558	-100.000
5 5	-100.000	-100.000	-101.116	-100.000	-101.116	-100.000

Fig.9. Computer output from a simulation test for the Wild autograph A-5. Error model: primary and secondary axes of the central gimbal joint are skew by an amount of 1 mm.

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