# Technique and Tool for Symbolic Representation and Manipulation of Stochastic Transition Systems

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## Abstract

*We present a new approach to the compact symbolic representation of stochastic transition systems, based on Decision Node BDDs, a novel stochastic extension of BDDs. Parallel composition of components can be performed on the basis of this new data structure. We also discuss symbolic state space reduction by Markovian bisimulation.*

In many areas of system design and analysis, there is the problem of generating, manipulating and analysing very large state spaces. We focus on stochastic labelled transition systems (SLTS) where each transition is labelled by an action name and an exponential delay. Such SLTSs (which originate, e.g., from Stochastic Process Algebra specifications [3]) can be interpreted as Markov chains and used for performability analysis.

We propose a novel approach to SLTS representation and manipulation which is based on symbolic techniques. This is motivated by the fact that, in recent years, the problem of representing and analysing large state spaces has been very successfully approached by using Binary Decision Diagrams (BDD) [1]. This work took place in the context of formal verification and model checking, i.e. dealing exclusively with functional behaviour, disregarding temporal aspects.

The success of symbolic techniques for functional analysis induced us to experiment with BDD-based representations of *stochastic* LTS. We introduce a novel data structure, Decision Node BDDs (DNBDD), which can capture not only functional, but also temporal (stochastic) information [4]. The idea is to employ binary encodings of states and transitions and store them in a canonical, graph-based format, a BDD. The information about transition rates is attached to a subset of the BDD nodes (the decision nodes) without modifying the basic structure of the BDD.

Complex systems can be most conveniently specified as a number of interacting components. Using traditional representations, parallel composition of components leads to an exponential growth of the state space, since all possible interleavings of events in different components are explicitely represented. Working with the symbolic representation, parallelism is represented implicitely, resulting in a growth of the (DN)BDD which is only linear in the number of parallel components (provided that an appropriate encoding and ordering of BDD variables is used). This is the main reason why BDD-based representations can be so compact.

We have developed an efficient algorithm for state space minimisation of SLTS, which is entirely (DN)BDD-based. The algorithm classifies states according to the concept of Markovian bisimulation [3] (which is related to Markov chain lumpability), and follows a partition refinement scheme with intelligent splitter management [2]. During minimisation, the cumulative rate from an individual state to a class of states has to be calculated. This step can be performed efficiently on the DNBDD data structure.

The combination of symbolic parallel composition and minimisation enables compositional reduction: After every composition step, the state space is minimised before the resulting component is used in further contexts. Thus, large intermediate state spaces are avoided as much as possible.

Our concepts have been realised in the form of a prototype tool which supports DNBDD generation, symbolic parallel composition and symbolic minimisation.

## References

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