Risk Analysis

Satellite-Navigation II Tutorial – 1
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Normal Distribution I

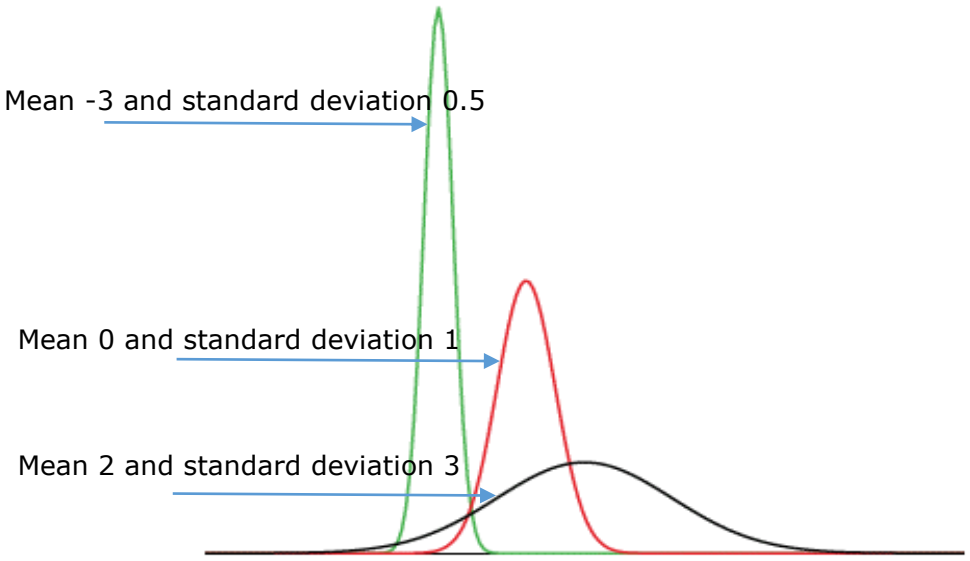
- Most Widely used distribution in the statistics
- Mathematically, $\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$
- Normal Distributions can differ in their
  - Mean
  - Standard Deviation

Mean

- $X \sim N(\mu, \sigma^2)$

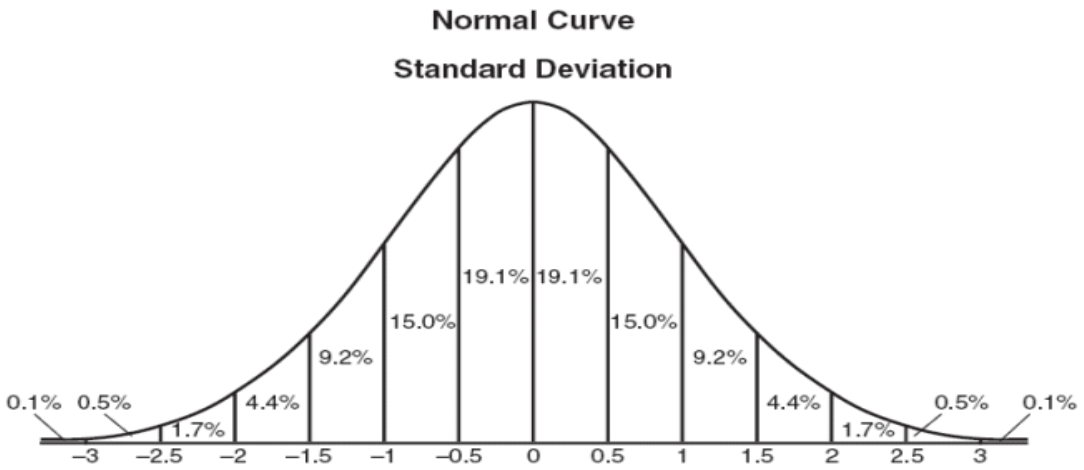
variance

- Normal Distribution with mean of 0 and a standard deviation of 1 is called standard normal deviation



Normal Distribution II

- The area under the normal curve is equal to 1.0
- 68% of the area of a normal distribution is within one standard deviation of the mean
- Approximately 95% of the area of a normal distribution is within two standard deviations of the mean
- Normal distributions are symmetric around their mean



Problem – Normal Distribution

Q1. A certain variety of pine tree has a mean trunk diameter of =150 cm and a standard deviation σ=30cm. Approximately what percent of these trees have a diameter greater than 210cm?

Q2. Assume that the mean weight of 1 year old in Germany is normally distributed with a mean of about 9.5 grams with a standard deviation of approx. 1.1 kilograms. Without using calculator, estimate the percentage of 1 year old girls in Germany that meet the following conditions. Draw a sketch and shade proper region.

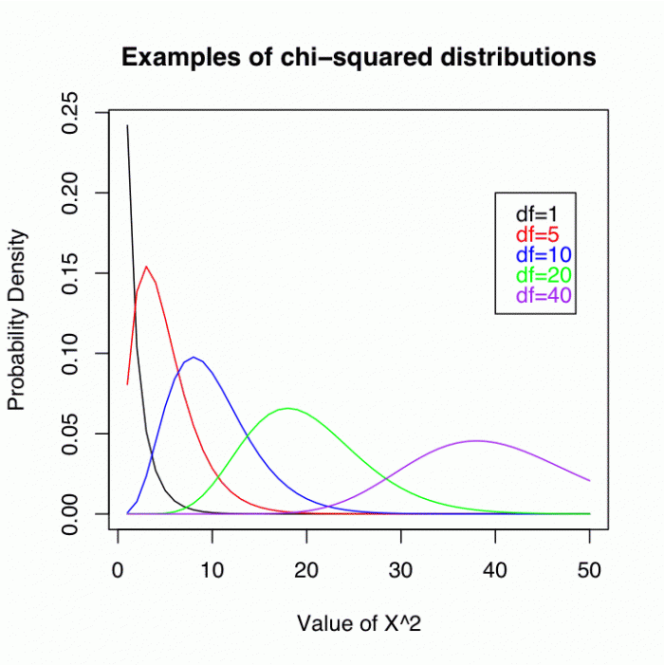
- Less than 8.4 kg
- Between 7.3 kg and 11 kg
- More than 12.8 kg

Chi Square Distribution

- Used in Hypothesis testing
  - Hypothesis/explanation for proposed phenomenon that is testable on the basis of observing a process that is modeled via set of random variables.
- Also refered as central chi-square distribution
- From Normal distribution we know :
  - $X \sim N(\mu,\sigma^2)$
  - Where X is a std. normal distributed random variable
- Now Chi Square distribution is :
  - $Q_1 = X^2$
  - $Q_1 \sim \chi_1^2$, where 1 in subscript is the degree of freedom
  - Where X is a std. normal distributed random variable
  - Q_1 is a Chi square distributed random variable

Chi Square Distribution

- Similarly :
  - $Q_2 = X_1 + X_2$
  - $Q_2 \sim \chi^2_2$, 2 in subscript is the degree of freedom
  - Where X_1 and X_2 are independent std. normal distributed random variable



Non Central Chi Square Distribution

- Chi-square test statistic is distributed when the null hypothesis is assumed to be true.

- The non-centrally chi-square distribution instead shows how the chi-square test statistic is distributed when the alternative hypothesis is assumed to be true (i.e. when the null hypothesis is assumed to be false).

Example – Chi-Squared Distribution

Q1. for a fair six-sided die, the probability of any given outcome on a single roll would be 1/6. The data in Table were obtained by rolling a six-sided die 36 times. However, as can be seen in Table, some outcomes occurred more frequently than others. For example, a "3" came up nine times, whereas a "4" came up only two times. Are these data consistent with the hypothesis that the die is a fair die? Naturally, we do not expect the sample frequencies of the six possible outcomes to be the same since chance differences will occur. So, the finding that the frequencies differ does not mean that the die is not fair. One way to test whether the die is fair is to conduct a significance test. The null hypothesis is that the die is fair. Given significance level is 0,10.

Outcome	Frequency
1	8
2	5
3	9
4	2
5	7
6	5

Problem – Chi-Squared Distribution

Q3. Imagine that you sample 12 scores from a standard normal distribution, square each score, and sum the squares. How many degrees of freedom does the Chi Square distribution that corresponds to this sum have?

Q4. What is the mean of a Chi Square distribution with 8 degrees of freedom?

Q5. Which Chi Square distribution looks the most like a normal distribution?

- A Chi Square distribution with 0 df
- A Chi Square distribution with 1 df
- A Chi Square distribution with 2 df
- A Chi Square distribution with 10 df

Q6. You buy a bag of 40 lollipops. This bag has 4 different colors of lollipops in it. You are curious if all 4 colors were equally likely to be put in the bag or whether certain colors were more likely. If all four colors were equally likely to be put in the bag, what would be the expected number of lollipops of each color?

Q7. Suppose now that you open the lollipops to find out that you have 8 red, 5 green, 12 orange, and 15 blue. Test the null hypothesis that the colors of the lollipops occur with equal frequency. What is the Chi Square value you get?

Problem – Chi-Squared Distribution

Q8. Suppose you plan to buy a restaurant and and the previous owner gives a data mentioning the percentage of his total customers coming to his restaurant over a period of week. But you want to make an anaylsis about how good is the data provided by the previous owner in order to make your deal successful. In the table below E is the expected data provided to you by the previous owner and O is the observed number of customer (your own data which you collected for hypothesis test). The Null hypothesis is that the owners distribution is correct. Given significance level is 0,05.

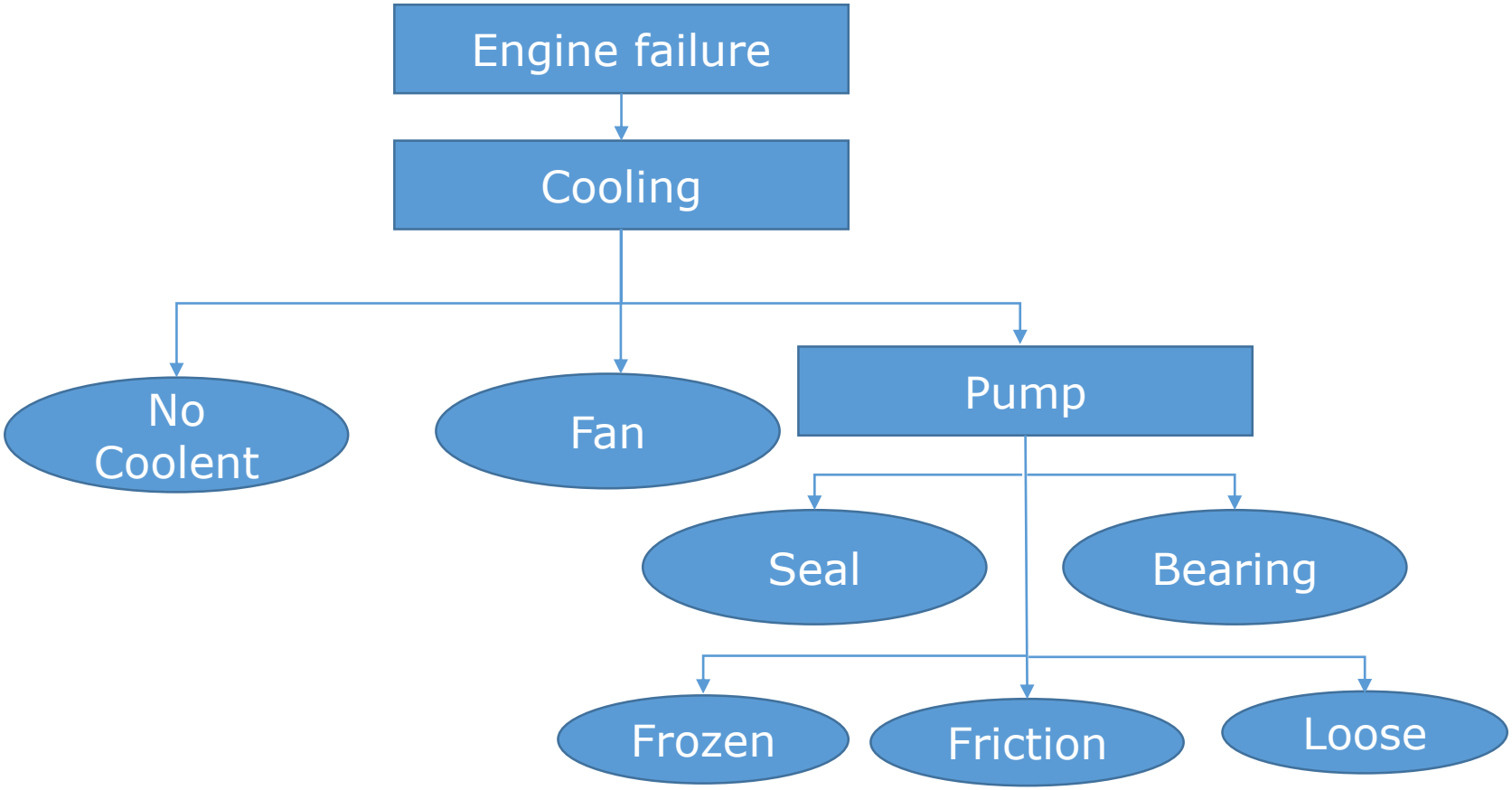
Weekday	Expected (%)	Observed (No. Of customers)
Monday	10	30
Tuesday	10	14
Wednesday	15	34
Thursday	20	45
Friday	30	57
Saturday	15	20

Fault Tree Analysis

- Popular and Produtive hazard identification tool
- Output is graphical presentation providing technical and administrative personal with a map of failure or hazard path.
- How to Design a fault Tree Analysis
  - Starts with the Development of top level event
  - Continue the top to down process until the root cause each branch is identified or untill further decomposition is not possible.
  - Assign possibilities of failure to the lowest level event in each branch of the tree via prediction, allocation or historical data
  - Design a boolean equation for the tree using boolean logic and evaluate the probabilty of the undesired top level event
  - Compare to the system level requirments. If the requirments are not met, implement correct action.

Fault Tree Analysis

- Design of a Fault Tree for Engine Failure due to Cooling malfunction.



Problem - Fault Tree Analysis

Q9. Prepare a Fault Tree Analysis for the Event "Engine Failure" due to

- Fuel

Thank You !

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