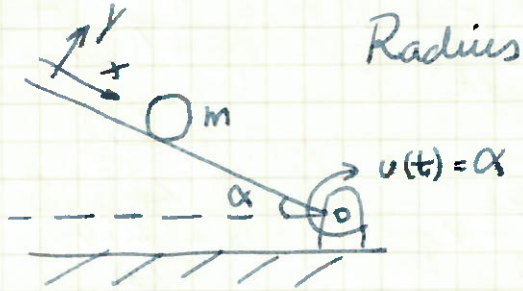
9. Übung Victor cheidde chaim

9.1)




Radius $R$, Masse $m$

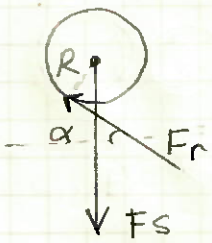
$F_r = m \cdot g \cdot \mu \cdot \cos\alpha$

$I_m = \frac{2}{5} \cdot m \cdot R^2$

die Kugel <u>rollt</u>.

i) $x = [p \;\; v]^T$, $u = \alpha$, $y = p$

Kräfte: * $F_r$ ist immer gegen die Bewegung.




Newton:

$\sum F_x$: $m \cdot a = F_s \cdot \sin\alpha - F_r$

$a \rightarrow$ Beschleunigung in X-Richtung

$\cancel{m} \cdot a = \cancel{m} \cdot g \cdot \sin\alpha - \cancel{m} \cdot g \cdot \mu \cdot \cos\alpha$

$$\boxed{a = g \cdot \sin\alpha - g \cdot \mu \cdot \cos\alpha} \quad (I)$$

die Kugel rollt!

↳ Euler: $\sum M_{CG}$: $I_m \cdot \ddot{\theta} = F_r \cdot R$

$\ddot{\theta} \rightarrow$ Winkelbeschleunigung auf der Kugel.

kinematic: $R\ddot{\theta} = a \rightarrow \ddot{\theta} = \frac{a}{R}$ ∴

$$I_m \cdot \frac{a}{R} = F_r \cdot R \rightarrow \frac{2}{5} \cancel{m} \cancel{R}^{\cancel{2}} \cdot \frac{a}{\cancel{R}} = \cancel{m} \cdot g \, \mu \cos\alpha \cdot \cancel{R}$$

$$\boxed{\mu = a \cdot \frac{2}{5} \cdot \frac{1}{g \cdot \cos\alpha}} \quad (II) \quad , \quad (II \rightarrow I):$$

$$a = g \sin\alpha - \cancel{g} \left( a \cdot \frac{2}{5} \cdot \frac{1}{\cancel{g} \cancel{\cos\alpha}} \right) \cdot \cancel{\cos\alpha} = g \sin\alpha - a \cdot \frac{2}{5}$$

$$a + a \frac{2}{5} = g \cdot \sin\alpha \rightarrow a \cdot \frac{7}{5} = g \sin\alpha \rightarrow \boxed{a = \frac{5}{7} \cdot g \cdot \sin\alpha}$$

Kinematic: $\dot{v} = a$, $\dot{p} = v$.

$$\dot{x} = \begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ \frac{5}{7} \cdot g \cdot \sin\alpha \end{bmatrix} = f(x,u) \quad //$$

$$y = p = g(x,u) \quad //$$

ii) Ruhelagen: $\dot{x} = [\dot{p} \quad \dot{v}] = 0$

$$\text{lok} = \begin{bmatrix} v_0 \\ \frac{5}{7} \cdot g \cdot \sin\alpha_0 \end{bmatrix} \rightarrow v_0 = 0, \quad \alpha_0 = 0$$

keine Bedingung für $p_0$.

$$x_0 = [p_0 \quad 0]^T, \quad u_0 = \alpha_0 = 0.$$

iii) $A := D_1 f(x_0,u_0)$ $\quad B := D_2 f(x_0,u_0)$

$C := D_1 g(x_0,u_0)$ $\quad D := D_2 g(x_0,u_0)$

$A_{11} = \frac{\partial f_1}{\partial p}(x_0,u_0) = 0$ $\quad A_{12} = \frac{\partial f_1}{\partial v}(x_0,u_0) = \underline{1}$

$A_{21} = \frac{\partial f_2}{\partial v}(x_0,u_0) = 0$ $\quad A_{22} = \frac{\partial f_2}{\partial v}(x_0,u_0) = 0$

$B_{11} = \frac{\partial f_1}{\partial \alpha}(x_0,u_0) = 0$ $\quad B_{21} = \frac{\partial f_2}{\partial \alpha}(x_0,u_0) = \frac{5}{7} \cdot g \cdot \cos 0 = \frac{5}{7} g$

$C_{11} = \frac{\partial g}{\partial p}(x_0,u_0) = 1$ $\quad C_{12} = \frac{\partial g}{\partial v}(x_0,u_0) = 0$

$D_{11} = \frac{\partial g}{\partial u}(x_0,u_0) = 0$ $\quad \therefore$ Zustandssystem:

$$\begin{cases} \Delta\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \Delta x + \begin{pmatrix} 0 \\ \frac{5}{7} \cdot g \end{pmatrix} \Delta u \\ \Delta y = \begin{pmatrix} 1 & 0 \end{pmatrix} \Delta x \end{cases} \quad \begin{cases} \Delta x := x(t) - x_0 \\ \Delta y := y(t) - y_0 \\ \Delta u := u(t) - u_0 \end{cases}$$

iv) $\Delta u = -[k_p \quad k_v]\,\Delta x, \quad k_p, k_v \in \mathbb{R}_+$

$$\Delta\dot{x} = A.\Delta x + B.\Delta u = A\,\Delta x - B.[k_p \quad k_v]\,\Delta x$$

$$\Delta\dot{x} = (A - B[k_p \quad k_v]).\Delta x$$

$$\bar{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{5}{7}\cdot g \end{pmatrix}(k_p \quad k_v) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ \frac{5}{7}g k_p & \frac{5}{7}g k_v \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} 0 & 1 \\ -\frac{5}{7}.g.k_p & -\frac{5}{7}.g.k_v \end{pmatrix}, \quad \bar{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \Delta\dot{x} = \bar{A}\,\Delta x + \bar{B}\,\Delta u$$

v) $k_v = 0, \quad x_0 \neq 0, \quad \psi(0{,}4, x_0, \cdot) = 0.$

$$\psi(t, x_0, \cdot) = C.e^{\bar{A}t}.x_0 + \int_0^t C\,e^{\bar{A}(t-\tau)}.\bar{B}.u(\tau)\,d\tau + D\,u(t)$$

$\rightarrow \bar{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ und $D = 0$: $\quad x_0 = (p_0 \quad 0)^T$.

$$\psi(t, x_0, \cdot) = C.\exp(\bar{A}.t).x_0, \quad \bar{A} = \begin{pmatrix} 0 & 1 \\ -\bar{k}_p & 0 \end{pmatrix}, \quad \bar{k}_p = \frac{5}{7}g k_p$$

$\exp(\bar{A}t) \rightarrow$ EW: $\det(\lambda id - \bar{A}) = 0$

$$\det\begin{pmatrix} \lambda & -1 \\ -\bar{k}_p & \lambda \end{pmatrix} = \lambda^2 - \bar{k}_p = 0 \xrightarrow{\bar{k}_p \geq 0} \lambda_1 = j\sqrt{\bar{k}_p}, \lambda_2 = -j\sqrt{\bar{k}_p}$$

Eigenwerte $\neq 0$, A ist nicht nilpotent.

Diagonaltransformation: $\bar{A} = T\Lambda T^{-1}, \quad T = [v_1 \quad v_2]$

$v_1 \rightarrow \quad \bar{A}v_1 = \lambda_1 v_1 \quad \rightarrow \quad (\bar{A} - \lambda_1 id)\,v_1 = 0$

$$(\bar{A} - \lambda_1 id)v_1 = \begin{pmatrix} -j\bar{k}_p^{1/2} & 1 \\ -\bar{k}_p & -j\bar{k}_p^{1/2} \end{pmatrix}\begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} = 0 \Rightarrow \begin{matrix} v_{11}\,j\,\bar{k}_p^{1/2} = v_{12} \\ -\bar{k}_p v_{11} = j\,\bar{k}^{1/2}.v_{12} \end{matrix}$$

$$v_{11} = \frac{v_{12}}{j\,\bar{k}_p^{1/2}} = \frac{-v_{12}\,j\,\bar{k}_p^{1/2}}{-j^2.\bar{k}_p} = -\frac{v_{12}\,j\,\bar{k}_p^{1/2}}{\bar{k}_p} \rightarrow -\bar{k}_p v_{11} = j\,\bar{k}^{1/2}.v_{12}$$

Bedingungen sind gleichwertig: $v_{12} = j\, v_{11}\, \bar{k}_p^{1/2}$

Wenn $v_{12} = 1$, $v_{12} = j\, \bar{k}_p^{1/2} \rightarrow v_1 = \begin{bmatrix} 1 & j\, \bar{k}_p^{1/2} \end{bmatrix}^T$

$v_2 \rightarrow \bar{A} v_2 = \lambda_2 v_2 \rightarrow (\bar{A} - \lambda_2 id) v_2 = 0$

$$(\bar{A} - \lambda_2 id) v_2 = \begin{pmatrix} j\, \bar{k}_p^{1/2} & 1 \\ -\bar{k}_p & j\, \bar{k}_p^{1/2} \end{pmatrix} \begin{pmatrix} v_{21} \\ v_{22} \end{pmatrix} = 0 \rightarrow \begin{matrix} v_{21}\, j\, \bar{k}_p^{1/2} = -v_{22} \\ v_{21}\, \bar{k}_p = j\, \bar{k}^{1/2} v_{12} \end{matrix}$$

$$v_{21}\, j\, \bar{k}_p^{1/2} \cdot (-j\, \bar{k}_p^{-1/2}) = -v_{22} \cdot (-j\, \bar{k}_p^{1/2}) \rightarrow v_{21}\, \bar{k}_p = j\, \bar{k}^{1/2} v_{12}$$

Bedingungen sind gleichwertig: $v_{22} = -v_{21}\, j\, \bar{k}_p^{1/2}$

Wenn $v_{21} = 1$, $v_{22} = -j\, \bar{k}_p^{1/2} \rightarrow v_2 = \begin{bmatrix} 1 & -j\, \bar{k}_p^{1/2} \end{bmatrix}^T$

$$T = \begin{pmatrix} 1 & 1 \\ j\, \bar{k}_p^{1/2} & -j\, \bar{k}_p^{1/2} \end{pmatrix}, \quad T^{-1} = \frac{1}{\det(T)} \begin{pmatrix} -j\, \bar{k}_p^{1/2} & -1 \\ -j\, \bar{k}_p^{1/2} & 1 \end{pmatrix}$$

$$\det(T) = -j\, \bar{k}_p^{1/2} - (j\, \bar{k}_p^{1/2}) = -2j\, \bar{k}_p^{1/2}$$

$$T^{-1} = \frac{1}{-2j\, \bar{k}_p^{1/2}} \begin{pmatrix} -j\, \bar{k}_p^{1/2} & -1 \\ -j\, \bar{k}_p^{1/2} & 1 \end{pmatrix} \rightarrow \exp(\bar{A} t) = \exp(T \Lambda T^{-1} t)$$

$$\exp(\bar{A} t) = \sum_{n=0}^{\infty} \frac{(T \Lambda T^{-1})^n t^n}{n!} = T \left( \sum_{n=0}^{\infty} \frac{\Lambda^n t^n}{n!} \right) T^{-1}$$

$$\exp(\bar{A} t) = T \begin{pmatrix} e^{j\sqrt{\bar{k}_p} t} & 0 \\ 0 & e^{-j\sqrt{\bar{k}_p} t} \end{pmatrix} \cdot T^{-1}$$

$$\exp(\bar{A} t) = \frac{1}{-2j\, \bar{k}_p^{1/2}} \begin{pmatrix} 1 & 1 \\ j\, \bar{k}_p^{1/2} & -j\, \bar{k}_p^{1/2} \end{pmatrix} \begin{pmatrix} e^{j\sqrt{\bar{k}_p} t} & 0 \\ 0 & e^{-j\sqrt{\bar{k}_p} t} \end{pmatrix} \begin{pmatrix} -j\, \bar{k}_p^{1/2} & -1 \\ -j\, \bar{k}_p^{1/2} & 1 \end{pmatrix} //$$

$$\psi(t, x_0, \cdot) = \frac{1}{-2j\, \bar{k}_p^{1/2}} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ j\, \bar{k}_p^{1/2} & -j\, \bar{k}_p^{1/2} \end{pmatrix} \begin{pmatrix} e^{j\sqrt{\bar{k}_p} t} & 0 \\ 0 & e^{-j\sqrt{\bar{k}_p} t} \end{pmatrix} \begin{pmatrix} -j\, \bar{k}_p^{1/2} & -1 \\ -j\, \bar{k}_p^{1/2} & 1 \end{pmatrix} \begin{pmatrix} p_0 \\ 0 \end{pmatrix}$$

$$\psi(t, x_0, \cdot) = \frac{1}{-2j\, \bar{k}_p^{1/2}} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} e^{j\sqrt{\bar{k}_p} t} & 0 \\ 0 & e^{-j\sqrt{\bar{k}_p} t} \end{pmatrix} \begin{pmatrix} -j\, \bar{k}_p^{1/2} p_0 \\ -j\, \bar{k}_p^{1/2} p_0 \end{pmatrix}$$

$$\psi(t, x_0, \cdot) = \frac{1}{-2j\, k_p^{1/2}} \left(e^{j\sqrt{k_p}t} - e^{-j\sqrt{k_p}t}\right) \begin{pmatrix} -j\, p_0\, k_p^{1/2} \\ -j\, p_0\, k_p^{1/2} \end{pmatrix}$$

$$\psi(t, x_0, \cdot) = \frac{\cancel{-j}\, p_0\, \cancel{k_p^{1/2}}}{-2\cancel{j}\, \cancel{k_p^{1/2}}} \left(e^{j\sqrt{k_p}} + e^{-j\sqrt{k_p}}\right) = \frac{p_0}{2}\left(e^{j\sqrt{k_p}t} + e^{-j\sqrt{k_p}t}\right)$$

Eulersche Formel: $e^{ia} = \cos(a) + j \cdot \sin(a)$

$$\psi(t, x_0, \cdot) = \frac{p_0}{2}\left(\cos(k_p^{1/2} \cdot t) + j \sin(k_p^{1/2} \cdot t) + \cos(-k_p^{1/2} t) + j \sin(-k_p^{1/2} t)\right)$$

$\cos(b) = \cos(-b)$ , $\sin(-b) = -\sin(b)$ $\therefore$

$$\psi(t, x_0, \cdot) = \frac{p_0}{2}\left(\cos(k_p^{1/2} t) + \cancel{j \sin(k_p^{1/2} \cdot t)} + \cos(k_p^{1/2} t) - \cancel{j \sin(k_p^{1/2} t)}\right)$$

$$\psi(t, x_0, \cdot) = \frac{p_0 \cdot \cancel{2} \cdot \cos(k_p^{1/2} \cdot t)}{\cancel{2}} = p_0 \cdot \cos(k_p^{1/2} \cdot t) \,//$$

Annahme: $\psi(0{,}4, x_0, \cdot) = 0$

$$\psi(0{,}4, x_0, \cdot) = p_0 \cdot \cos(k_p^{1/2} \cdot 0{,}4) = 0 \Rightarrow k_p^{1/2} \cdot 0{,}4 = \frac{\pi}{2}$$

$$\sqrt{\frac{5}{7} k_p \cdot g} \cdot 0{,}4 = \frac{\pi}{2} \quad , \quad \sqrt{\frac{5}{7} k_p \cdot g} = \frac{\pi}{0{,}8} = \frac{10 \cdot \pi}{8}$$

$$k_p \cdot g \cdot \frac{5}{7} = \frac{100}{64} \cdot \pi^2 \rightarrow k_p = \frac{700}{320} \cdot \frac{\pi^2}{g} = \frac{70}{32} \frac{\pi^2}{g} = \frac{35}{16} \cdot \frac{\pi^2}{g} \,//$$

vi) $k_p, k_v = 0$ : $\bar{A} = \begin{pmatrix} 0 & 1 \\ -k_p \frac{5}{7} g & -k_v \frac{5}{7} g \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{100}{64}\pi^2 & 0 \end{pmatrix}$

$$EW(\bar{A}) \Rightarrow \det(\lambda\, id - \bar{A}) = 0$$

$$\det\begin{pmatrix} \lambda & -1 \\ +\frac{25}{16}\pi^2 & \lambda \end{pmatrix} = \lambda^2 + \frac{25}{16}\pi^2 \Rightarrow \cancel{\lambda = \sqrt{\frac{25}{16}\pi^2} \Rightarrow \lambda_{1,2} = \frac{5}{4}\pi^2}$$

$\lambda_1 = j\frac{5}{4}\pi$ , $\lambda_2 = -j\frac{5}{4}\pi$ //

• Das Zustandsraumsystem ist nicht asymptotisch stabil, weil $Re(\lambda_1)$ und $Re(\lambda_2)$ ~~positiv sind~~ gleich Null sind.

vii) Z5 zwei gleiche negative reele EW.

EW: $\bar{A} = \begin{pmatrix} 0 & 1 \\ -\frac{25}{16}\pi^2 & -k_v \cdot \frac{5}{7} g \end{pmatrix} \rightarrow \bar{A} = \begin{pmatrix} 0 & 1 \\ -\bar{k}_p & -\bar{k}_v \end{pmatrix}$

$\bar{k}_p = \frac{25}{16} \cdot \pi^2 \quad , \quad \bar{k}_v = k_v \cdot \frac{5}{7} \cdot g$

$\det(\lambda \, id - A) = 0 \rightarrow \det \begin{pmatrix} \lambda & -1 \\ \bar{k}_p & \lambda + \bar{k}_v \end{pmatrix} = 0$

$\lambda(\lambda + \bar{k}_v) + \bar{k}_p = 0, \quad \lambda^2 + \lambda \bar{k}_v + \bar{k}_p = 0$

$\lambda = \frac{-\bar{k}_v \pm \sqrt{\bar{k}_v^2 - 4\bar{k}_p}}{2}$ , für $\lambda_1 = \lambda_2$ :

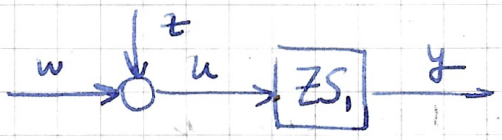
$\bar{k}_v^2 = 4\bar{k}_p \rightarrow \bar{k}_v = 2 \cdot \bar{k}_p^{1/2} \rightarrow k_v \cdot \frac{5}{7} \cdot g = 2 \cdot \sqrt{\frac{25}{16}\pi^2}$

$k_v = \frac{2 \cdot \cancel{5}\pi}{4} \cdot \frac{7}{\cancel{5} \cdot g} = \frac{7}{2} \cdot \frac{\pi}{g}$ //

9. Übung, Victor Cheidde Chaim

9.3 Aufgabe: $A_1 = \begin{bmatrix} 0 & 1 \\ 6 & -5 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_1 = [1 \;\; 0]$, $D_1 = [0]$

$x_1(0) = 0$.




i) $w = \sigma(t)$, $z = 0$

$ZS_1 \rightarrow H_1 = C_1 (sid - A_1)^{-1} \cdot B_1 + \cancel{D_1}^{\,0}$

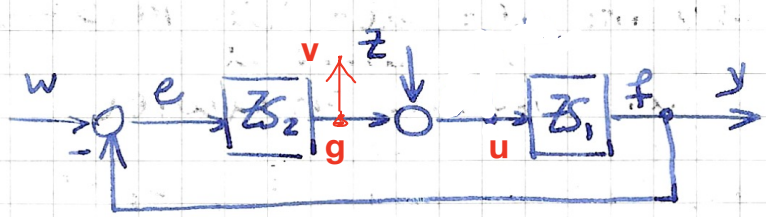
$(sid - A_1)^{-1} = \begin{bmatrix} s & -1 \\ -6 & s+5 \end{bmatrix}^{-1} = \frac{1}{s(s+5)-6} \begin{bmatrix} s+5 & 1 \\ 6 & s \end{bmatrix} = \frac{1}{s^2+5s-6} \begin{bmatrix} s+5 & 1 \\ 6 & s \end{bmatrix}$

CLCP: $s^2+5s-6 \rightarrow$ Pole von $H_1(s) = \frac{-5 \pm \sqrt{25+24}}{2} \Rightarrow p_1 = -6$, $p_2 = 1$

$H_1$ nicht BIBO-stabil, da $p_2 = 1$ ($Re(p_2) \geq 0$)
↳ kein Endwert

oder: (ohne Berechnung der Pole)
CLCP: $s^2+5s-6 \rightarrow$ nicht Hurwitz, da die Koeffizienten unterschiedliche Vorzeichen haben
(hinreichend Bedingung für ein Polynom 2. Ordnung)
↳ ∴ $H_1$ nicht BIBO stabil, kein Endwert.

ii)




$C_2 = 0$, $D_2 = 10$

$x_2(0) = 0$

$w = \sigma$, $z = 0$: $H_2 = \cancel{\overset{0}{C_2}(sid - A_2)B_2}^{\,0} + D_2 = \overset{\text{P-Regler}}{10} = \frac{Z_2}{N_2}$, $Z_2 = 10$, $N_2 = 1$

Führungsübertragungsfunktion: $FÜF = \frac{Z_1 Z_2}{Z_1 Z_2 + N_1 N_2}$

$H_1(s) = \frac{1}{s^2+5s-6} [1 \;\; 0] \begin{bmatrix} s+5 & 1 \\ 6 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{s^2+5s-6} = \frac{Z_1}{N_1}$

∴ $FÜF(s) = \frac{10}{s^2+5s-6+10} = \frac{10}{s^2+5s+4}$, Pole: $\frac{-5 \pm \sqrt{25-16}}{2}$

$p_1 = -4$, $p_2 = -1$

$Re(p_1, p_2) < 0 \rightarrow$ BIBO-stabil! Endwert:

$\psi(\infty, 0, w \cancel{+z}^{\,z=0}) = \psi(\infty, 0, w) = FÜF(0)\, \overset{1}{w(\infty)} = \frac{10}{4} = 2{,}5$ //

iii) $w = \sigma$, $z = k.\sigma$

Störungsübertragungsfunktion: $SÜF = \dfrac{Z_1 N_2}{\underbrace{Z_1 Z_2 + N_1 N_2}_{\hookrightarrow CLCP}}$

$SÜF = \dfrac{1}{s^2+5s+4}$ → BIBO-stabil! Endwert:

$$\psi(\infty, 0, w+z) = \underbrace{FÜF(0)\, w(\infty) + SÜF(0)\, z(\infty)}_{\text{linearität}} = 2{,}5 + \frac{k}{4}$$

Je größer die Störung, desto größer der Abweichung

iv) P-Regler hat Stabilität mitgebracht, aber keine stationäre Genauigkeit.

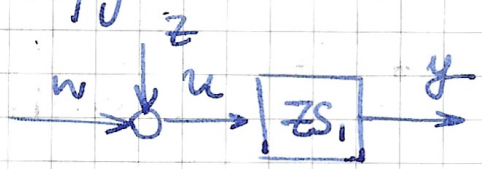
$$e(\infty) = w(\infty) - \psi(\infty, 0, w+z) = 1 - (2{,}5 + k/4) = -1{,}5 - \frac{k}{4} \;//$$

Auch wenn keine Störung vorliegt, gibt es keine stationäre Genauigkeit.

$$e(\infty) = w(\infty) - \psi(\infty, 0, w) = -1{,}5 \;//$$

Wir könnten sofort beantworten, dass der Regelkreis keine stationäre Genauigkeit hat, weil der Regler keine Nullstelle $= 0$ hat (Keinen Integrator).

9.2 Aufgabe: $A_1 = -1$, $B_1 = 1$, $C_1 = 1$, $D_1 = 0$

i) 


$w = \sigma$, $z = 0$, $x_1(0) = 0$

Eigenwert von $A_1 = -1$ → asymp stabil → BIBO-stabil

$$H_1(s) = C_1 (s\,id - A_1)^{-1} B_1 + \cancel{D_1}^{\,0} = \frac{1}{s+1} \;//$$

Endwert: $\psi(\infty, 0, w + \cancel{z}^{\,0}) = \psi(\infty, 0, w) = H_1(0)\, \overset{1}{w(\infty)} = 1 \;//$

ii) Anstiegszeit $t_r$ für $90\%\ w(\infty)$.

Da es sich um $PT_1$ handelt, bleibt die Regelgröße innerhalb des Wertes $0{,}9\, w(\infty)$, sobald dieser erreicht ist.

$$\psi(t,0,\cancel{w}^{0}+z) = \psi(t,0,w) = \psi(t,0,v) = (g_1 * v)(t)$$

Impulsantwort: $g_1(t) = C_1 \exp(A_1 t) B_1 v(t) + \cancel{D_1 \cdot \delta(t)}^{\,0}$

$$g_1(t) = 1 \cdot \exp(-t) \cdot 1 = e^{-t}$$

$$\psi(t,0,v) = v(t) \int_0^t e^{-\tau} d\tau = v(t)\,(1 - e^{-t})$$

$$\psi(t_r,0,v) = 1 - e^{-t_r} = 0{,}9 \cdot w(\infty) = 0{,}9$$

$$1 - e^{-t_r} = 0{,}9 \rightarrow e^{-t_r} = 0{,}1 \rightarrow \ln(e^{-t_r}) = \ln(0{,}1) \rightarrow t_r \cong 2{,}3\,\text{sek}$$

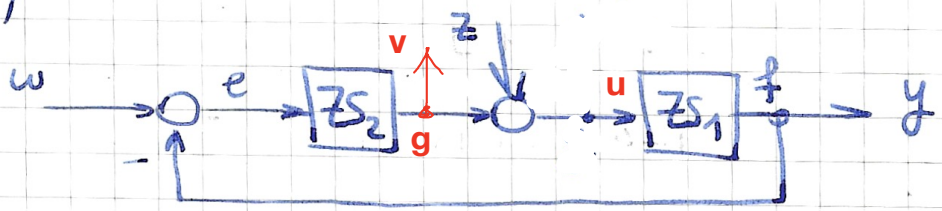
iii) $w = v$, $z = k v$

$$\psi(\infty,0,w+z) = H_1(0)\, \overset{=1}{w(\infty)} + H_1(0)\, \overset{=k}{z(\infty)} = 1 + k.$$

$$e(\infty) = w(\infty) - y(\infty) = 1 - (1+k) = -k$$

Bei einer konstanten Störung geht der Fehler nicht gegen null. Es gibt keine stationäre Genauigkeit.

iv)




$A_2 = 0$ $B_2 = 1$
$C_2 = 1$ $D_2 = 4$

$x_2(0) = 0$ PI-Regler

$w = v$, $z = 0$: $H_2(s) = C_2 (s\,id - A_2)^{-1} B_2 + D_2 = \frac{4}{s} + 4 = \frac{4+4s}{s}$

$$\text{FÜF} = \frac{Z_1 Z_2}{Z_1 Z_2 + N_1 N_2} = \frac{4+4s}{4+4s+s(s+1)} = \frac{4+4s}{s^2+5s+4}$$

Pole: $p_1 = -1$, $p_2 = -4$
BIBO-stabil!

$$\text{FÜF} = \frac{4(1+s)}{(1+s)(4+s)} = \frac{4}{(s+4)}$$

Endwert: $\psi(\infty,0,w+\cancel{z}^{0}) = \psi(\infty,0,w) = \text{FÜF}(0) \cdot \overset{=1}{w(\infty)} = 1$

v) Anstiegszeit $t_r$ für $90\%\, w(\infty)$

$$\psi(t,0,w+\cancel{z}^{0}) = \psi(t,0,w) = \psi(t,0,v) = (g_F * v)(t)$$

$g_F(t) = C_F \exp(A_F t) B_F + D_F$ $\left\{ \begin{array}{l} A_F = -4,\ B_F = 1 \\ C_F = 4,\ D_F = 0 \end{array} \right.$ Regelungsnormalform FÜF

$$g_F(t) = 4 e^{-4t} \cdot 1 = 4 e^{-4t}$$

$$\psi(t,0,v) = v(t)\int_0^t 4e^{-4\tau}\,d\tau = \frac{v(t)\cdot 4}{4}\left(1-e^{-4t}\right) = v(t)\left(1-e^{-4t}\right)$$

$$\psi(t_r,0,w) = 1-e^{-4t_r} = 0{,}9\,w(\infty) = 0{,}9$$

$$\ln(e^{-4t_r}) = \ln(0{,}1) \Rightarrow -4t_r \cong -2{,}3 \rightarrow t_r \cong 0{,}58\ \text{Sek}$$

$\Rightarrow$ 4-Mal schneller mit dem Regler!

vi) $w = v$, $z = kv$

$$\text{SÜF} = \frac{Z_1 N_2}{Z_1 Z_2 + N_1 N_2} = \frac{s}{(s+1)(s+4)} = \frac{s}{s^2+5s+4}$$

$$g_s(t) = C_s \exp(A_s t) B_s + D_s \qquad A_s = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix},\ B_s = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\exp(A_s t) \rightarrow \mathrm{Ew}(A_s) \Rightarrow \begin{matrix} \lambda_1 = -1 \\ \lambda_2 = -4 \end{matrix} \qquad \underbrace{C_s = [0\ \ 1],\ D_s = 0}_{\text{Regelungsnormalform SÜF}}$$

$\hookrightarrow$ zwei verschiedene Eigenwerte:

nicht notwendig!

Diagonalform $A_s = T\Lambda T^{-1}$

$$\lambda_1 = -1,\quad [A_s - \lambda_1 id]v_1 = 0 \quad \begin{bmatrix} 1 & 1 \\ -4 & 4 \end{bmatrix} v_1 = 0 \quad v_1 = [1\ \ -1]^T$$

$$[A_s - \lambda_2 id]v_2 = 0 \quad \begin{bmatrix} 4 & 1 \\ -4 & -1 \end{bmatrix} v_2 = 0 \quad v_2 = [1\ \ -4]^T$$

$$A_s = \underbrace{\begin{bmatrix} 1 & 1 \\ -1 & -4 \end{bmatrix}}_{T} \cdot \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -4 \end{bmatrix}}_{\Lambda} \begin{bmatrix} -4 & -1 \\ 1 & 1 \end{bmatrix} \cdot \frac{1}{(-3)}$$

$$\exp(A_s t) = \sum_{n=0}^{\infty} \frac{A_s^n \cdot t^n}{n!} = \sum_{n=0}^{\infty} \frac{(T\Lambda T^{-1})^n t^n}{n!} = \sum_{n=0}^{\infty} \frac{T\Lambda^n T^{-1} \cdot t^n}{n!} =$$

$$\exp(A_s t) = T\left(\sum_{n=0}^{\infty} \frac{\Lambda^n t^n}{n!}\right)T^{-1} = T\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-4t} \end{pmatrix}T^{-1}$$

$$g_s(t) = [0\ \ 1]\begin{bmatrix} 1 & 1 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-4t} \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 1 & 1 \end{bmatrix}\frac{1}{(-3)}\cdot\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$g_s(t) = -\frac{1}{3}[1\ \ -4]\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-4t} \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = [1\ \ 4]\begin{bmatrix} -e^{-t} \\ e^{-4t} \end{bmatrix}$$

$$g_s(t) = (4e^{-4t} - e^{-t})\frac{1}{3}$$

Einfach, $\psi(\infty, 0, w+z) = FÜF(0)\, \underset{\overset{\shortparallel}{1}}{w(\infty)} + SÜF(0) \cdot \underset{=k}{z(\infty)}$

Endwert: $\psi(\infty, 0, w+z) = 1 \cdot 1 + 0 \cdot k = 1$ //

vii) Wir haben Schnelligkeit, die Regelstrecke erreicht mit dem Regler schneller den gewünschten Wert (Führungsgröße) und hat jetzt stationäre Genauigkeit.