6. Übung   Victor Cheidde Chaim

6.1 Aufgabe $A = \begin{pmatrix} 1 & \alpha \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} \beta \\ 2 \end{pmatrix}$, $C = (\gamma \;\; 1)$, $D = \alpha$

* Ist das ZS asympt. stabil?

Eigenwerte von A: $(1-\lambda)(1-\lambda) = 0 \;\; < \begin{matrix} \lambda_1 = 1 \\ \lambda_2 = -1 \end{matrix}$

$Re(\lambda_1) > 0 \rightarrow$ nicht stabil!

* Satz 7.9 (5): BIBO-Stabil $\Leftrightarrow Re(p) < 0$

$H(s) = C(s\,id - A)^{-1} \cdot B + D$ $\rightarrow$ D spielt keine Rolle bei BIBO-Stabilität.

wir setzen $D = 0$

$H^*(s) = C(s\,id - A)^{-1} \cdot B$

$$(s\,id - A) = \begin{bmatrix} s-1 & -\alpha \\ 0 & s+1 \end{bmatrix}$$

$$(s\,id - A)^{-1} = \frac{1}{\det(s\,id - A)} \begin{bmatrix} s+1 & \alpha \\ 0 & s-1 \end{bmatrix} = \frac{1}{(s+1)(s-1)} \begin{bmatrix} s+1 & \alpha \\ 0 & s-1 \end{bmatrix}$$

$$H^*(s) = C(s\,id - A)^{-1} \cdot B = \frac{1}{(s+1)(s-1)} \cdot [\gamma \;\; 1] \begin{bmatrix} s+1 & \alpha \\ 0 & s-1 \end{bmatrix} \begin{bmatrix} \beta \\ 2 \end{bmatrix}$$

$$H^*(s) = [\gamma \;\; 1] \begin{bmatrix} \beta(s+1) + 2\alpha \\ 2(s-1) \end{bmatrix} \cdot \frac{1}{(s+1)(s-1)}$$

$$H^*(s) = \frac{\gamma\beta(s+1) + 2\alpha\gamma + 2(s-1)}{(s-1)(s+1)}$$

Wir müssen den instabilen Pol 1 eliminieren:

Zähler $= 0$ oder Zähler $= k(s-1)$

1) Zähler $= k(s-1)$: $\rightarrow \boxed{\gamma = 0}$ $\rightarrow H^*(s) = \frac{2}{(s+1)}$

$\downarrow$

BIBO-stabil.

oder $s(2+\gamma\beta) + (\gamma\beta + 2\gamma\alpha - 2) = k(s-1)$

$$\begin{cases} 2+\gamma\beta = k \\ \gamma\beta + 2\gamma\alpha - 2 = -k \end{cases} \quad \begin{matrix} \gamma\beta = -2+k \\ \gamma\alpha = -k+2 \end{matrix} \rightarrow \boxed{\alpha = -\beta}$$

2) Zähler $= 0$: $\begin{cases} 2+\gamma\beta = 0 \\ \gamma\beta + 2\gamma\alpha - 2 = 0 \end{cases} \quad \begin{matrix} \gamma\beta = -2 \\ \gamma\alpha = 2 \end{matrix} \rightarrow \boxed{\alpha = -\beta}$

BIBO stabil für $\alpha = -\beta$ oder $\gamma = 0$ oder ($\gamma\beta = -2+k$ und $\gamma\alpha = -k+2$, $k \in \mathbb{R}$).

$\rightarrow$ Überprüfung:

- $\alpha = -\beta$: $H^*(s) = \dfrac{\gamma\beta(s+1) - 2\gamma\beta + 2(s-1)}{(s-1)(s+1)} = \dfrac{(2+\gamma\beta)(s-1)}{(s-1)(s+1)}$

$$H^*(s) = \frac{(2+\gamma\beta)}{(s+1)} \Rightarrow \text{BIBO-stabil} \quad p = -1 < 0 .$$

- $\gamma = 0$: $H^*(s) = \dfrac{2(s-1)}{(s-1)(s+1)} = \dfrac{2}{(s+1)}$ BIBO-stabil

- $\gamma\beta = -2+k$ und $\gamma\alpha = -k+2$:

$$H^*(s) = \frac{(-2+k)(s+1) + 2(2-k) + 2(s-1)}{(s+1)(s-1)} =$$

$$= \frac{-2s + ks \cancel{-2} + k + \cancel{4} - 2k + 2s \cancel{-2}}{(s+1)(s-1)} =$$

$$= \frac{s(\cancel{-2}+k\cancel{+2}) - k}{(s+1)(s-1)} = \frac{k\cancel{(s-1)}}{\cancel{(s-1)}(s+1)} = \frac{k}{s+1} \quad \text{BIBO stabil}$$

6.2 Aufgabe $A = -2$, $B = 1$, $C = 1$, $D = 1$

i) $\phi(t) = \exp(At) = \displaystyle\sum_{n=0}^{\infty} \frac{(-2t)^n}{n!} = e^{-2t}$ //

ii) $H(s) = C(s\,id - A)^{-1} \cdot B + D = 1 \cdot (s-(-2))^{-1} \cdot 1 + 1 =$

$$= \frac{1}{s+2} + 1 = \frac{1+s+2}{s+2} = \frac{s+3}{s+2} \; //$$

iii) $g(t) = \sigma(t) \cdot C \cdot \exp(At)\, B + \delta(t) \cdot D$

$$g(t) = \sigma(t) \cdot 1 \cdot e^{-2t} \cdot 1 + \delta(t) \cdot 1 = \sigma(t)\, e^{-2t} + \delta(t)$$

iv) Sprungantwort.

$$\psi(t,0,\sigma) = (g * \sigma)(t) = \sigma(t) \int_0^t e^{-2\tau}\, d\tau + \sigma(t) \overbrace{\int_0^t \delta(\tau)\, d\tau}^{1}$$

$$\psi(t,0,\sigma) = \sigma(t) \left. \frac{e^{-2\tau}}{-2} \right|_0^t + \sigma(t) =$$

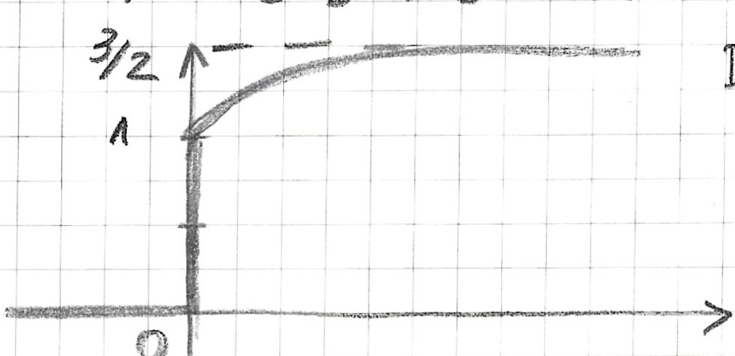
$$= \sigma(t) \left(1 + \frac{1}{2} - \frac{e^{-2t}}{2} \right) = \sigma(t) \cdot \left(\frac{3}{2} - \frac{e^{-2t}}{2} \right)$$

Anfangswert: $\psi(0,0,\sigma) = 1 \left(\frac{3}{2} - \frac{1}{2} \right) = 1$ //

(oder $s \to \infty$ $\quad H(\infty) = \lim_{s \to \infty} \frac{s+3}{s+2} = 1$)

Endwert: $\psi(\infty,0,\sigma) = \lim_{t \to \infty} \sigma(t) \left(\frac{3}{2} - \frac{e^{-2t}}{2} \right) = \frac{3}{2}$

(oder $s \to 0$ $\quad H(0) = 3/2$)



$D_1 \psi(0,0,\sigma) = \frac{-2}{-2} \cancelto{1}{e^{-2 \cdot 0}} = 1$ //

$D_1 \psi(0,0,\sigma) > 0$.

6.3 **Aufgabe** $A = \begin{pmatrix} -3 & 0 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $C = (1 \;\; 0)$, $D = 0$

i) $H(s) = C(s\,id - A)^{-1}B + \cancel{D}^{\,0}$

$(s\,id - A) = \begin{bmatrix} s+3 & 0 \\ 0 & s+1 \end{bmatrix}$, $(s\,id - A)^{-1} = \frac{1}{(s+3)(s+1)} \begin{bmatrix} s+1 & 0 \\ 0 & s+3 \end{bmatrix}$

$H(s) = \frac{1}{(s+3)(s+1)} [1 \;\; 0] \begin{bmatrix} s+1 & 0 \\ 0 & s+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{(s+3)}$ //

ii) $u(t) = \cos(3t)$ $\quad u(t) = Re(e^{3it})$, $\tilde{u} = e^{3it}$

$y(t) = Re(\tilde{y}(t))$, $\tilde{y}(t) = H(\lambda)\,\tilde{u}(t)$

- Eigenwerte von A : $-1, -3 \neq 3i$ $\quad$ 1. Bedingung erfüllt ✓

$\tilde{y}(t) = H(3i)\,\tilde{u}(t) \Rightarrow H(3i) = \frac{1}{3i+3} = \frac{3i-3}{9i^2-9} = \frac{-1}{6}(i-1)$

$H(3i) = -\frac{1}{6}(e^{\frac{\pi}{2}i} - 1)$ $\quad (e^{ki} = \cos(k) + i\sin(k))$.

$y(t) = Re(\tilde{y}(t)) = Re\left(\frac{-1}{6}(e^{\frac{\pi}{2}i} - 1) \cdot e^{3it}\right) = Re\left(-\frac{1}{6}(e^{(\pi/2 + 3t)i} - e^{3it})\right)$

$y(t) = \frac{1}{6}(\cos(3t) - \cos(3t + \pi/2))$ //