2. Übung

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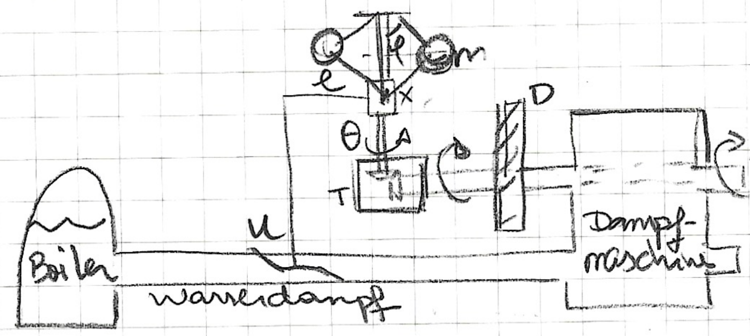
Aufgabe 2.1: $M_r = -b\dot{\varphi}(t)$ $\quad \theta(t) = n\Omega(t)$

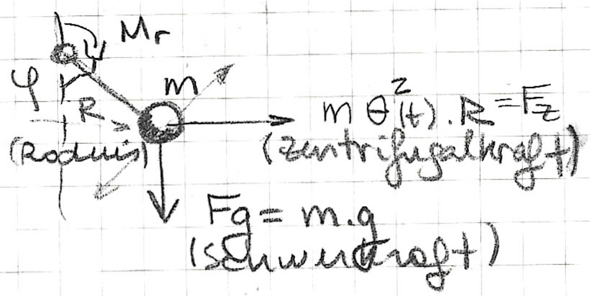
$u(t) = k(x(t) - x_{ref})$

1) Modellierungsziel: Zusammenhang zw. $\varphi(t)$ und $\theta(t)$ $(\Omega(t))$.

2) Blöcke: Masse, Trägheitsmoment, Dämpfer, Getriebeverhältnis

Verbindende Phänomene:





F_g, F_z, M_r, I_D, u

3)a) Verhalten der Blöcke:

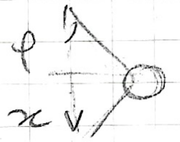
Masse: $M_I \overset{(I)}{=} I_m . \ddot{\varphi}(t)$, $M_{Fg} \overset{(II)}{=} \sin(\varphi(t)) . l . mg$, $M_{Fz} \overset{(III)}{=} R m \theta^2(t) \cos(\varphi) l$

Dämpfer: $M_r = -b\dot{\varphi}(t)$ (IV)

Getriebeverhältnis: $\theta(t) = n\Omega(t)$ (V)

Trägheitsmoment: $M_D = u = k(x(t) - x_{ref})$ (VI)

3)b) Verbindungen der Blöcke:



$M_I = M_{Fz} - M_{Fg} - M_r$ (VII)

$M_D = k(x(t) - x_{ref}) = k(2 . l . \cos(\varphi(t)) - x_{ref})$ (VIII)

3)c) Vereinfachung und Elimination:

(VII) $M_I = I_m \ddot{\varphi} = R m \theta^2 \cos\varphi . l - \sin\varphi . l . m g - b\dot{\varphi}$

$R = l . \sin\varphi$ und $I_m = m l^2$

$m l^2 \ddot{\varphi} = l^2 m \theta^2 \cos\varphi \sin\varphi - \sin\varphi\, l m g - b\dot{\varphi}$

(IX): $\ddot{\varphi} = \theta^2 \cos\varphi \sin\varphi - \sin\varphi \frac{g}{l} - b^* \dot{\varphi}$, $\quad b^* = b/ml^2$

(VIII): $I_D . \dot{\Omega} = k(2 l \cos\varphi - x_{ref})$

(V) $\rightarrow$ (VIII): $\frac{\dot{\theta}}{n} I_D = k(2l\cos\varphi - x_{ref})$

$$\dot{\theta} = \frac{k \cdot n \cdot 2 \cdot (l\cos\varphi)}{I_D} - \frac{x_{ref} \cdot k \cdot n}{I_D} = k^* \cdot \cos\varphi - k_1$$

wobei: $k^* = \frac{k \cdot n \cdot 2 \cdot l}{I_D}$, $k_1 = \frac{x_{ref} \cdot k \cdot n}{I_D}$

$$x = [\varphi \quad \dot{\varphi} \quad \theta]^T$$

$$\dot{x} = \begin{pmatrix} \dot{\varphi} \\ \theta^2\cos\varphi\sin\varphi - \sin\varphi\frac{g}{l} - b^*\dot{\varphi} \\ k^*\cos\varphi - k_1 \end{pmatrix}$$

3e) Linearisierung:

Ruhelage: $\dot{\varphi} = 0$, $\theta = \theta_0$, $\varphi = \varphi_0$

$\dot{\theta} = 0$, $\ddot{\varphi} = 0$

$\rightarrow k^*\cos\varphi_0 = k_1$, $\cos\varphi_0 = \frac{k_1}{k^*}$ //

$\rightarrow \theta_0^2\cos\varphi_0 \cancel{\sin\varphi_0} - \cancel{\sin\varphi_0} \cdot \frac{g}{l} - \cancel{b^*\dot{\varphi}_0} = 0$

$$\theta_0^2 = \frac{g}{l} \cdot \frac{1}{\cos(\varphi_0)} = \frac{g}{l} \cdot \frac{k^*}{k_1} \text{ //} \quad (\cos\varphi_0 \neq 0)$$

$$A = D_1 f(x_0, u_0)$$

$A_{11} = \frac{\partial f_1}{\partial \varphi} = 0$, $A_{12} = \frac{\partial f_1}{\partial \dot{\varphi}} = 1$, $A_{13} = \frac{\partial f_1}{\partial \theta} = 0$

$$A_{21} = \frac{\partial f_2}{\partial \varphi} = \theta_0^2(\underbrace{\cos^2\varphi_0 - \sin^2\varphi_0}_{=\cos 2\varphi_0}) - \cos\varphi_0 \frac{g}{l}$$

$A_{22} = \frac{\partial f_2}{\partial \dot{\varphi}} = -b^*$, $A_{23} = \frac{\partial f_2}{\partial \theta} = \theta_0 \overbrace{2\cos\varphi_0\sin\varphi_0}^{=\sin 2\varphi_0}$

$A_{31} = \frac{\partial f_3}{\partial \varphi} = -k^*\sin\varphi_0$, $A_{32} = \frac{\partial f_3}{\partial \dot{\varphi}} = 0$, $A_{33} = \frac{\partial f_3}{\partial \theta} = 0$

$\rightarrow \dot{x} = Ax$

$$\Rightarrow \begin{bmatrix} \dot{\varphi} \\ \ddot{\varphi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \theta_0^2\cos 2\varphi_0 - \cos\varphi_0 \cdot \frac{g}{l} & -b^* & \theta_0 \sin 2\varphi_0 \\ -k^*\sin\varphi_0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi \\ \dot{\varphi} \\ \theta \end{bmatrix} \text{ //}$$