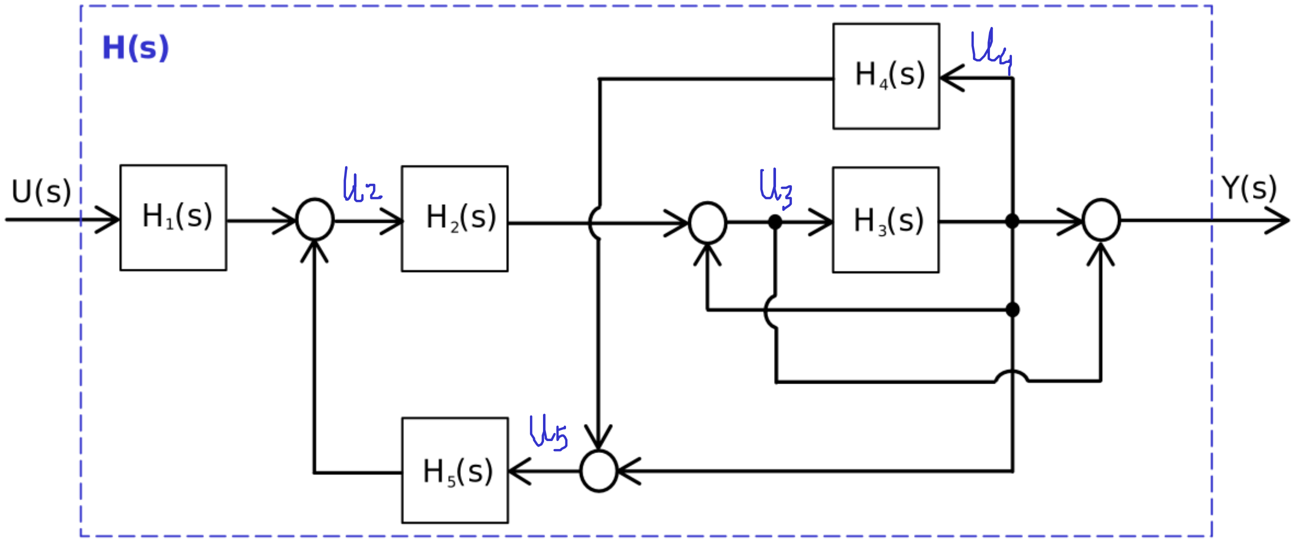
Victor Cheidde Chaim

→ Probeklausuraufgaben SRT

8. Aufgabenblatt

8.2)



$Y(s) = U_3 + H_3 U_3 = U_3(1 + H_3)$ (I)

$U_2 = U_5 H_5 + U H_1$ (II)

$U_3 = H_3 U_3 + U_2 H_2$ (III)

$U_4 = H_3 U_3$ (IV)

$U_5 = H_3 U_3 + H_4 U_4$ (V)

2 Punkte

(II) → (III) : $U_3 = H_3 U_3 + H_2(U_5 H_5 + U H_1)$

$U_3 = H_3 U_3 + H_2 H_1 U + H_2 H_5 U_5$ (VI) 1 Punkt

(IV) → (V) : $U_5 = H_3 U_3 + H_4 H_3 U_3$

$U_5 = H_3 U_3 (1 + H_4)$ (VII) 1 Punkt

$U_3 = H_3 U_3 + H_2 H_1 U + H_2 H_5 (H_3 U_3 (1 + H_4))$

$U_3 = H_3 U_3 + H_2 H_5 H_3 U_3 + H_3 H_4 H_2 H_5 U_3 + H_2 H_1 U = U_3 H_3 (1 + H_2 H_5 + H_4 H_5 H_2) + H_2 H_1 U$

$U_3 (1 - H_3 (1 + H_2 H_5 + H_4 H_5 H_2)) = U H_2 H_1$

$$U_3 = \frac{U H_2 H_1}{(1 - H_3(1 + H_2 H_5 + H_4 H_5 H_2))} \quad \text{(VIII)}$$ 1 Punkt

(VIII) → (I) : $Y(s) = \dfrac{U H_2 H_1 (1 + H_3)}{(1 - H_3(1 + H_2 H_5 + H_4 H_5 H_2))}$

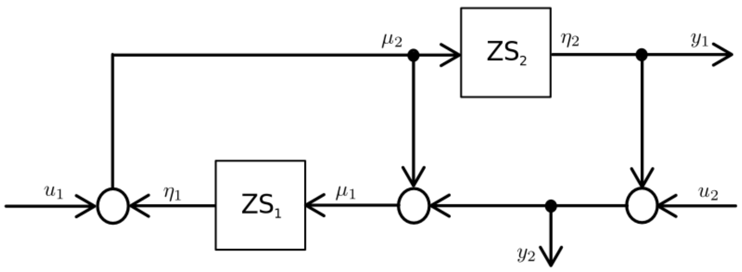
$$\boxed{H(s) = \frac{Y(s)}{U(s)} = \frac{H_2 H_1 (1 + H_3)}{(1 - H_3(1 + H_2 H_5 + H_4 H_5 H_2))}}$$

2 Punkte

8.3) $A_1 = 1 \quad B_1 = 1$
$C_1 = 1 \quad D_1 = 0$

$A_2 = 2 \quad B_2 = 3$
$C_2 = 1 \quad D_2 = 0$

$\bar{A} = \mathrm{diag}(1,2)$
$\bar{B} = \mathrm{diag}(1,3)$
$\bar{C} = \mathrm{diag}(1,1) = id$
$\bar{D} = \mathrm{diag}(0,0)$



Verhalten der Blöcke

$\dot{x}_1 = A_1 x_1 + B_1 \mu_1$
$y = C_1 x_1 + D_1 \mu_1$

$\dot{x}_2 = A_2 x_2 + B_2 \mu_2$
$y = C_2 x_2 + D_2 \mu_2$

Verbindungen der Blöcke:

$$\begin{cases} \mu_1 = u_2 + \eta_2 + \mu_2 = u_2 + \eta_2 + u_1 + \eta_1 \\ \mu_2 = \eta_1 + u_1 \end{cases}$$

1 Punkt

$$\begin{cases} y_1 = \eta_2 \\ y_2 = u_2 + \eta_2 \end{cases}$$

1 Punkt

1 Punkt

$$\mu = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}_{=K} \eta + \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}_{=L} u$$

$$y = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}}_{=E} \eta + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{=F} u$$

• Bedingung: $(id - K\bar{D})$ invertierbar

$(id - K\bar{D}) = id - K\,\mathrm{diag}(0,0) = id$ 1 Punkt

$\det(id) = 1 \neq 0 \Rightarrow$ invertierbar!

$\Rightarrow$ Zusammenfassung: (Ergebnis von Prof. Reißig, Vorlesung 8)

• $A = \bar{A} + \bar{B}(id - K\bar{D})^{-1} K\bar{C} = \bar{A} + \bar{B}\, id^{-1} K \bar{C} = \bar{A} + \bar{B}\, id\, K\bar{C} = \bar{A} + \bar{B} K \overset{=id}{\bar{C}} = \bar{A} + \bar{B}K$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \quad //$$

1 Punkt

• $B = \bar{B}\underbrace{(id - K\bar{D})}_{=id}^{-1} L = \bar{B} \cdot L = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \quad //$ 1 Punkt

• $C = E(id + \cancel{\bar{D}(id - K\bar{D})^{-1}K})\bar{C} = E\, id\, \overset{=id}{\bar{C}} = E = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \quad //$ ($\bar{D} = 0$) 1 Punkt

• $D = F + \cancel{E\bar{D}(id - K\bar{D})^{-1} L} = F = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad //$ ($\bar{D} = 0$) 1 Punkt

8.4) Verbindungen und Verhalten der Blöcke:

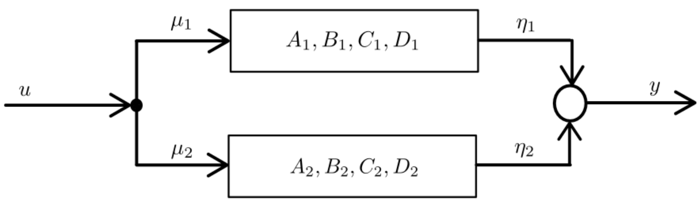
$\mu_1 = u$
1 Punkt $\mu_2 = u$

$y = \eta_1 + \eta_2$
1 Punkt

$\dot{x}_1 = A_1 x_1 + B_1 \mu_1$
$\eta_1 = C_1 x_1 + D_1 \mu_1$

$\dot{x}_2 = A_2 x_2 + B_2 \mu_2$
$\eta_2 = C_2 x_2 + D_2 \mu_2$

1 Punkt



$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}}_{=A} x + \underbrace{\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}}_{=B} u \;,$$

$$y = [C_1 \;\; 0]\, x + D_1 \overset{=u}{\mu_1} + [0 \;\; C_2]\, x + D_2 \overset{=u}{\mu_2}$$

$$y = [C_1 \;\; C_2]\, x + (D_1 + D_2) u$$

2 Punkte

$H_1(s) = C_1(s\,id - A_1)^{-1} B_1 + D_1, \quad H_2(s) = C_2(s\,id - A_2)^{-1} B_2 + D_2$

$\Rightarrow H(s) = C(s\,id - A)^{-1} B + D =$

$$= [C_1 \;\; C_2]\begin{bmatrix} (s\,id - A_1)^{-1} & 0 \\ 0 & (s\,id - A_2)^{-1} \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + (D_1 + D_2)$$

$$= \big(\underbrace{C_1(s\,id - A_1)^{-1} B_1 + D_1}_{=H_1}\big) + \big(\underbrace{C_2(s\,id - A_2)^{-1} B_2 + D_2}_{=H_2}\big)$$

$$= H_1(s) + H_2(s) \quad //$$

2 Punkte