

Vector chain

→ Probeklausuraufgaben WT - SRT 4. Aufgabenblatt

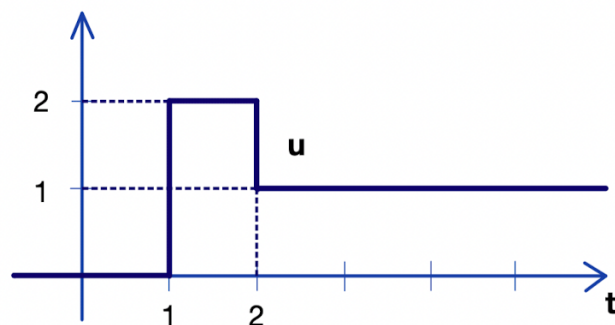
4.2) $\dot{\Phi}(At) = A \cdot \exp(At)$ 1 Punkt

$\dot{\Phi}(0) = A \cdot \exp(0) = A \cdot \text{id} = A //$

4.3) X nilpotent. 1 Punkt

4.4)
$$\begin{cases} \dot{x} = -zx + u \\ y = x \end{cases}$$

 $x(0) = 0$



Impulsantwort: $g(t) = \nabla(t) C e^{At} B$, $\nabla(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{sonst} \end{cases}$

$C = 1, B = 1, A = -z \rightarrow g(t) = \nabla(t) e^{-zt} //$ 1 Punkt

$$y(t) = \int_{-\infty}^{\infty} g(z) u(t-z) dz = \int_0^t g(z) u(t-z) dz$$

3 Fällen: $0 < t < 1, 1 < t < 2, 2 < t$:

$u(t) = z \nabla(t-1) - \nabla(t-2)$ 1 Punkt

• Linearität: $y(t) = \psi(t, 0, u) = z \psi(t, 0, u_1) - \psi(t, 0, u_2)$

• Kausalität: $\psi(t, 0, u_1) = \psi(t-1, 0, \nabla) = (g * \nabla)(t-1)$
 Zeitinvariant $\psi(t, 0, u_2) = \psi(t-2, 0, \nabla) = (g * \nabla)(t-2)$

$$y(t) = z(g * \nabla)(t-1) - (g * \nabla)(t-2)$$

$$(g * v) = \int_{-\infty}^{\infty} g(z) v(t-z) dz = \int_{-\infty}^{\infty} \underbrace{v(z)}_{=0 \text{ für } z < 0} e^{-zz} \underbrace{v(t-z)}_{=0 \text{ für } z > t} dz =$$

$$= v(t) \int_0^t e^{-zz} dz = v(t) \left(-e^{-z} / z \right) \Big|_0^t = \frac{1}{z} v(t) (1 - e^{-zt})$$

1 Punkt

$$\Rightarrow y(t) = v(t-1) (1 - e^{-z(t-1)}) - \frac{1}{z} v(t-z) (1 - e^{-z(t-z)}) //$$

2 Punkte

4.5) $A = \begin{pmatrix} z & 0 & 0 \\ -1 & z & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Hauptfundamentalmatrix:

$\phi(t) = \exp(At)$

1 Punkt

$$\exp(At) \Rightarrow A = \text{diag} \left(\underbrace{\begin{pmatrix} z & 0 \\ -1 & z \end{pmatrix}}_{A_1}, \underbrace{1}_{A_2} \right) \Rightarrow e^{At} = \text{diag} \left(e^{A_1 t}, e^{A_2 t} \right)$$

1 Punkt

$$A_1 = z \cdot \text{id} + \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \Rightarrow e^{A_1 t} = e^{zt} \exp \left(t \underbrace{\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}}_{\text{mit } p} \right) = e^{zt} (\text{id} + t \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix})$$

$$e^{A_1 t} = e^{zt} \begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix} //$$

2 Punkte

$$\exp(At) = \text{diag} \left(e^{zt} \begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix}, e^t \right) = \begin{pmatrix} e^{zt} & 0 & 0 \\ -t e^{zt} & e^{zt} & 0 \\ 0 & 0 & e^t \end{pmatrix} //$$