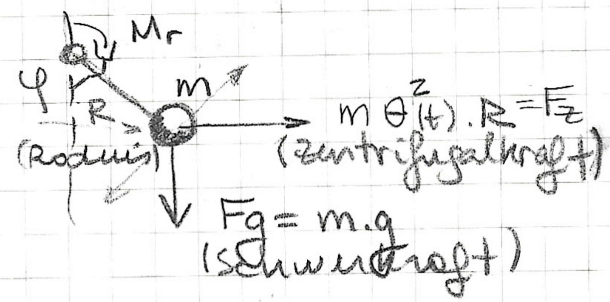
2. Übung Victor C. Chaim

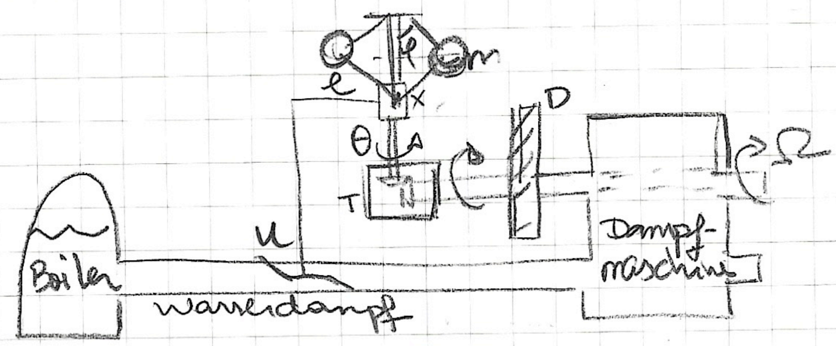
Aufgabe 2.1 : $M_r = -b\dot{\varphi}(t)$ $\theta(t) = n\Omega(t)$
$u(t) = k(x(t) - x_{ref})$

1) Modellierungsziel: Zusammenhang zw. $\varphi(t)$ und $\theta(t)$ $(\Omega(t))$.

2) Blöcke: Masse, Trägheitsmoment, Dämpfer, Getriebeverhältnis



Verbindende Phänomene:

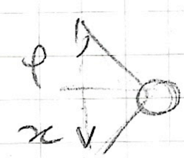


F_g, F_z, M_r, I_D, u

3) a) Verhalten der Blöcke:

Masse: $M_I \overset{(I)}{=} I_m.\ddot{\varphi}(t)$, $M_{Fg} \overset{(II)}{=} \sin(\varphi(t)).\,l.\,mg$, $M_{Fz} \overset{(III)}{=} R m \theta^2(t) \cos\varphi\, l$
Dämpfer: $M_r = -b\dot{\varphi}(t)$ (IV)
Getriebeverhältnis: $\theta(t) = n\Omega(t)$ (V)
Trägheitsmoment: $M_D = u = k(x(t) - x_{ref})$ (VI)

3) b) Verbindungen der Blöcke:



$M_I = M_{Fz} - M_{Fg} - M_r$ (VII)
$M_D = k(x(t) - x_{ref}) = k(2.l.\cos(\varphi(t)) - x_{ref})$ (VIII)

3) c) Vereinfachung und Elimination:

(VII) $M_I = I_m\,\ddot{\varphi} = R\,m\,\theta^2 \cos\varphi\,.\,l - \sin\varphi\; l.m\,g - b\dot{\varphi}$

$R = l.\sin\varphi$ und $I_m = m l^2$

$m l^2 \ddot{\varphi} = l^2 m\, \theta^2 \cos\varphi \sin\varphi - \sin\varphi\, l m g - b\dot{\varphi}$

(IX): $\ddot{\varphi} = \theta^2 \cos\varphi \sin\varphi - \sin\varphi\, \frac{g}{l} - b^* \dot{\varphi}$, $b^* = b/ml^2$

(VIII): $I_D.\dot{\Omega} = k(2l\cos\varphi - x_{ref})$

$(\mathrm{V}) \to (\mathrm{VIII})$: $\frac{\dot{\theta}}{n} I_D = k(2l\cos\varphi - x_{ref})$

$$\dot{\theta} = \frac{k \cdot n \cdot 2 \cdot (l\cos\varphi)}{I_D} - \frac{x_{ref} \cdot k\, n}{I_D} = k^* \cdot \cos\varphi - k_1$$

wobei: $k^* = \frac{k \cdot n \cdot 2 \cdot l}{I_D}$, $k_1 = \frac{x_{ref} \cdot k \cdot n}{I_D}$

$$x = [\varphi \quad \dot{\varphi} \quad \theta]^T$$

$$\dot{x} = \begin{pmatrix} \dot{\varphi} \\ \theta^2 \cos\varphi \sin\varphi - \sin\varphi \frac{g}{l} - b^* \dot{\varphi} \\ k^* \cos\varphi - k_1 \end{pmatrix}$$

3e) Linearisierung:

Ruhelage: $\dot{\varphi} = 0$, $\theta = \theta_0$, $\varphi = \varphi_0$

$\dot{\theta} = 0$, $\ddot{\varphi} = 0$

$\to k^* \cos\varphi_0 = k_1$, $\cos\varphi_0 = \frac{k_1}{k^*}$ //

$\to \theta_0^2 \cos\varphi_0 \cancel{\sin\varphi_0} - \cancel{\sin\varphi_0} \cdot \frac{g}{l} - \cancel{b^* \dot{\varphi}_0} = 0$

$$\theta_0^2 = \frac{g}{l} \frac{1}{\cos(\varphi_0)} = \frac{g}{l} \cdot \frac{k^*}{k_1} \text{ // } \quad (\cos\varphi_0 \neq 0)$$

$A = D_1 f(x_0, u_0)$

$A_{11} = \frac{\partial f_1}{\partial \varphi} = 0$, $A_{12} = \frac{\partial f_1}{\partial \dot{\varphi}} = 1$, $A_{13} = \frac{\partial f_1}{\partial \theta} = 0$

$A_{21} = \frac{\partial f_2}{\partial \varphi} = \theta_0^2 \underbrace{(\cos^2\varphi_0 - \sin^2\varphi_0)}_{= \cos 2\varphi_0} - \cos\varphi_0 \frac{g}{l}$

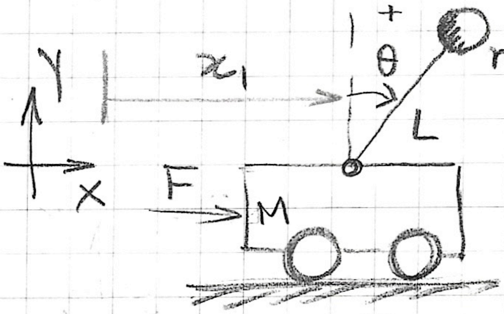
$A_{22} = \frac{\partial f_2}{\partial \dot{\varphi}} = -b^*$, $A_{23} = \frac{\partial f_2}{\partial \theta} = \theta_0 \overbrace{2\cos\varphi_0 \sin\varphi_0}^{= \sin 2\varphi_0}$

$A_{31} = \frac{\partial f_3}{\partial \varphi} = -k^* \sin\varphi_0$, $A_{32} = \frac{\partial f_3}{\partial \dot{\varphi}} = 0$, $A_{33} = \frac{\partial f_3}{\partial \theta} = 0$

$\to \dot{x} = Ax$

$$\Rightarrow \begin{bmatrix} \dot{\varphi} \\ \ddot{\varphi} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \theta_0^2 \cos 2\varphi_0 - \cos\varphi_0 \frac{g}{l} & -b^* & \theta_0 \sin 2\varphi_0 \\ -k^* \sin\varphi_0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varphi \\ \dot{\varphi} \\ \theta \end{bmatrix} \text{ //}$$

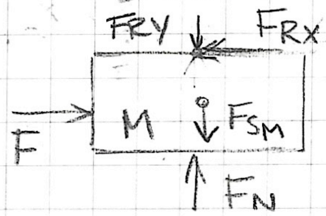
Aufgabe 2.2 $x(t) \to$ Position des Wagens
$F \to$ Eingang



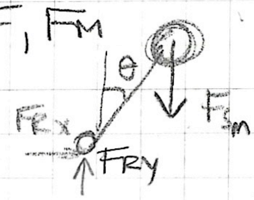
1) Modellierungsziel: Zusammenhang zw. Kraft auf Wagen und $x(t)$, $\theta(t)$

2) Blöcke: 2 Massen

3a)



Verhalten der Blöcke:
Masse M: F_{RX}, F_N, F_{Ry}, F_{S_M}, F, F_M
Masse m: F_{RX}, F_{Ry}, F_{Sm}, F_m



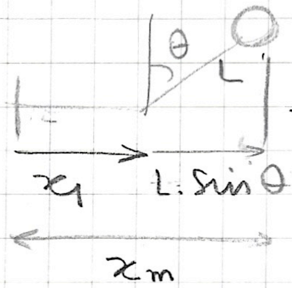
3b) Verbindung der Blöcke:

x: $F_{M_x} = M \cdot \ddot{x}_1 = F - F_{RX}$ (I)

Drehmoment: $F_{RX} \cdot \cos\theta \cdot \cancel{L} = F_{Ry} \sin\theta \, L$ (II)
x: $F_{mx} = m \cdot \ddot{x}_m = F_{RX}$ (III)
y: $F_{my} = m \cdot \ddot{y}_m = F_{Ry} - F_{Sm}$ (IV)

3)c) Vereinfachung und Elimination:

- kinematische Gleichungen:



$y_m = L \cos\theta$

(V) $\dot{x}_m = \dot{x}_1 + L\dot{\theta}\cos\theta$
(VI) $\dot{y}_m = -L\sin\theta \cdot \dot{\theta}$

(VII) $\ddot{x}_m = \ddot{x}_1 + L(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)$
(VIII) $\ddot{y}_m = -L(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$

- (VII) $\to$ (III) $\to$ (I): $M\ddot{x}_1 = F - m\ddot{x}_m = F - m(\ddot{x}_1 + L(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)$

$$\boxed{(M+m)\ddot{x}_1 = F - mL\cos\theta \cdot \ddot{\theta} - mL\dot{\theta}^2\sin\theta \quad (IX)}$$

- (IV) und (III) $\to$ (II): $\cancel{m}\,\ddot{x}_m \cos\theta \cancel{L} = (\cancel{m}\,\ddot{y}_m + \cancel{m}\,g) \cdot \sin\theta \cancel{L}$ (X)

- (VII) und (VIII) $\to$ (X): $(\ddot{x}_1 + L(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta))\cos\theta =$
$= (-L(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) + g)\sin\theta$

$\ddot{x}_1\cos\theta + L\ddot{\theta}\cos^2\theta - \cancel{L\dot{\theta}^2\sin\theta\cos\theta} = g\sin\theta - L\ddot{\theta}\sin^2\theta - \cancel{L\dot{\theta}^2\sin\theta\cos\theta}$

$\ddot{x}_1\cos\theta + L\ddot{\theta}(\underbrace{\cos^2\theta + \sin^2\theta}_{1}) - g\sin\theta = 0$

$\Rightarrow \quad \ddot{x}_1 \cos\theta + L\ddot{\theta} - g\sin\theta = 0 \quad \text{(XI)}$

$$\boxed{\ddot{\theta} = -\frac{\ddot{x}_1 \cos\theta}{L} + \frac{g}{L}\sin\theta} \quad \text{(XII)}$$

(XII) → (IX): $(M+m)\ddot{x}_1 = F - mL\cos\theta\left(\frac{g}{L}\sin\theta - \frac{\ddot{x}_1\cos\theta}{L}\right) - mL\dot{\theta}^2\sin\theta$

$$(M+m-m\cos^2\theta)\ddot{x}_1 = F - m\cos\theta\sin\theta\, g - mL\dot{\theta}^2\sin\theta \quad \text{(XIII)} \;/\!/$$

(XIII) → (IX): $\ddot{\theta} = -\left(\frac{F - mg\cos\theta\sin\theta - mL\dot{\theta}^2\sin\theta}{M+m(1-\cos^2\theta)}\right)\cdot\frac{\cos\theta}{L} + \frac{g}{L}\sin\theta$

$$\ddot{\theta} = -\frac{(F - mg\cos\theta\sin\theta - mL\dot{\theta}^2\sin\theta)}{M+m\sin^2\theta}\cdot\frac{\cos\theta}{L} + \frac{g}{L}\sin\theta$$

$x = [\theta, \dot{\theta}, x_1, \dot{x}_1]^T$, $u = F$

$$\dot{x} = \begin{pmatrix} \dot{\theta} \\ -\frac{(F - mg\cos\theta\sin\theta - mL\dot{\theta}^2\sin\theta)}{M+m\sin^2\theta}\cdot\frac{\cos\theta}{L} + \frac{g}{L}\sin\theta \\ \dot{x}_1 \\ \frac{F - m\cos\theta\sin\theta\, g - mL\dot{\theta}^2\sin\theta}{M+m\sin^2\theta} \end{pmatrix}$$

3e) Linearisierung:

→ Ruhelage: $\dot{x} = 0$, $\dot{\theta}_0 = 0$, $\dot{x}_1 = 0$, $x_1 = x_0$, $u_0 = F_0$

$\dot{x}_4 = 0 \rightarrow \dot{x}_2 = 0 = \frac{g}{L}\sin\theta \;\therefore\; \theta_0 = 0$

$A = D_1 f(x_0, u_0)$

$A_{11} = \frac{\partial f_1}{\partial\theta} = 0$, $A_{12} = \frac{\partial f_1}{\partial\theta} = 1$, $A_{13} = 0$, $A_{14} = 0$

$$A_{21} = mg\left(\frac{\cos^2\theta_0 - \cancel{\sin^2\theta_0} - \cancel{2\sin\theta_0\cos\theta_0}}{(M+m\cancel{\sin^2\theta_0})^2}\right)\cdot\frac{\cos\theta_0}{L} - \frac{mg\cos\theta_0\cancel{\sin\theta_0}}{(M+\cancel{m\sin^2\theta_0})^2}\,\frac{\sin\theta_0}{L} +$$

$$+\left(\frac{\cancel{mL\dot{\theta}^2\cos\theta_0} - \cancel{2\sin\theta_0\cos\theta_0}}{(M+m\sin^2\theta_0)^2}\right)\frac{\cos\theta_0}{L} - \frac{mL\dot{\theta}^2\cancel{\sin\theta_0}}{(M+\cancel{m\sin^2\theta_0})^2\cdot L} + \frac{g}{L}\cos\theta_0$$

$$A_{21} = \frac{mg}{ML} + \frac{g}{L} = \frac{g}{L}\left(\frac{M+m}{M}\right)$$

- $A_{22} = \frac{\partial f_2}{\partial \dot\theta} = \frac{mL\,2.\cancel{\dot\theta_0}\sin\theta_0}{L\,\cancel{M+m\sin^2\theta_0}} \cdot \cos\theta = 0$

- $A_{23} = \frac{\partial f_2}{\partial x_1} = 0 \quad , \quad A_{24} = \frac{\partial f_2}{\partial \dot x_1} = 0$

- $A_{31} = \frac{\partial f_3}{\theta} = 0 \quad , \quad A_{32} = \frac{\partial f_3}{\dot\theta} = 0 \quad , \quad A_{33} = \frac{\partial f_3}{x_1} = 0 \, , \quad A_{34} = \frac{\partial f_3}{\dot x_1} = 1$

- $A_{41} = \frac{\partial f_4}{\theta} = \frac{-m\,(\cos^2\theta_0 - \cancel{\sin^2\theta_0}).g - \cancel{mL\dot\theta^2\cos\theta_0}}{(M + m\sin^2\theta_0)} -$

$- \frac{2\sin\theta_0 \cdot \cos\theta_0\, m \ldots 2}{(M + \cancel{m\sin^2\theta_0})}$

- $A_{41} = \frac{\partial f_4}{\theta}(x_0, u_0) = -\frac{mg}{M}$ //

- $A_{42} = \frac{\partial f_4}{\dot\theta} = \frac{-mL2\cancel{\dot\theta_0\sin\theta_0}}{M+\cancel{m\sin^2\theta_0}} = 0$ //

- $A_{43} = \frac{\partial f_4}{x_1} = 0 \, , \quad A_{44} = \frac{\partial f_4}{\partial \dot x_1} = 0$

$B = D_2 f(x_0, u_0) \Rightarrow \quad B_{11} = \frac{\partial f_1}{\partial u} = 0 , \quad B_{21} = \frac{\partial f_2}{\partial u} = -\frac{1}{\cancel{(M+m\sin^2\theta_0).L}} \cos\theta$

$B_{21} = -\frac{1}{ML}$ // $\quad B_{31} = \frac{\partial f_3}{\partial u} = 0 , \quad B_{41} = \frac{\partial f_4}{\partial u} = \frac{1}{M}$

$$\begin{pmatrix} \dot\theta \\ \ddot\theta \\ \dot x_1 \\ \ddot x_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{g}{L}\frac{(M+m)}{M} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -g\,m/M & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \theta \\ \dot\theta \\ x_1 \\ \dot x_1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1/(ML) \\ 0 \\ 1/M \end{pmatrix} \cdot u$$ //

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