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→ Probeklausuraufgaben | 4. Aufgabenblatt

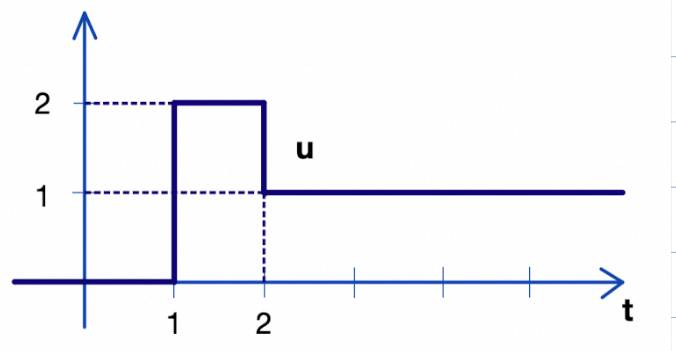
4.2) $\Phi'(At) = A \cdot \exp(At)$ 1 Punkt

$\Phi'(0) = A \cdot \exp(0) = A \cdot id = A$ //

4.3) X nilpotent. 1 Punkt

4.4) $\begin{cases} \dot{x} = -2x + u \\ y = x \end{cases}$

$x(0) = 0$



Impulsantwort: $g(t) = \sigma(t) C e^{At} B$, $\sigma(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{sonst} \end{cases}$

$C = 1$, $B = 1$, $A = -2$ → $g(t) = \sigma(t) e^{-2t}$ // 1 Punkt

$y(t) = \int_{-\infty}^{t} g(\tau) u(t-\tau) d\tau = \int_{0}^{t} g(\tau) u(t-\tau) d\tau$

3 Fällen: $0 < t < 1$, $1 < t < 2$, $2 < t$:

$u(t) = 2\sigma(t-1) - \sigma(t-2)$ 1 Punkt

- Linearität: $y(t) = \psi(t, 0, u) = 2\psi(t, 0, u_1) - \psi(t, 0, u_2)$

- Kausilät, Zeitinvariant : $\psi(t, 0, u_1) = \psi(t-1, 0, \sigma) = (g * \sigma)(t-1)$
  $\psi(t, 0, u_2) = \psi(t-2, 0, \sigma) = (g * \sigma)(t-2)$

$y(t) = 2(g * \sigma)(t-1) - (g * \sigma)(t-2)$

$$(g * v) = \int_{-\infty}^{\infty} g(\tau)\, v(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \underbrace{v(\tau)}_{=0 \text{ für } \tau<0} e^{-2\tau} \underbrace{v(t-\tau)}_{=0 \text{ für } \tau>t} d\tau =$$

$$= v(t) \int_0^t e^{-2\tau} d\tau = v(t) \left(-\frac{e^{-2\tau}}{2} \right) \Big|_0^t = \frac{1}{2} v(t) (1 - e^{-2t})$$

1 Punkt

$$\Rightarrow y(t) = v(t-1)(1 - e^{-2(t-1)}) - \frac{1}{2} v(t-2)(1 - e^{-2(t-2)}) \quad //$$

2 Punkte

4.5) $A = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, Hauptfundamentalmatrix: $\phi(t) = \exp(At)$

1 Punkt

$$\exp(At) \Rightarrow A = \text{diag}\Big(\underbrace{\begin{pmatrix} 2 & 0 \\ -1 & 2 \end{pmatrix}}_{A_1}, \underbrace{1}_{A_2}\Big) \Rightarrow e^{At} = \text{diag}(e^{A_1 t}, e^{1t})$$

1 Punkt

$$A_1 = 2 \cdot id + \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \Rightarrow e^{A_1 t} = e^{2t} \exp\Big(t \underbrace{\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}}_{nilp}\Big) = e^{2t}\Big(id + t\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}\Big)$$

$$e^{A_1 t} = e^{2t} \begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix} \quad //$$

2 Punkte

$$\exp(At) = \text{diag}\left(e^{2t}\begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix}, e^t\right) = \begin{pmatrix} e^{2t} & 0 & 0 \\ -te^{2t} & e^{2t} & 0 \\ 0 & 0 & e^t \end{pmatrix} \quad //$$