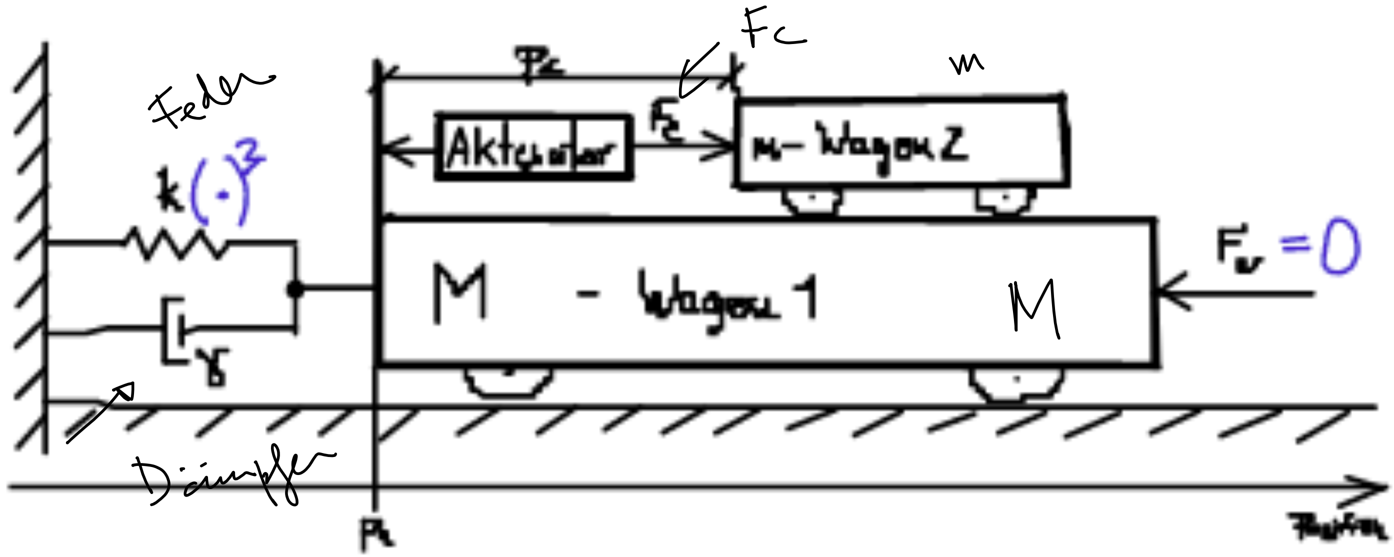
3.3 Beispiel: Masse-Feder-Dämpfung mit Nichtlinearität und zwei Massen



1.) Modellierungsziel: Zusammenhang zw. Kraft auf Wagen 2 und Position von Wagen 1.

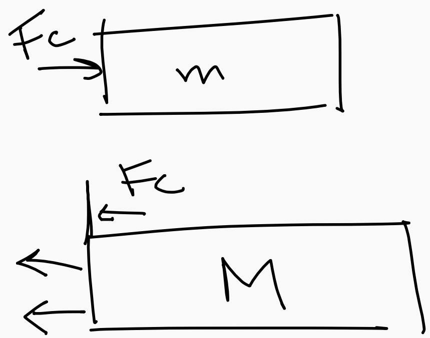
$F_c \rightarrow$ Eingang (u)

$P_1 \rightarrow$ Ausgang (y)

2) Blöcke: 2 Massen, Feder, Dämpfer

3 a-c) Aufstellen von (2): Kräfte auf Wagen 2: $F_{c_1} - m(\ddot{p}_1 + \ddot{p}_2)$

Kräfte auf Wagen 1: $-F_{c_1} - F_{k_1} - F_D ; M\ddot{p}_1$



$$\begin{cases} F_k = k\cdot(p_1)^3 \text{ (nicht linear)} \\ F_D = \gamma \cdot \dot{p}_1 \end{cases}$$

Newton: $M\ddot{p}_1 = -F_c - kp_1^3 - \gamma\dot{p}_1$ Wagen 1

(I) $\ddot{p}_1 = \frac{1}{M}\left(-F_c - kp_1^3 - \gamma\dot{p}_1\right)$

$m(\ddot{p}_1 + \ddot{p}_2) = F_c$ Wagen 2

Newton:

$$\left.\begin{aligned} M\ddot{p}_1 &= -F_c - kp_1^3 - \gamma\dot{p}_1 \\ \text{(I)}\quad \ddot{p}_1 &= \frac{1}{M}\left(-F_c - kp_1^3 - \gamma\dot{p}_1\right) \end{aligned}\right\} \text{Wagen 1}$$

$$\left. m(\ddot{p}_1 + \ddot{p}_2) = F_c \right\} \text{Wagen 2}$$

Zustände:

$$\left\{\begin{aligned} \vec{x} &= [p_1, \dot{p}_1, p_2, \dot{p}_2] \\ u &= F_c \end{aligned}\right.$$

$$m\ddot{p}_2 = F_c - m\ddot{p}_1 = F_c - m\left(\frac{1}{M}\left(-F_c - kp_1^3 - \gamma\dot{p}_1\right)\right)$$

$$\ddot{p}_2 = \frac{F_c}{m} + \frac{1}{M}\left(F_c + kp_1^3 + \gamma\dot{p}_1\right)$$

$$\underline{\ddot{p}_2 = F_c\left(\frac{1}{m} + \frac{1}{M}\right) + \frac{k}{M}p_1^3 + \frac{\gamma}{M}\dot{p}_1} \quad \text{(II)}$$

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$$\vec{x} = [\underset{x_1}{p_1}, \underset{x_2}{\dot{p}_1}, \underset{x_3}{p_2}, \underset{x_4}{\dot{p}_2}], \quad u = F_C$$

$$y = x_1$$

$$(\mathrm{I}) \quad \ddot{p}_1 = \frac{1}{M}\left(-F_C - k p_1^3 - \gamma \dot{p}_1\right)$$

$$\ddot{p}_2 = F_C\left(\frac{1}{m} + \frac{1}{M}\right) + \frac{k}{M} p_1^3 + \frac{\gamma}{M} \dot{p}_1 \quad (\mathrm{II})$$

$$\begin{cases} \dot{\vec{x}} = \begin{bmatrix} \dot{p}_1 \\ \ddot{p}_1 \\ \dot{p}_2 \\ \ddot{p}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{k}{M} \cdot x_1^3 - \frac{\gamma}{M} \cdot x_2 - \frac{1}{M} \cdot u \\ x_4 \\ \frac{k}{M} \cdot x_1^3 + \frac{\gamma}{M} \cdot x_2 + u\left(\frac{1}{m} + \frac{1}{M}\right) \end{bmatrix} = f(x, u) \\ y = x_1 = g(x, u) \end{cases}$$

3e) Linearisierung: Ruhelagen bestimmen:

$$y = x_1 = g(x,u)$$

$$\dot{x} = 0 \rightarrow$$

$$\dot{\vec{x}} = \begin{bmatrix} \dot{p}_1 \\ \ddot{p}_1 \\ \dot{p}_2 \\ \ddot{p}_2 \end{bmatrix} = \begin{bmatrix} x_2 & = f_1 \\ -\frac{k}{M}\cdot x_1^3 - \frac{\gamma}{M}\cdot x_2 - \frac{1}{M}\cdot u & = f_2 \\ x_4 & = f_3 \\ \frac{k}{M}\cdot x_1^3 + \frac{\gamma}{M}\cdot x_2 + u\left(\frac{1}{m} + \frac{1}{M}\right) & = f_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} \boxed{x_{2_0} = x_{4_0} = 0} \\ -\frac{k}{M}\cdot x_{1_0}^3 - \frac{1}{M}\cdot u_0 = 0 \\ \frac{k}{M}\cdot x_{1_0}^3 + u_0\left(\frac{1}{m} + \frac{1}{M}\right) = 0 \end{cases} \oplus \quad \cancel{-\frac{1}{M}u_0} + u_0\cdot\frac{1}{m} + \cancel{u_0\cdot\frac{1}{M}} = 0, \quad \boxed{u_0 = 0}$$

$m > 0$

$$\boxed{x_{3_0} \in \mathbb{R}}$$

$$-\frac{k}{M}\cdot x_{1_0}^3 - \cancel{\frac{1}{M}\cdot 0} = 0 \rightarrow \boxed{x_{1_0} = 0}$$

$k > 0$
$M > 0$

$$\underline{A} = D_1 f(x_0, u_0) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(x_0,u_0) & \cdots & \frac{\partial f_1}{\partial x_4}(x_0,u_0) \\ \vdots & \ddots & \vdots \\ \frac{\partial f_4}{\partial x_1}(x_0,u_0) & \cdots & \frac{\partial f_4}{\partial x_4}(x_0,u_0) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{M}\cdot 3x_{1_0}^2 & -\gamma/M & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{M}\cdot 3x_{1_0}^2 & \gamma/M & 0 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -\gamma/M & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \gamma/M & 0 & 0 \end{pmatrix}$$

$$\underline{B} = D_2 f(x_0, u_0) = \begin{pmatrix} \frac{\partial f_1(x_0,u_0)}{\partial u} \\ \frac{\partial f_2(x_0,u_0)}{\partial u} \\ \vdots \\ \frac{\partial f_4(x_0,u_0)}{\partial u} \end{pmatrix} = \begin{pmatrix} 0 \\ -1/M \\ 0 \\ 1/m + 1/M \end{pmatrix},$$

$$\underline{C} = D_1 g(x_0, u_0) = \left[\frac{\partial g}{\partial x_1}(x_0,u_0) \;\cdots\; \frac{\partial g}{\partial x_4}(x_0,u_0)\right] = [1 \quad 0 \quad 0 \quad 0]$$

$$\underline{D} = D_2 g(x_0, u_0) = \left[\frac{\partial g(x_0,u_0)}{\partial u}\right] = 0$$