11.02.22, Victor Cheidde Chaim

→ **Probeklausuraufgaben WTZZ-SRT** — 4. Aufgabenblatt

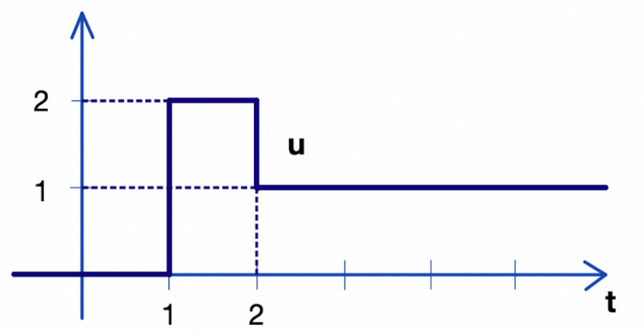
4.2) $\Phi'(At) = A \cdot \exp(At)$ — 1 Punkt

$\Phi'(0) = A \cdot \exp(0) = A \cdot id = A$ //

4.3) X nilpotent. — 1 Punkt

4.4) $\begin{cases} \dot{x} = -2x + u \\ y = x \end{cases}$

$x(0) = 0$



Impulsantwort: $g(t) = \sigma(t) C e^{At} B$, $\sigma(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{sonst} \end{cases}$

$C = 1$, $B = 1$, $A = -2 \rightarrow g(t) = \sigma(t) e^{-2t}$ // — 1 Punkt

$y(t) = \int_{-\infty}^{t} g(\tau) u(t-\tau) d\tau = \int_{0}^{t} g(\tau) u(t-\tau) d\tau$

3 Fällen: $0 < t < 1$, $1 < t < 2$, $2 < t$:

$u(t) = 2\sigma(t-1) - \sigma(t-2)$ — 1 Punkt

- Linearität: $y(t) = \psi(t,0,u) = 2\psi(t,0,u_1) - \psi(t,0,u_2)$

- Kausilät, Zeitinvariant: $\psi(t,0,u_1) = \psi(t-1,0,\sigma) = (g * \sigma)(t-1)$
  $\psi(t,0,u_2) = \psi(t-2,0,\sigma) = (g * \sigma)(t-2)$

$y(t) = 2(g * \sigma)(t-1) - (g * \sigma)(t-2)$

$$(g*\sigma) = \int_{-\infty}^{\infty} g(\tau)\,\sigma(t-\tau)\,d\tau = \int_{-\infty}^{\infty} \underbrace{\sigma(\tau)}_{=0 \text{ für } \tau<0} e^{-2\tau} \underbrace{\sigma(t-\tau)}_{=0 \text{ für } \tau>t} d\tau =$$

$$= \sigma(t)\int_0^t e^{-2\tau} d\tau = \sigma(t)\left(-\frac{e^{-2\tau}}{2}\right)\Big|_0^t = \frac{1}{2}\sigma(t)(1-e^{-2t})$$

1 Punkt

$$\Rightarrow y(t) = \sigma(t-1)(1-e^{-2(t-1)}) - \frac{1}{2}\sigma(t-2)(1-e^{-2(t-2)})$$

2 Punkte

4.5)

$$A = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{Hauptfundamentalmatrix: } \phi(t) = \exp(At)$$

1 Punkt

$$\exp(At) \Rightarrow A = \operatorname{diag}\Big(\underbrace{\begin{pmatrix} 2 & 0 \\ -1 & 2 \end{pmatrix}}_{A_1}, \underbrace{1}_{A_2}\Big) \Rightarrow e^{At} = \operatorname{diag}(e^{A_1 t}, e^{1t})$$

1 Punkt

$$A_1 = 2\cdot id + \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \Rightarrow e^{A_1 t} = e^{2t} \exp\Big(t \underbrace{\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}}_{nilp}\Big) = e^{2t}\left(id + t\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}\right)$$

$$e^{A_1 t} = e^{2t}\begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix}$$

2 Punkte

$$\exp(At) = \operatorname{diag}\left(e^{2t}\begin{pmatrix} 1 & 0 \\ -t & 1 \end{pmatrix}, e^t\right) = \begin{pmatrix} e^{2t} & 0 & 0 \\ -te^{2t} & e^{2t} & 0 \\ 0 & 0 & e^t \end{pmatrix}$$