Regelungstechnik

2. Übung

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$u(t) = \sin(t) \rightarrow \boxed{K} \rightarrow y(t) = K \cdot \sin(t)$

$u(t) \rightarrow \boxed{\frac{1}{1+s}} \rightarrow y(t) = K_1$ [illegible] $= K_1$

$y(t) = K_1 \sin(t + \varphi)$

Frequenzgang:

$$G(j\omega) = \frac{y_0(\omega)}{u_0} e^{j\varphi(\omega)}$$

beschreibt wie ein dynamisches System eine sinus-förmige Eingangsgröße überträgt (stationäres Verhalten).

Amplitudengang:

$$|G(j\omega)| = \sqrt{\mathrm{Re}(G(j\omega))^2 + \mathrm{Im}(G(j\omega))^2}$$

ist ein Maß für die Amplitudenveränderung (frequenzabhängiger Verstärkungsfaktor).

Phasengang:

$$\varphi(\omega) = \arg G(j\omega) = \tan^{-1} \frac{\mathrm{Im}(G(j\omega))^*}{\mathrm{Re}(G(j\omega))}$$

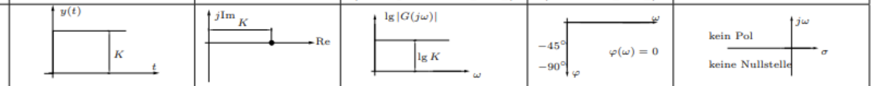
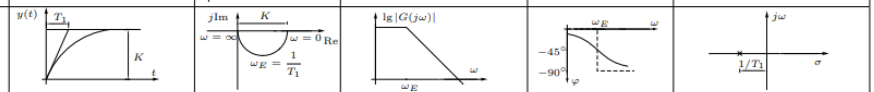
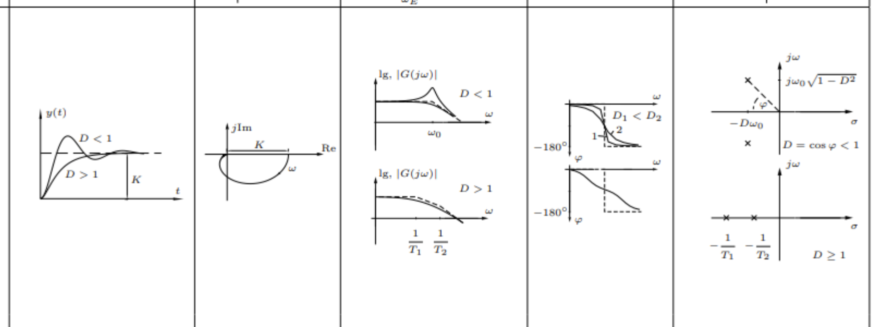
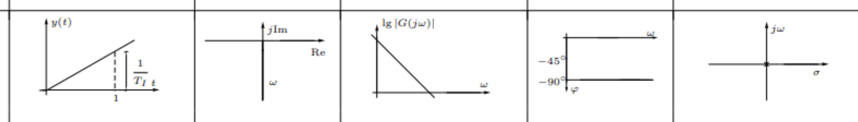
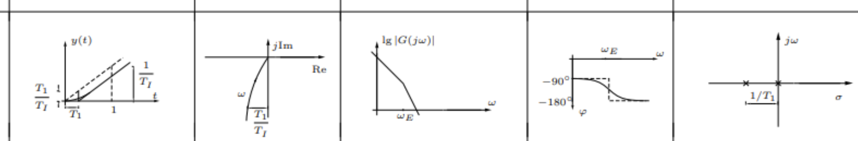
gibt an mit welcher Verspätung das Ausgangssignal dem Eingangssignal folgt.

* Mehrdeutigkeit von arctan entsprechend der Vorzeichen von Im $G(j\omega)$ und Re $G(j\omega)$ muß berücksichtigt werden.

Prof. Dr.-Ing. Ferdinand Svaricek — Regelungstechnik

Skizzierregeln Bodediagramm

Tabelle 2.3: Verhalten der wichtigsten Regelkreisglieder

System	Zeitbereich Bildbereich	Übergangsfunktion	Ortskurve	Bode–Diagramm (Amplitudengang)	(Phasengang)	s–Ebene × Pol ○ Nullstelle
P	$y(t) = K\,u(t)$ $G(s) = K$					
PT_1	$T_1\,\dot y(t) + y(t) = K\,u(t)$ $G(s) = K\frac{1}{1+T_1\,s}$					
PT_2	$\frac{1}{\omega_0^2}\ddot y(t) + \frac{2\,D}{\omega_0}\dot y(t) + y(t) = K\,u(t)$ $G(s) = K\frac{1}{\frac{1}{\omega_0^2}s^2 + \frac{2\,D}{\omega_0}s + 1}$ $D < 1$: konjugiert komplexe Wurzeln der char. Gleichung $\lambda_{1,2} = -\omega_0(D \pm j\sqrt{1-D^2})$ $D \geq 1$: reelle Wurzeln der char. Gleichung $\lambda_{1,2} = -\omega_0(D \pm \sqrt{D^2-1}) = -1/T_{1,2}$					
I	$y(t) = \frac{1}{T_I}\int u\mathrm{d}t$ $G(s) = \frac{1}{T_I\,s}$					
IT_1	$T_1\,\dot y(t) + y(t) = \frac{1}{T_I}\int u(t)\mathrm{d}t$ $G(s) = \frac{1}{T_I\,s(1+T_1\,s)}$					

Svaricek, 2017 – 30

Aufgabe 2.1

$$PT_1 \Rightarrow G(s) = \frac{1}{1+s/\omega_E} \rightarrow \text{Eckfrequenz}$$

$\rightarrow 0° \rightarrow 90°$

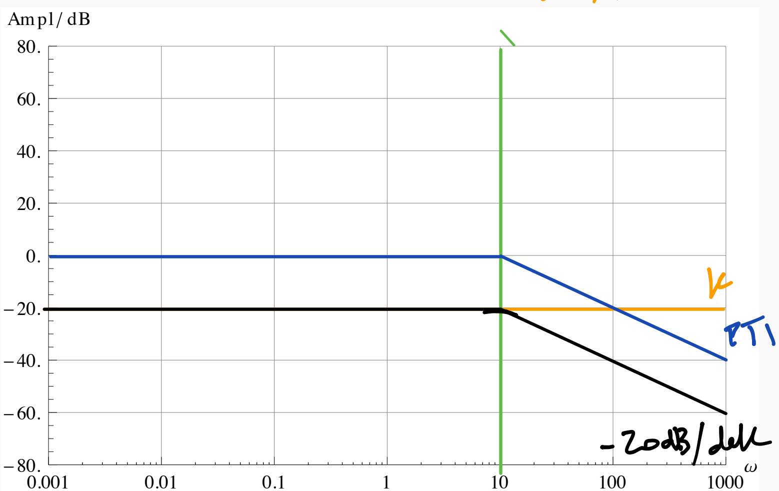
$PT_1 \rightarrow \omega_E = -10\,\text{rad/s}$ $(+\omega\,\text{rad/s}) \rightarrow 0° \rightarrow -90°$

$$\text{(i)}\quad G(s) = \frac{1}{10-s} = \frac{1}{10} \cdot \frac{1}{1-s/10}$$

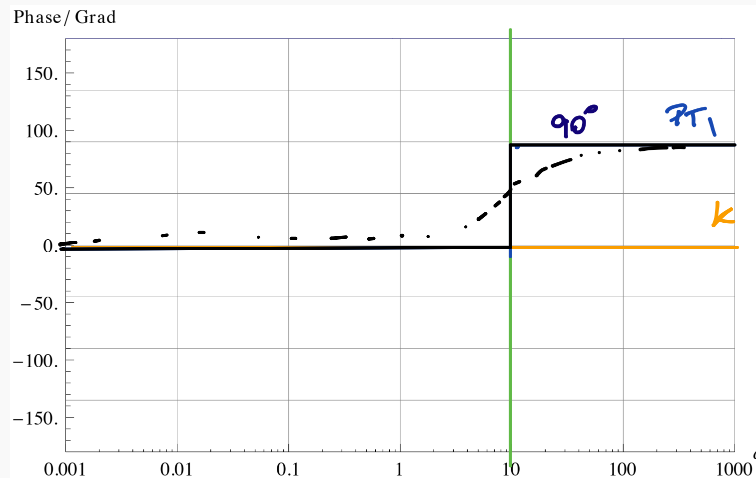
P-Glied

$k = 1/10$

$\rightarrow 20 \log(k) = 20 \log(1/10) = -20\,\text{dB}$



ω_E



ω_E

Aufgabe 2.1

Phase $-90°$

I

P-Glied

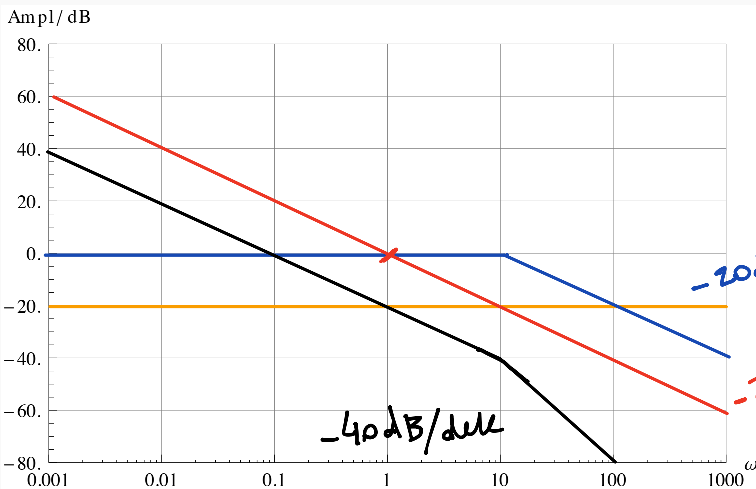
PT_1

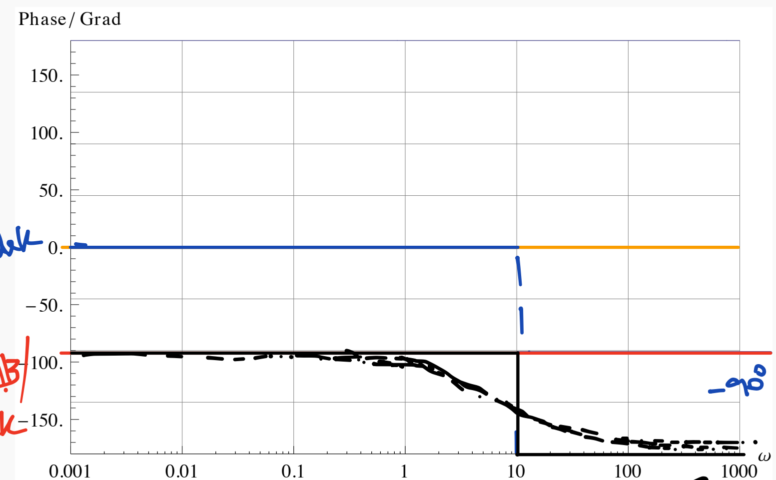
$20\log(K) = 20\log(1/10) = -20\,dB$

$\rightarrow 0° \rightarrow -90°$

(ii) $G(s) = \frac{1}{s(10+s)} = \frac{1}{s} \cdot \frac{1}{10} \cdot \frac{1}{1+s/10}$

$\rightarrow \omega_E = 10\ rad/s$





Aufgabe 2.1

$1+s=0$
$s=-1\,\mathrm{rad/s}$

P-Glied

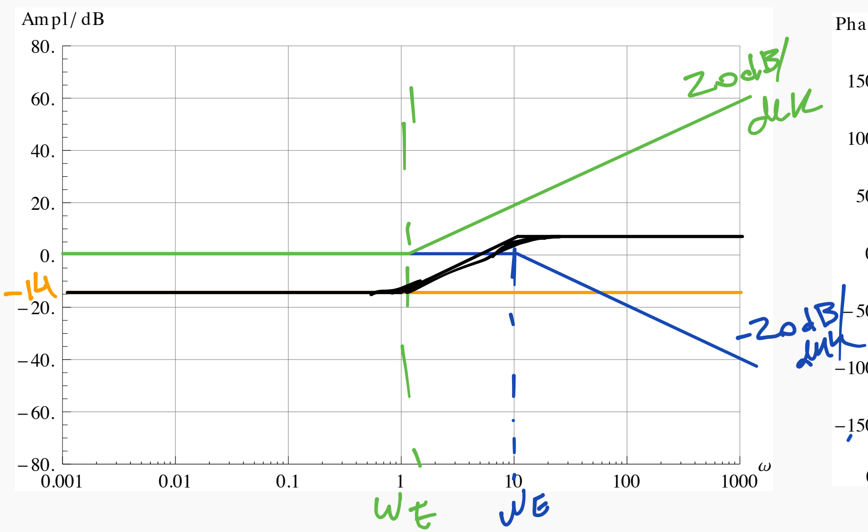
PD-Glied
$1+s/\omega_E$

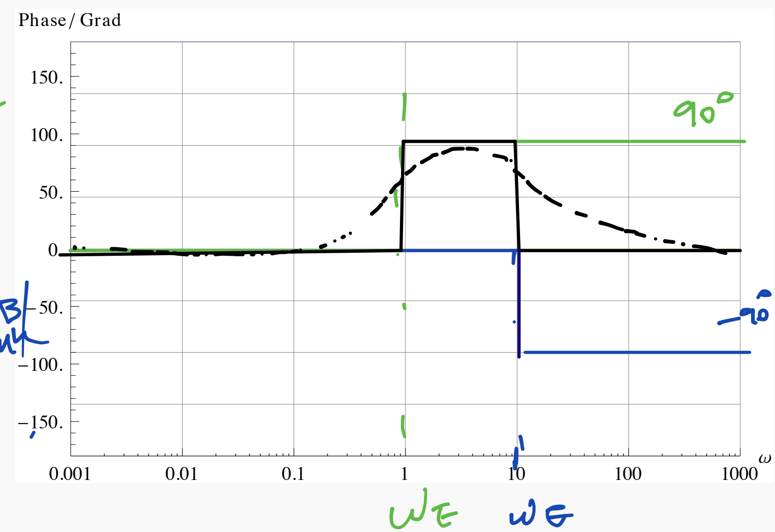
$20\log(K) = 20\log(1/5) \approx -14\,\mathrm{dB}$

PT1 System
$\omega_E = 10\,\mathrm{rad/s}$
$\to 0° \to -90°$

(iii) $G(s) = \frac{2(1+s)}{10+s} = \frac{1}{5}\cdot(1+s)\cdot\frac{1}{1+s/10}$

$\hookrightarrow \omega_E = 1\,\mathrm{rad/s}$





Aufgabe 2.1

(iv) $G(s) = \frac{2(1-s/4)}{1+s/4} = 2 \cdot \left(1 - \frac{s}{4}\right) \cdot \frac{1}{1+s/4}$

PGlied

$0^\circ \rightarrow -90^\circ$

$20\log(2) \approx 6dB$

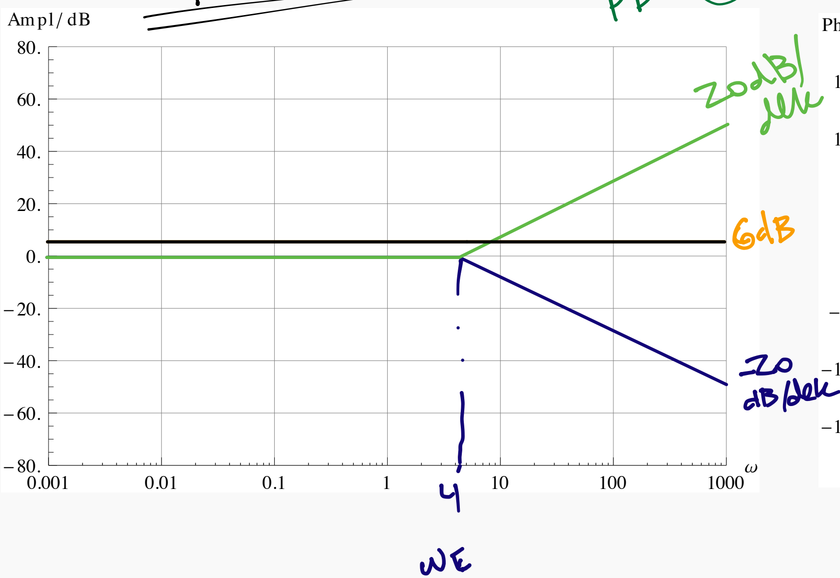
PT_1-System

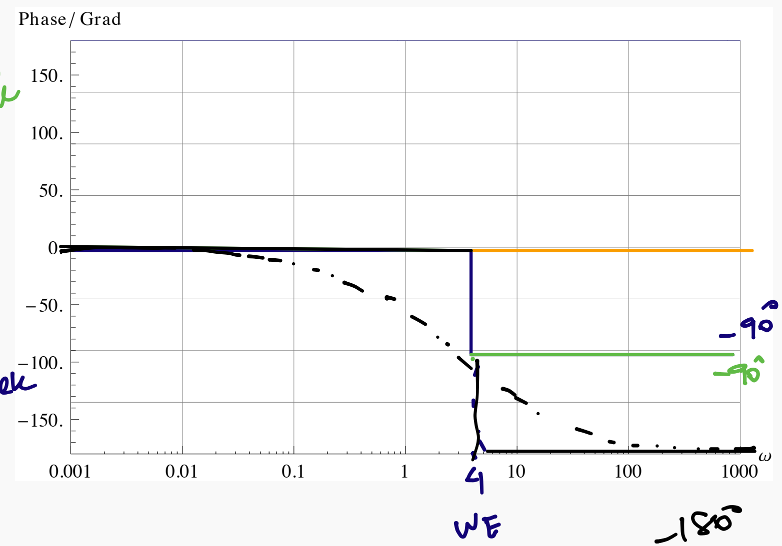
Phase: $0^\circ \rightarrow -90^\circ$

$\omega_E = 4$ rad/s

PD = (-4) rad/s

Allpassglied





Aufgabe 2.1

$P_1, P_2 = -0{,}5 \pm 0{,}87 i$

$\rightarrow$ Phase: $0° \rightarrow -180°$

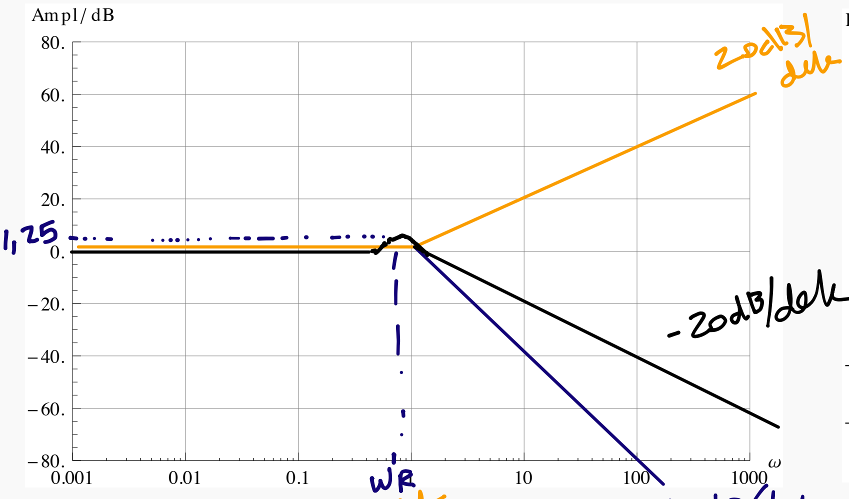
PD-Glied $\omega_G = 1$ rad/s $0° \rightarrow 90°$

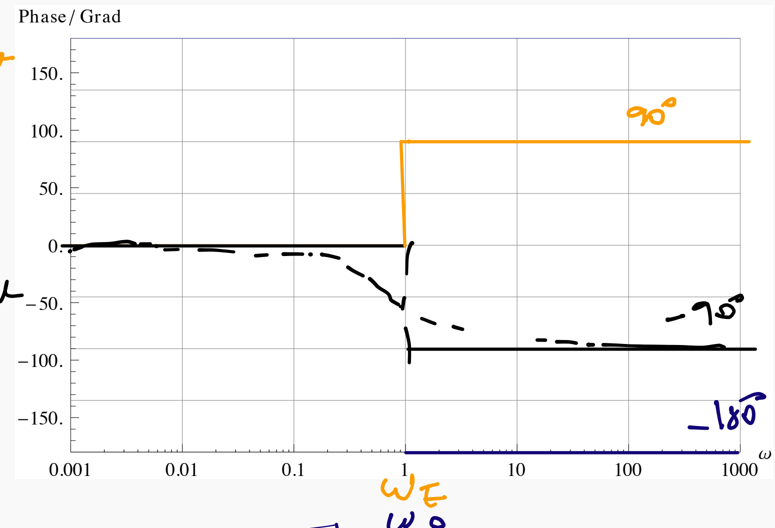
(v) $G(s) = \frac{1+s}{1+s+s^2} = (1+s) \cdot \frac{1}{(1+s+s^2)}$ PT2

PT2: $G(s) = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{2D}{\omega_0} \cdot s + 1}$

$\frac{2D}{\omega_0} := 1 \rightarrow \boxed{D = 1/2}$

$\omega_0^2 = 1 \rightarrow \boxed{\omega_0 = 1 \text{ rad/s}}$





Resonanzfrequenz: ω_0

$\omega_R = \omega_0 \sqrt{1-2D^2} \rightarrow$ max.

$\omega_R = 1 \sqrt{1 - 2(1/2)^2} = 0{,}7$ rad/s

$M_r = 1/(2D\sqrt{1-D^2})$

$M_r = 1/(2D\sqrt{1-D^2}) \approx 1{,}15 \rightarrow 20 \log(1{,}15) \approx 1{,}25$ dB

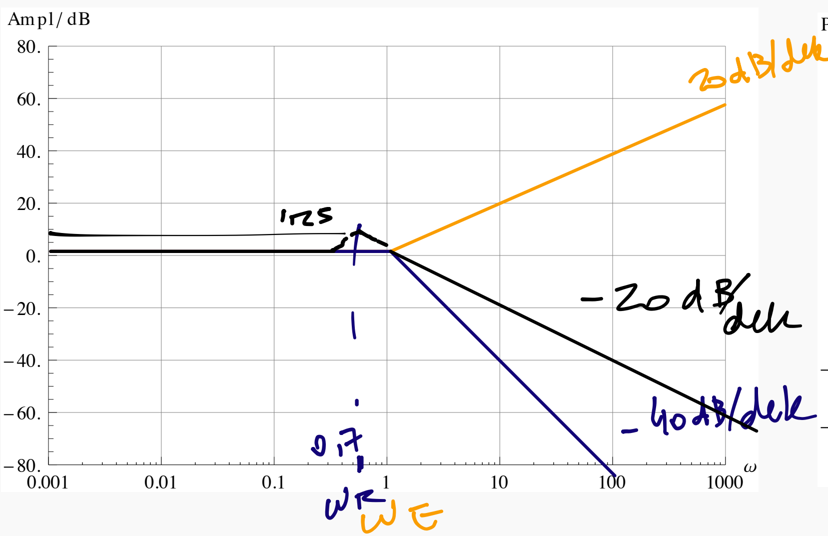
Aufgabe 2.1

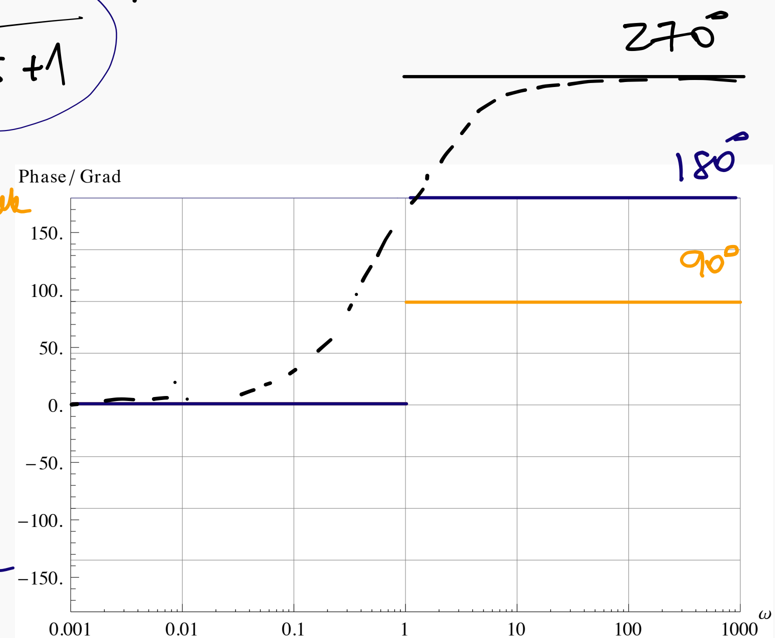
$P_1, P_2 = +0{,}5 \pm 0{,}87j$

↳ $0° \to 180°$

PD-Glied PT_2 $\omega_0 = 1\,\text{rad/s}$

(vi) $G(s) = \frac{1+s}{1-s+s^2} = (1+s)\,\frac{1}{s^2-s+1}$





PD $\Rightarrow$ $\omega_E = 3\,\text{rad/s}$

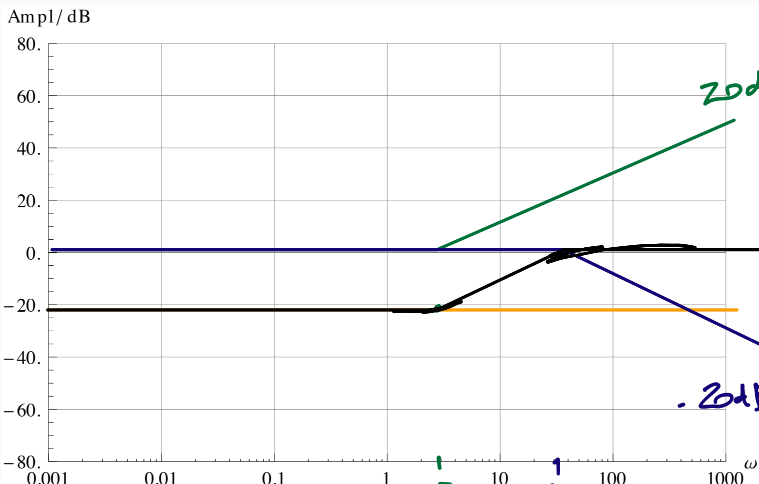
$0^\circ \rightarrow 90^\circ$

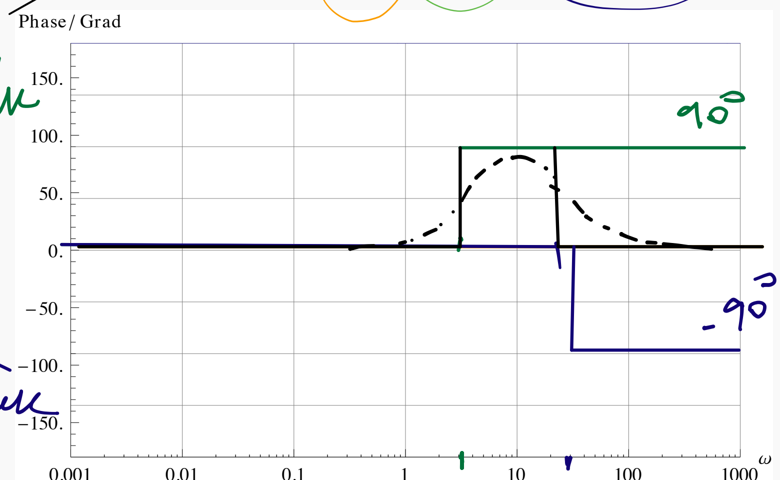
$12 + 7s + s^2 = 0 \rightarrow n_1 = -4,\ n_2 = -3$

$120 + 34s + s^2 = 0 \rightarrow p_1 = -4,\ p_2 = -30$

(vii) $G(s) = \frac{12+7s+s^2}{120+34s+s^2}$ $\Rightarrow$

$$G(s) = \frac{12+7s+s^2}{120+34s+s^2} = \frac{\cancel{(s+4)}(s+3)}{\cancel{(s+4)}(s+30)} = \underbrace{\frac{1}{10}}_{\text{P glied}} \cdot \underbrace{\left(1+\frac{s}{3}\right)}_{\text{PD}}\underbrace{\left(\frac{1}{1+s/30}\right)}_{\text{PT1}}$$





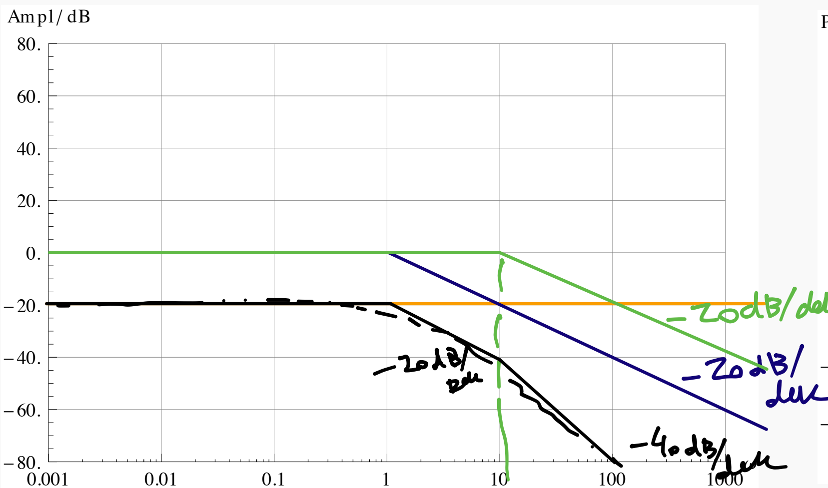
$20 \cdot \log(1/10) = -20\,\text{dB}$

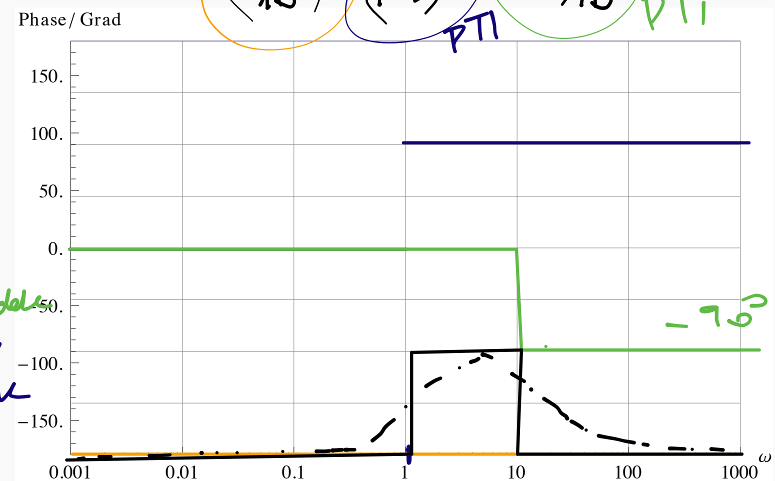
PT1 $\Rightarrow$ $\omega_0 = 30\,\text{rad/s}$

$0^\circ \rightarrow -90^\circ$

Aufgabe 2.1

(viii) $G(s) = \frac{1}{s^2+9s-10}$ $\Rightarrow$ $P_1 = -10$, $P_2 = 1$ $\rightarrow$ $G(s) = \frac{1}{s^2+9s-10} = \frac{1}{(s+10)(s-1)} = \underbrace{\left(\frac{-1}{10}\right)}_{P} \cdot \underbrace{\frac{1}{(1-s)}}_{PT_1} \cdot \underbrace{\frac{1}{1+s/10}}_{PT_1}$





neg. Vorzeichen → Phase: −180° (oder 180°)

$20 \log\left(\left|-\frac{1}{10}\right|\right) = -20\,dB$

PT_1: $\omega_E = 1\,rad/s$

Phase: $0° \rightarrow 90°$ (instabil)

PT_1: $\omega_E = -10\,rad/s$

Phase $0° \rightarrow -90°$ (stabil)

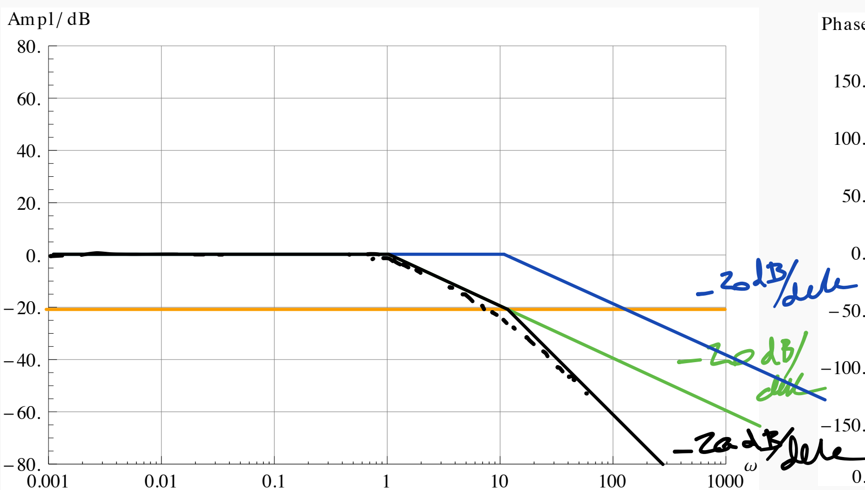
Aufgabe 2.1

(ix) $G(s) = \frac{1}{s^2+11s+10}$ $\Rightarrow$

Pole: $p_1 = -10$, $p = -1$

$G(s) = \frac{1}{(s+1)} \cdot \frac{1}{(s+10)} = \frac{1}{10} \cdot \frac{1}{(s+1)} \cdot \frac{1}{(s+1/10)}$

P Glied PT1 PT1



ω_G ω_E

$20\log(1/10) = 20\,\mathrm{dB}$
Phase $= 0°$

$\omega_0 = 1$ rad/s
stabil → $0° - 90°$

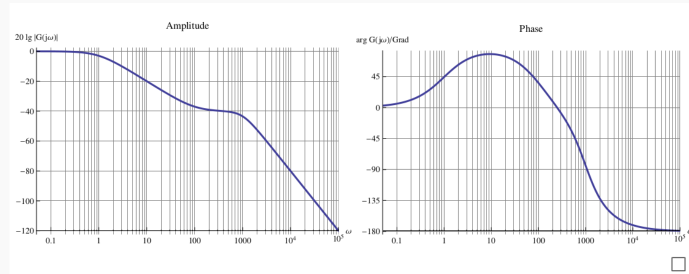
$\omega_0 = 10$ rad/s
stabil → $0° \to -90°$

2.2 Aufgabe (Klausuraufgabe HT 2013). Stellen Sie fest, zu welcher der angegebenen Übertragungsfunktionen das dargestellte Bode-Diagramm gehört. Geben Sie dazu für 5 der 6 Fälle jeweils ein Merkmal der Übertragungsfunktion und ein Merkmal des Bode-Diagramms an, die nicht miteinander verträglich sind.

$$G_1(s) = -\frac{\frac{s}{100} - 1}{(s+1)\left(\frac{s^2}{1000000} + \frac{3s}{2000} + 1\right)}, \qquad G_2(s) = \frac{\frac{s}{100} - 1}{(s-1)\left(\frac{s^2}{1000000} + \frac{s}{2000} + 1\right)},$$

$$G_3(s) = -\frac{\frac{s}{100} + 1}{(s-1)\left(\frac{s^2}{1000000} + \frac{3s}{2000} + 1\right)}, \qquad \boxed{G_4(s) = \frac{\frac{s}{100} - 1}{(s-1)\left(\frac{s^2}{1000000} + \frac{3s}{2000} + 1\right)}},$$

$$G_5(s) = \frac{s-2}{(s-1)\left(\frac{s^2}{1000000} + \frac{3s}{2000} + 1\right)}, \qquad G_6(s) = \frac{\frac{s}{100} + 1}{(s+1)\left(\frac{s^2}{1000000} + \frac{3s}{2000} + 1\right)}.$$



$G_1(s)$: Polstelle -1 müsste Phase nach unten knicken;

$G_2(s)$: Resonanzüberhöhung müsste auftreten;

$G_3(s)$: Nullstelle -100 müsste Phase nach oben knicken;

$G_4(s)$: Alles stimmt!

$G_5(s)$: Nullstelle 2 müsste Phase nach unten knicken
oder: Gleichverstärkung müsste größer als $0\,dB$ sein;

$G_6(s)$: Nullstelle -100 müsste Phase nach oben knicken,
oder: Polstelle -1 müsste Phase nach unten knicken.