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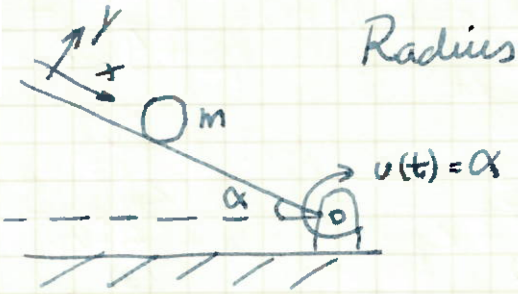


Radius R, Masse m

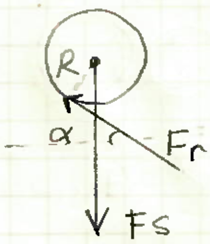
$F_r = m.g.\mu.\cos\alpha$

$I_m = \frac{2}{5}.m.R^2$

die Kugel rollt.

i) $x = [p \;\; v]^T$, $u = \alpha$, $y = p$

Kräfte: * Fr ist immer gegen die Bewegung.



Newton:

$\sum F_x : \; m.a = F_s.\sin\alpha - F_r$

$a \rightarrow$ Beschleunigung in X-Richtung

$\cancel{m}.a = \cancel{m}.g.\sin\alpha - \cancel{m}.g.\mu.\cos\alpha$

$\boxed{a = g.\sin\alpha - g.\mu.\cos\alpha}$ (I)

die Kugel rollt!

↳ Euler: $\sum M_{CG}$: $I_m.\ddot{\theta} = F_r.R$

$\ddot{\theta} \rightarrow$ Winkelbeschleunigung auf der Kugel.

kinematic: $R\ddot{\theta} = a \rightarrow \ddot{\theta} = \frac{a}{R}$ ∴

$I_m.\frac{a}{R} = F_r.R \rightarrow \frac{2}{5}\cancel{m}\cancel{R^2}.\frac{a}{\cancel{R}} = \cancel{m}.g\mu\cos\alpha.\cancel{R}$

$\boxed{\mu = a.\frac{2}{5}.\frac{1}{g.\cos\alpha}}$ (II) , (II → I):

$a = g\sin\alpha - g.\left(a.\frac{2}{5}.\frac{1}{g.\cancel{\cos\alpha}}\right).\cancel{\cos\alpha} = g\sin\alpha - a.\frac{2}{5}$

$a + a\frac{2}{5} = g.\sin\alpha \rightarrow a.\frac{7}{5} = g\sin\alpha \rightarrow \boxed{a = \frac{5}{7}.g.\sin\alpha}$

Kinematic: $\dot{v} = a$, $\dot{p} = v$.

$$\dot{x} = \begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ \frac{5}{7} \cdot g \cdot \sin\alpha \end{bmatrix} = f(x,u) \quad //$$

$$y = p = g(x,u) \quad //$$

ii) Ruhelagen: $\dot{x} = [\dot{p} \quad \dot{v}] = 0$

$$\{0\} = \begin{bmatrix} v_0 \\ \frac{5}{7} \cdot g \cdot \sin\alpha_0 \end{bmatrix} \rightarrow v_0 = 0, \quad \alpha_0 = 0$$

keine Bedingung für p_0.

$$x_0 = [p_0 \quad 0]^T, \quad u_0 = \alpha_0 = 0.$$

iii) $A := D_1 f(x_0,u_0)$ $\qquad$ $B := D_2 f(x_0,u_0)$

$C := D_1 g(x_0,u_0)$ $\qquad$ $D := D_2 g(x_0,u_0)$

$A_{11} = \frac{\partial f_1}{\partial p}(x_0,u_0) = 0$ $\qquad$ $A_{12} = \frac{\partial f_1}{\partial v}(x_0,u_0) = \underline{1}$

$A_{21} = \frac{\partial f_2}{\partial v}(x_0,u_0) = 0$ $\qquad$ $A_{22} = \frac{\partial f_2}{\partial v}(x_0,u_0) = 0$

$B_{11} = \frac{\partial f_1}{\partial \alpha}(x_0,u_0) = 0$ $\qquad$ $B_{21} = \frac{\partial f_2}{\partial \alpha}(x_0,u_0) = \frac{5}{7} \cdot g \cdot \cos 0 = \frac{5}{7} g$

$C_{11} = \frac{\partial g}{\partial p}(x_0,u_0) = 1$ $\qquad$ $C_{12} = \frac{\partial g}{\partial v}(x_0,u_0) = 0$

$D_{11} = \frac{\partial g}{\partial u}(x_0,u_0) = 0$ $\quad \therefore$ Zustandssystem:

$$\begin{cases} \Delta\dot{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \Delta x + \begin{pmatrix} 0 \\ \frac{5}{7} \cdot g \end{pmatrix} \Delta u \\ \Delta y = \begin{pmatrix} 1 & 0 \end{pmatrix} \Delta x \end{cases} \qquad \begin{cases} \Delta x := x(t) - x_0 \\ \Delta y := y(t) - y_0 \\ \Delta u := u(t) - u_0 \end{cases}$$

* Regelungstechnik WT24

→ Übung - Experiment

iv) $$\Delta\dot{x} = A\Delta x + B\Delta u \quad , \quad A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \frac{5}{7}g \end{pmatrix}$$
$$\Delta y = C\Delta x + D\Delta u$$
$$C = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad D = 0.$$

Steuerbarkeit: $\text{Rang } Q_S = \text{Rang}\left[B \;\; AB \;\; A^2B \dots A^{n-1}B\right] = n$
(Kalman-Kriterium)

n → Ordnung des ZS.

$$Q_S = \left[B \;\; AB\right], \quad AB = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{5}{7}g \end{pmatrix} = \begin{pmatrix} \frac{5}{7}g \\ 0 \end{pmatrix}$$

$$Q_S = \begin{bmatrix} 0 & \frac{5}{7}g \\ \frac{5}{7}g & 0 \end{bmatrix}$$

die Matrix Q_S hat vollen Rang, das heißt $\text{Rang } Q_S = n$, wenn $\det(Q_S) \neq 0$.

$$\det(Q_S) = \det\begin{pmatrix} 0 & \frac{5}{7}g \\ \frac{5}{7}g & 0 \end{pmatrix} = -\left(\frac{5}{7}g\right)^2 \neq 0 \rightarrow \text{voller Rang!}$$

∴ Das linearisierte Zustandsraumsystem ist steuerbar.

v) Beobachtbarkeit: $\text{Rang } Q_B = \text{Rang}\left[C^T \;\; A^TC^T \dots (A^T)^{n-1}C^T\right] = n$
(Kalman-Kriterium)

$$Q_B = \left[C^T \;\; A^TC^T\right], \quad A^TC^T = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\det(Q_B) = \det\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1 \neq 0 \rightarrow \text{voller Rang!}$$

∴ Das linearisierte Zustandsraumsystem ist beobachtbar.

vi) $\Delta u = -[k_p \;\; k_v] \cdot \Delta e_x \;, \Delta e_x = \Delta x - \begin{bmatrix} z(t) \\ 0 \end{bmatrix}$

$z(t) \rightarrow$ Sollwert der Position

$k_p, k_v \in \mathbb{R}_+$

$$\Delta \dot{x} = A \Delta x + B \cdot \Delta u \rightarrow \Delta \dot{x} = A \Delta x + B(-[k_p \;\; k_v] \Delta e_x)$$

$$\Delta \dot{x} = A \cdot \Delta x - B[k_p \;\; k_v]\left(\Delta x - \begin{bmatrix} z(t) \\ 0 \end{bmatrix}\right)$$

$$\Delta \dot{x} = A \Delta x - B[k_p \;\; k_v] \Delta x + B[k_p \;\; k_v] \cdot \begin{bmatrix} z(t) \\ 0 \end{bmatrix}$$

$$\Delta \dot{x} = (A - B[k_p \;\; k_v]) \Delta x + B[k_p \;\; k_v] \begin{bmatrix} z(t) \\ 0 \end{bmatrix}$$

$$\Delta \dot{x} = \left(\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 \\ \frac{5}{7}g \end{pmatrix} (k_p \;\; k_v)\right) \Delta x + \begin{pmatrix} 0 \\ \frac{5}{7}g \end{pmatrix} (k_p \;\; k_v) \cdot \begin{bmatrix} z(t) \\ 0 \end{bmatrix}$$

$$\Delta \dot{x} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\frac{5}{7}g k_p & -\frac{5}{7}g k_v \end{pmatrix}}_{\bar{A}} \cdot \Delta x + \underbrace{\begin{pmatrix} 0 & 0 \\ \frac{5}{7}g k_p & \frac{5}{7}g k_v \end{pmatrix}}_{\bar{B}} \underbrace{\begin{bmatrix} z(t) \\ 0 \end{bmatrix}}_{u_z}$$

$\Delta y = \underbrace{(1 \;\; 0)}_{C} \Delta x$ (keine Änderung, da $D = 0$).

vii) $k_p, k_v \rightarrow p_1, p_2 = -\frac{5}{4}\pi$.

$$G(s) = C \cdot (s\,id - \bar{A})^{-1} \bar{B} + \cancel{D} \quad (D=0)$$

$$G(s) = (1 \;\; 0)(s\,id - \bar{A})^{-1} \cdot \bar{B}$$

$$(s\,id - \bar{A})^{-1} = \frac{1}{\det(s\,id - \bar{A})} \cdot adj(s\,id - \bar{A})$$

$$\det(s\,id-\bar{A}) = \det\begin{pmatrix} s & -1 \\ \frac{5}{7}g k_p & s+\frac{5}{7}g k_v \end{pmatrix} = s^2 + s\cdot\frac{5}{7}g k_v + \frac{5}{7}g k_p$$

$$\operatorname{adj}(s\,id-\bar{A}) = \begin{pmatrix} s+\frac{5}{7}g\cdot k_v & 1 \\ -\frac{5}{7}g k_p & s \end{pmatrix}$$

$$(s\,id-\bar{A})^{-1} = \frac{1}{s^2+s\cdot\frac{5}{7}g k_v + \frac{5}{7}g k_p}\begin{pmatrix} s+\frac{5}{7}g k_v & 1 \\ -\frac{5}{7}g k_p & s \end{pmatrix}$$

$$G(s) = \begin{pmatrix} 1 & 0 \end{pmatrix}\cdot\begin{pmatrix} s+\frac{5}{7}g k_v & 1 \\ -\frac{5}{7}g k_p & s \end{pmatrix}\cdot\bar{B}\cdot\frac{1}{s^2+s\cdot\frac{5}{7}g k_v+\frac{5}{7}g k_p}$$

$$G(s) = \begin{pmatrix} s+\frac{5}{7}g k_v & 1 \end{pmatrix}\cdot\begin{pmatrix} 0 & 0 \\ \frac{5}{7}g k_p & \frac{5}{7}g k_v \end{pmatrix}\cdot\frac{1}{s^2+s\cdot\frac{5}{7}g k_v+\frac{5}{7}g k_p}$$

$$G(s) = \begin{pmatrix} \underbrace{\frac{5}{7}g k_p}_{\frac{Z(s)}{Y(s)}} & \underbrace{\frac{5}{7}g k_v}_{\frac{V}{Y}} \end{pmatrix}\frac{1}{s^2+s\cdot\frac{5}{7}g k_v+\frac{5}{7}g k_p}$$

$\frac{V}{Y}$ (= 0, weil $u_z = [z(t) \quad 0]$)

$$\frac{Z(s)}{Y(s)} = G_1(s) = \frac{5/7\cdot g\cdot k_p}{s^2+s\cdot\frac{5}{7}g k_v+\frac{5}{7}g k_p}$$

Pole: $s^2+s\cdot\frac{5}{7}g k_v+\frac{5}{7}g k_p = 0$

$$p_{1,2} = \frac{-\frac{5}{7}g k_v \pm\sqrt{\left(\frac{5}{7}g k_v\right)^2-4\cdot\frac{5}{7}g k_p}}{2}$$

$p_1 = p_2$, nur wenn $\left(\frac{5}{7}g k_v\right)^2 = 4\cdot\frac{5}{7}g k_p$:

$$p_{1,2} = -\frac{5}{4}\cdot\pi = \frac{-\frac{5}{7}g k_v}{2} \rightarrow \boxed{k_v = \frac{7}{2}\frac{\pi}{g}}$$

$$\left(\frac{5}{7} g\, k_V\right)^2 = 4 \cdot \frac{5}{7} \cdot g \cdot k_P \rightarrow \left(\frac{5}{7} g \frac{7}{2}\frac{\pi}{g}\right)^2 = 4 \cdot \frac{5}{7} \cdot g\, k_P$$

$$k_P \cdot 4 \cdot \frac{5}{7} \cdot g = \left(\frac{5\pi}{2}\right)^2 \rightarrow k_P = \frac{25}{4}\pi^2 \frac{7}{20g} = \frac{35}{16}\frac{\pi^2}{g}$$

viii) $L_1, L_2 \rightarrow$ Ew: $(A - [L_1\ L_2]^T \cdot C) = -20$.

$$\left.\begin{aligned} \Delta \dot{x} &= A \Delta x + B \Delta u \\ \Delta y &= C \Delta x \end{aligned}\right\} \text{ZS Punkt iii).}$$

$$\left(\underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{A} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \cdot \underbrace{\begin{pmatrix} 1 & 0 \end{pmatrix}}_{C}\right) = \begin{pmatrix} -L_1 & 1 \\ -L_2 & 0 \end{pmatrix}$$

Ew: $\det\left(\lambda\, id - \left(A - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \cdot C\right)\right) = 0$

$$\det \begin{pmatrix} \lambda + L_1 & -1 \\ L_2 & \lambda \end{pmatrix} = \lambda^2 + L_1 \lambda + L_2 = 0$$

$$\lambda_{1,2} = \frac{-L_1 \pm \sqrt{L_1^2 - 4L_2}}{2}, \quad \lambda_1 = \lambda_2 \text{ wenn } L_1^2 = 4L_2$$

$$\lambda_{1,2} = -20 = -\frac{L_1}{2} \rightarrow \boxed{L_1 = 40}$$

$$4L_2 = L_1^2 \rightarrow 4L_2 = 1600 \rightarrow \boxed{L_2 = 400}$$