Regelungstechnik

2. Übung

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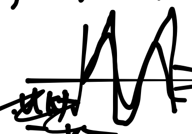
$u(t)=\sin(t) \rightarrow \boxed{K} \rightarrow y(t)=K\cdot\sin(t)$

$u(t) \rightarrow \boxed{1/(K+s)} \rightarrow y(t)=K_1 \cdot \ldots = K_1$



$y(t)=K_1\sin(t+\varphi)$

Frequenzgang:

$$G(j\omega)=\frac{y_0(\omega)}{u_0}e^{j\varphi(\omega)}$$

beschreibt wie ein dynamisches System eine sinusförmige Eingangsgröße überträgt (stationäres Verhalten).

Amplitudengang:

$$|G(j\omega)| = \sqrt{\mathrm{Re}(G(j\omega))^2 + \mathrm{Im}(G(j\omega))^2}$$

ist ein Maß für die Amplitudenveränderung (frequenzabhängiger Verstärkungsfaktor).

Phasengang:

$$\varphi(\omega) = \arg G(j\omega) = \tan^{-1}\frac{\mathrm{Im}(G(j\omega))^*}{\mathrm{Re}(G(j\omega))}$$

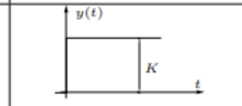
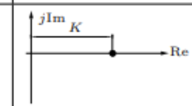
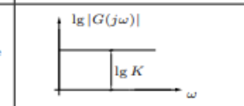
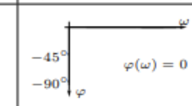
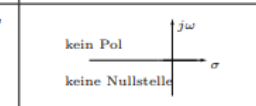
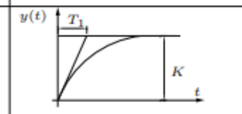
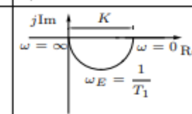
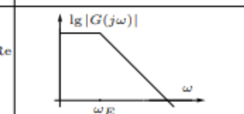
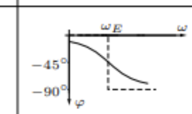
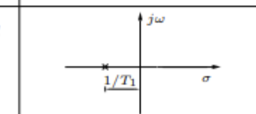
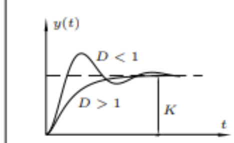
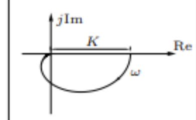
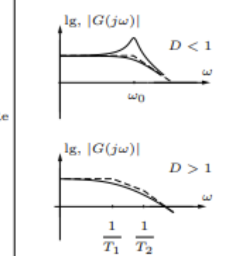
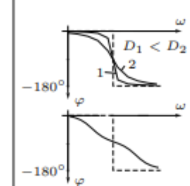
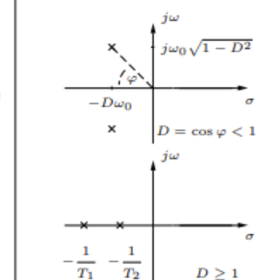
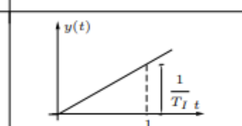
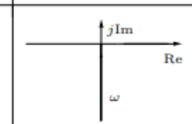
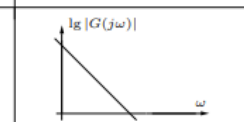
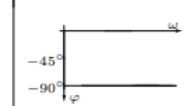
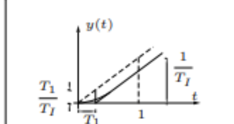
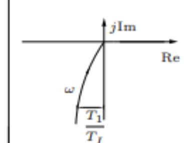
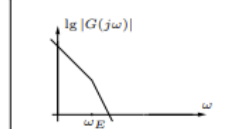
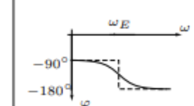
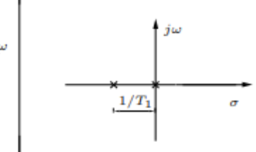
gibt an mit welcher Verspätung das Ausgangssignal dem Eingangssignal folgt.

* Mehrdeutigkeit von arctan entsprechend der Vorzeichen von Im $G(j\omega)$ und Re $G(j\omega)$ muß berücksichtigt werden.

Prof. Dr.-Ing. Ferdinand Svaricek Regelungstechnik

Skizzierregeln Bodediagramm

Tabelle 2.3: Verhalten der wichtigsten Regelkreisglieder

System	Zeitbereich Bildbereich	Übergangsfunktion	Ortskurve	Bode–Diagramm (Amplitudengang)	(Phasengang)	s–Ebene $\times$ Pol $\circ$ Nullstelle
P	$y(t) = K\,u(t)$ $G(s) = K$					
PT_1	$T_1\,\dot{y}(t) + y(t) = K\,u(t)$ $G(s) = K\dfrac{1}{1+T_1\,s}$					
PT_2	$\dfrac{1}{\omega_0^2}\ddot{y}(t) + \dfrac{2\,D}{\omega_0}\dot{y}(t) + y(t) = K\,u(t)$ $G(s) = K\dfrac{1}{\dfrac{1}{\omega_0^2}s^2 + \dfrac{2\,D}{\omega_0}s + 1}$ $D < 1$: konjugiert komplexe Wurzeln der char. Gleichung $\lambda_{1,2} = -\omega_0(D \pm j\sqrt{1-D^2})$ $D \geq 1$: reelle Wurzeln der char. Gleichung $\lambda_{1,2} = -\omega_0(D \pm \sqrt{D^2-1}) = -1/T_{1,2}$					
I	$y(t) = \dfrac{1}{T_I}\displaystyle\int u\mathrm{d}t$ $G(s) = \dfrac{1}{T_I\,s}$					
IT_1	$T_1\,\dot{y}(t) + y(t) = \dfrac{1}{T_I}\displaystyle\int u(t)\mathrm{d}t$ $G(s) = \dfrac{1}{T_I\,s(1+T_1\,s)}$					

Aufgabe 2.1

PT$_1$ $\Rightarrow$ $G(s) = \frac{1}{1+s/\omega_E}$ $\rightarrow$ Eckfrequenz

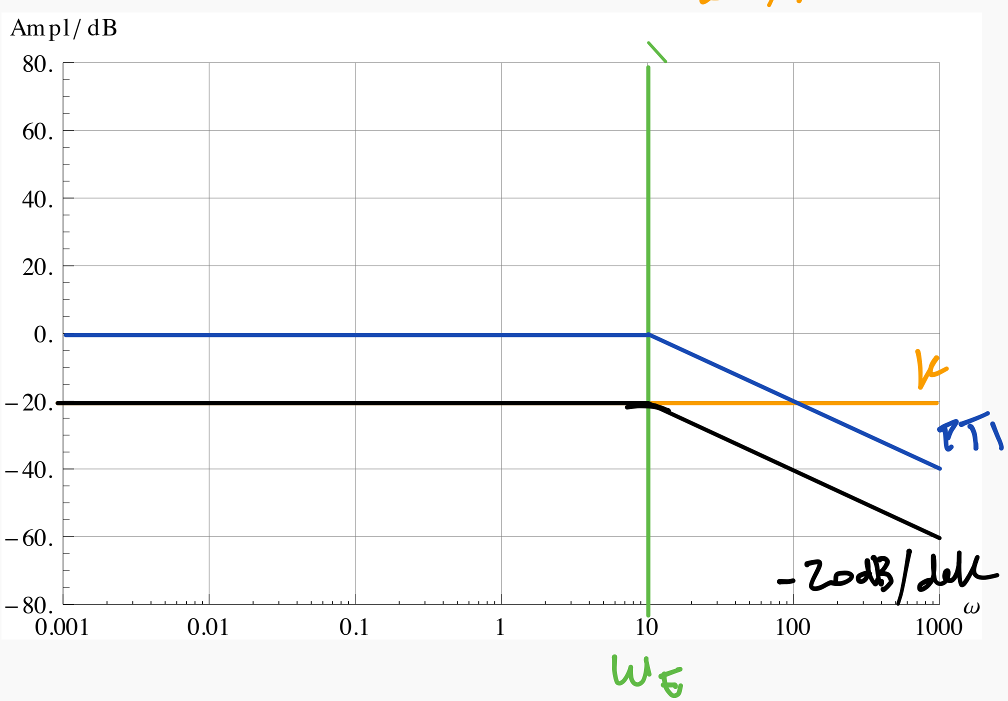
$\rightarrow$ $0° \rightarrow 90°$

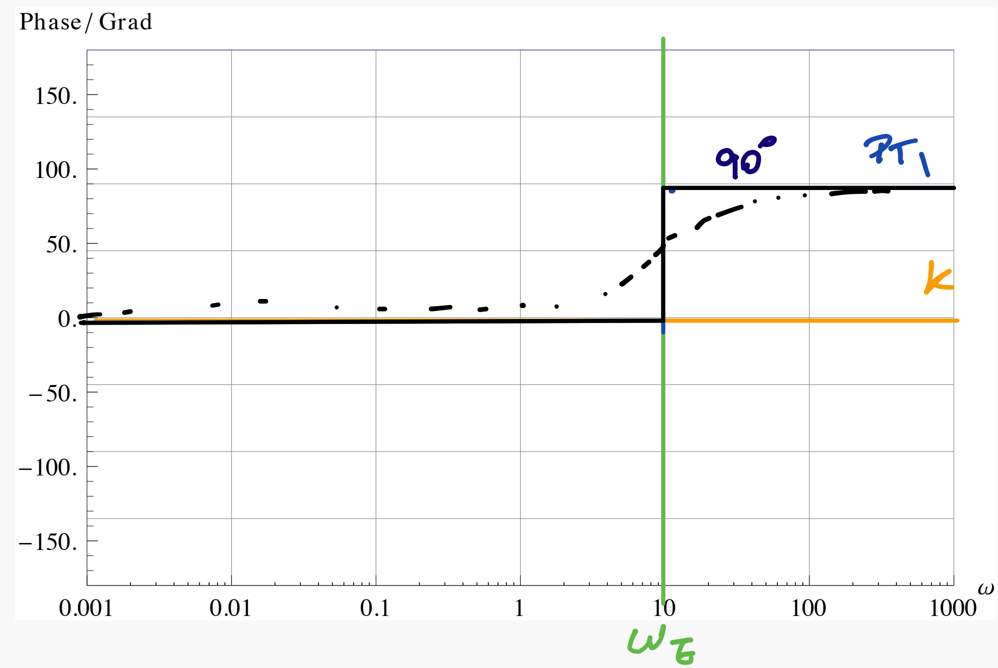
PT$_1$ $\rightarrow$ $\omega_E = -10$ rad/s

(+ ω rad/s) $\rightarrow$ $0° \rightarrow -90°$

(i) $G(s) = \frac{1}{10-s} = \underbrace{\frac{1}{10}}_{\text{P-Glied}} \cdot \underbrace{\frac{1}{1-s/10}}_{k=1/10}$

$\rightarrow$ $20 \log(k) = 20 \log(1/10) = -20\,\text{dB}$





Aufgabe 2.1

Phase: $-90°$

I

P-Glied

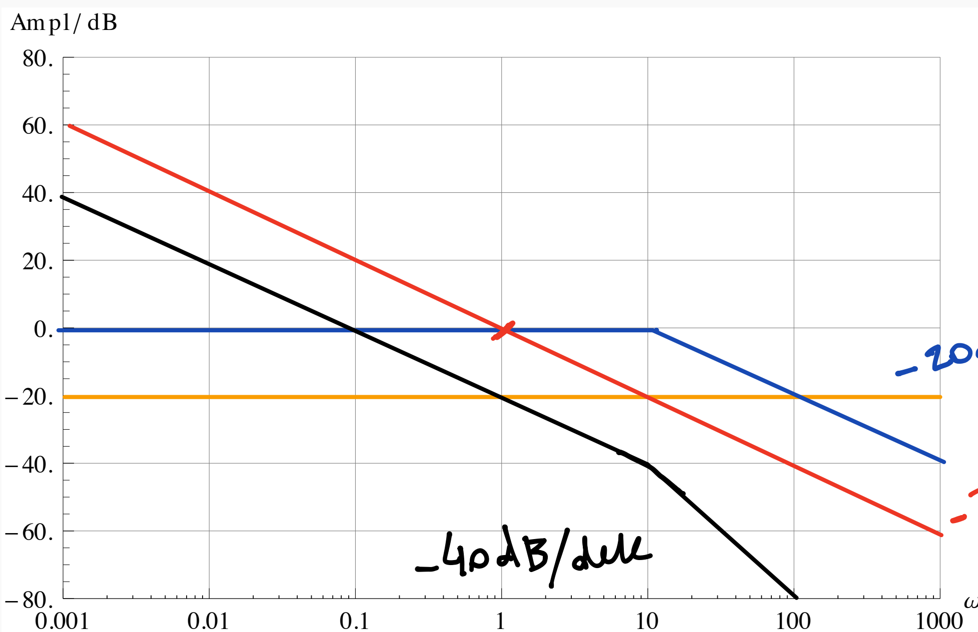
PT_1

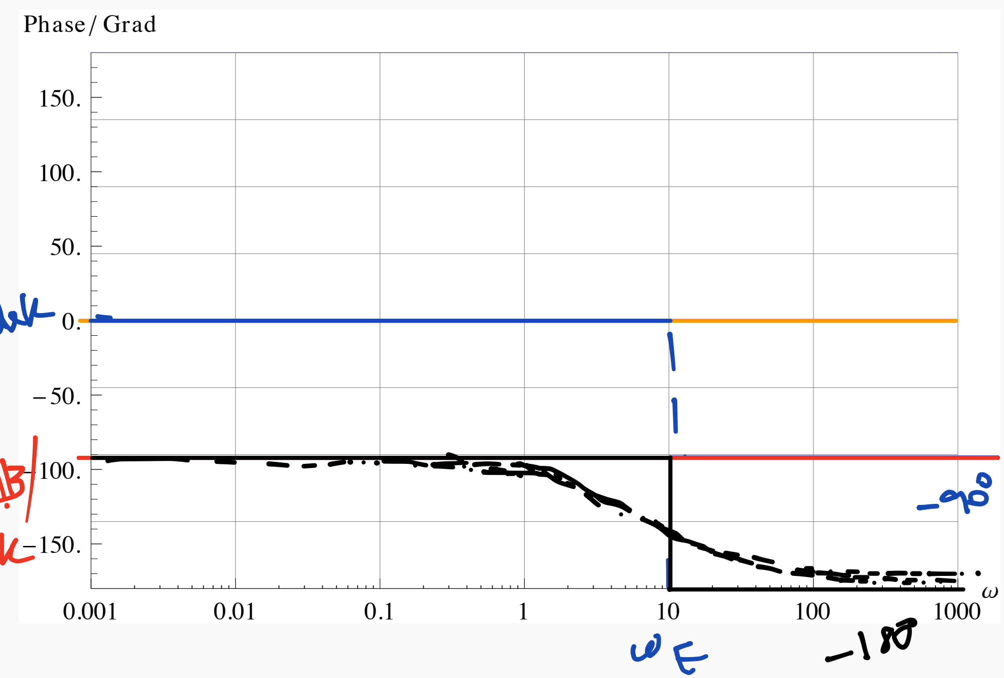
$20\log(K) = 20\log(1/10) = -20\,dB$

$\to 0° \to -90°$

(ii) $G(s) = \frac{1}{s(10+s)} = \frac{1}{s} \cdot \frac{1}{10} \cdot \frac{1}{1+s/10}$

$\to \omega_E = 10\ rad/s$





Aufgabe 2.1

$1+s=0$

$s=-1\,\mathrm{rad/s}$

P-Glied   PD-Glied $1+s/\omega_E$

$20\log(K) = 20\log(1/5) \hat{=} -14\,\mathrm{dB}$

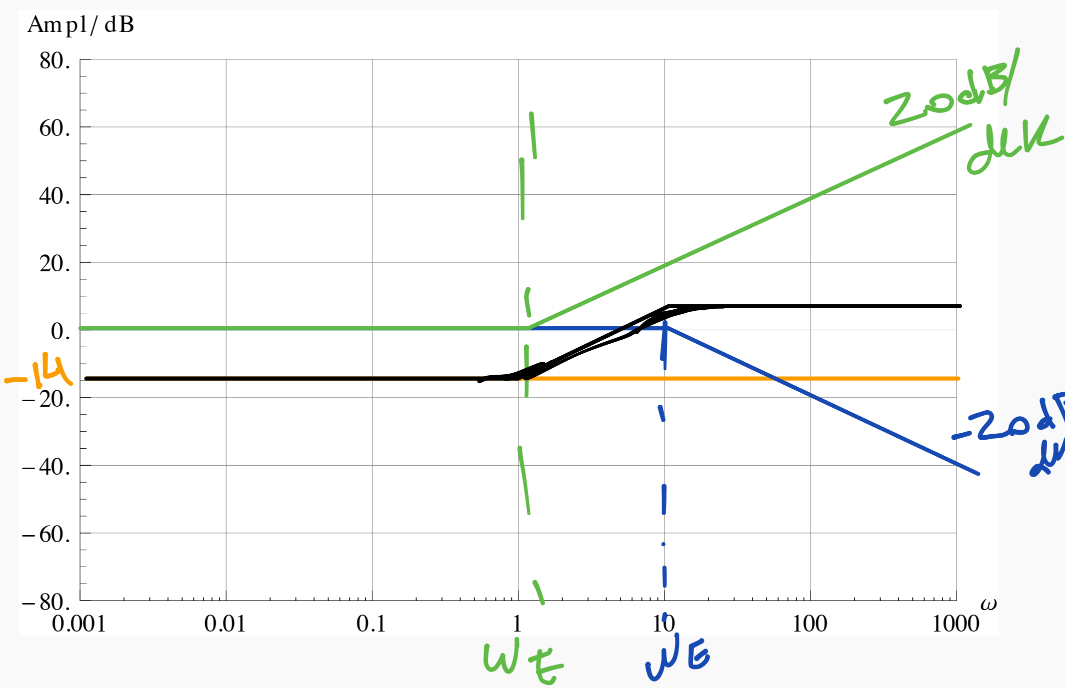
PT1 System

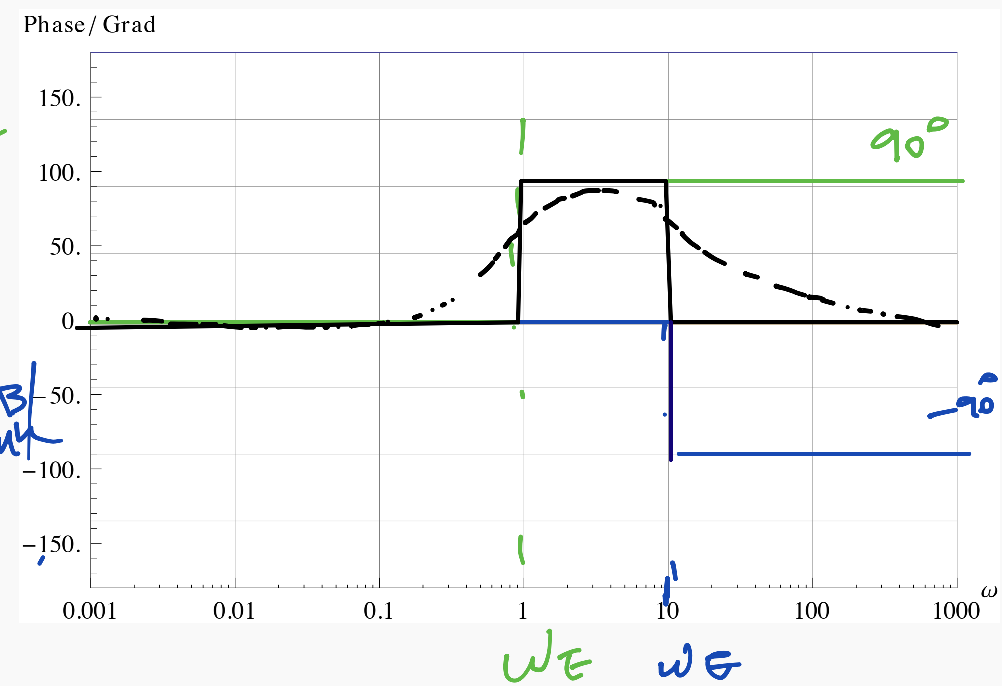
$\rightarrow 0^\circ \rightarrow -90^\circ$

$\omega_G = 10\,\mathrm{rad/s}$

$$\text{(iii)}\;\; G(s) = \frac{2(1+s)}{10+s} = \frac{1}{5}\cdot(1+s)\cdot\frac{1}{1+s/10}$$

↳ $\omega_E = 1\,\mathrm{rad/s}$





Aufgabe 2.1

(iv) $G(s) = \frac{2(1-s/4)}{1+s/4} = 2 \cdot \left(1 - \frac{s}{4}\right) \cdot \frac{1}{1+s/4}$

PGlied

$0° \rightarrow -90°$

$20\log(2) \approx 6dB$

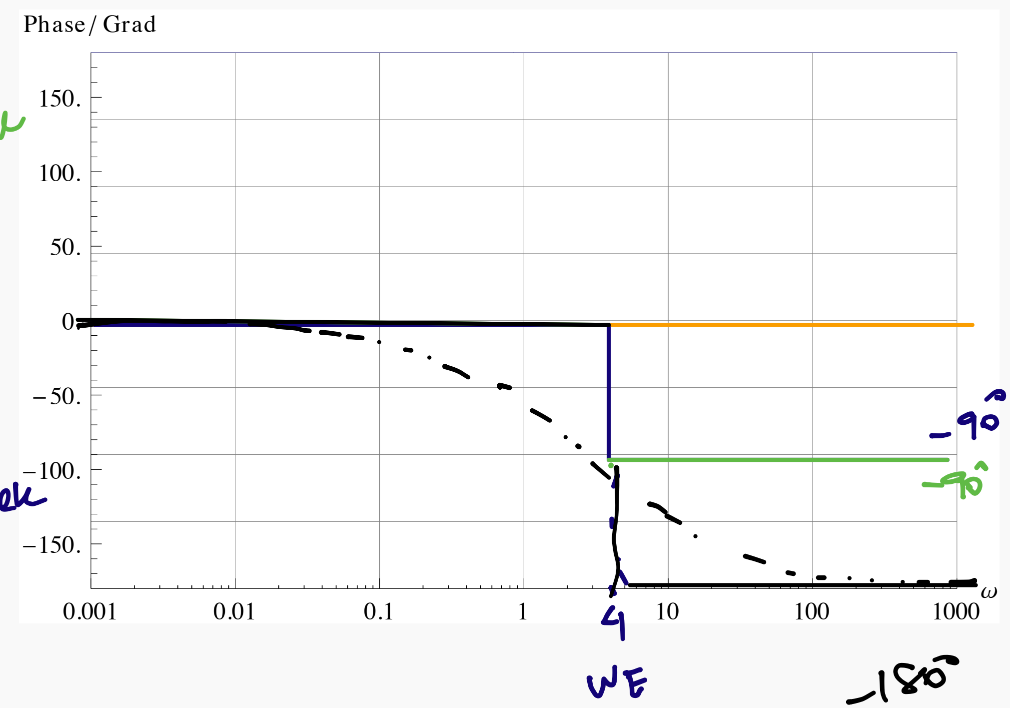
PT_1-System

Phase: $0° \rightarrow -90°$

$\omega_E = 4$ rad/s

PD = -4 rad/s

Allpassglied



Aufgabe 2.1

$P_1, P_2 = -0{,}5 \pm 0{,}87i$

$\rightarrow$ Phase: $0° \rightarrow -180°$

(v) $G(s) = \frac{1+s}{1+s+s^2} = (1+s) \cdot \frac{1}{1+s+s^2}$

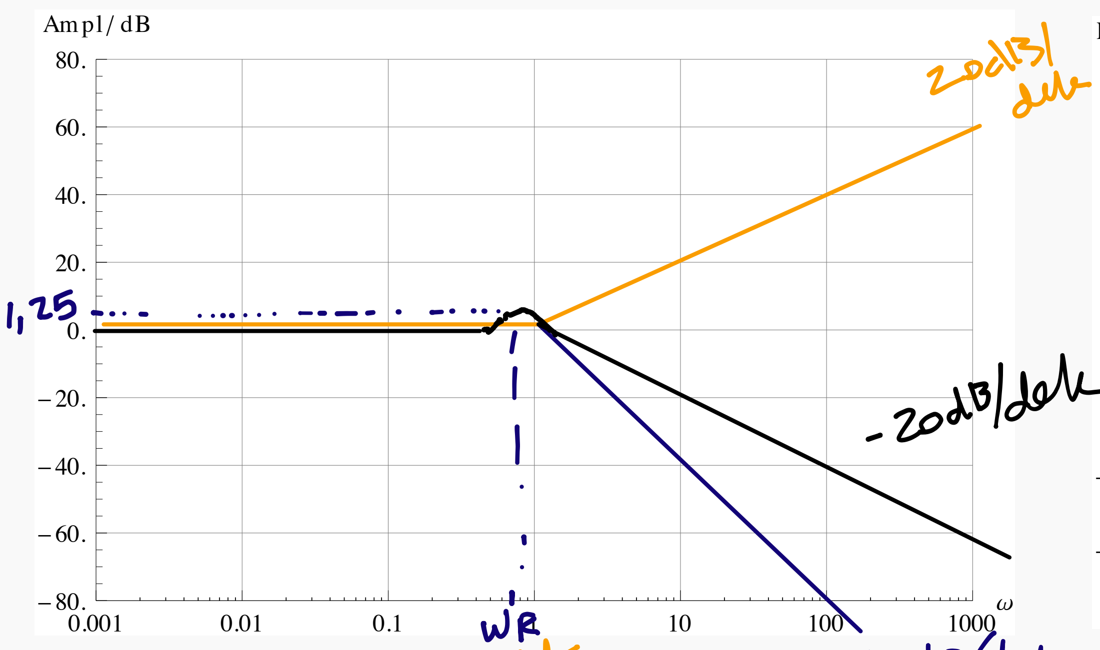
PD-Glied $\omega_E = 1$ rad/s $0° \rightarrow 90°$

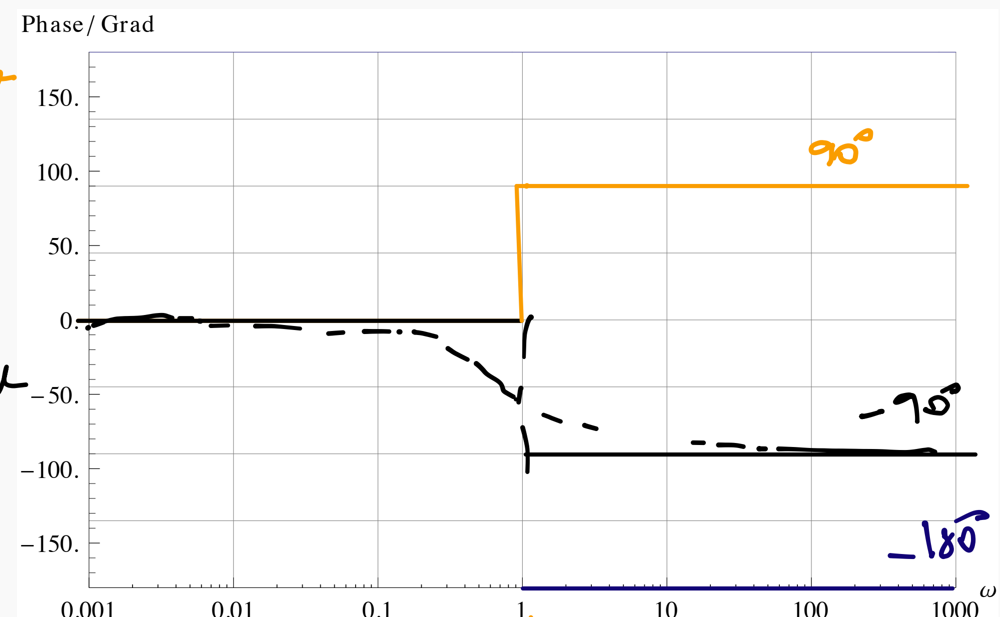
PT2

PT2: $G(s) = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{2D}{\omega_0} s + 1}$

$\frac{2D}{\omega_0} := 1 \rightarrow \boxed{D = 1/2}$

$\omega_0^2 = 1 \rightarrow \boxed{\omega_0 = 1\,\text{rad/s}}$





ωE   ω0   -40dB/dek   ωE   ω0

Resonanzfrequenz:

$\omega_R = \omega_0\sqrt{1-2D^2} \rightarrow$ max.

$M_r = 1/(2D\sqrt{1-D^2})$

$\omega_R = 1\sqrt{1-2(1/2)^2} = 0{,}7$ rad/s

$M_r = 1/(2D\sqrt{1-D^2}) \approx 1{,}15 \rightarrow 20\log(1{,}15) \approx 1{,}25$ dB

$P_{1,P} = +0{,}5 \pm 0{,}87j$

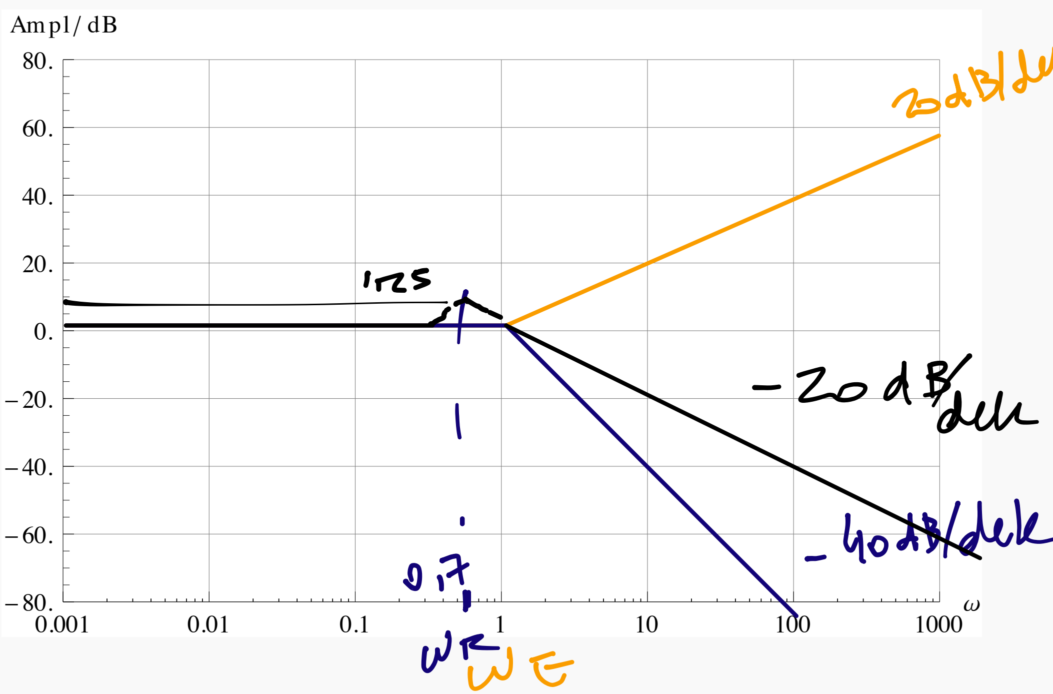
↳ $0° \rightarrow 180°$

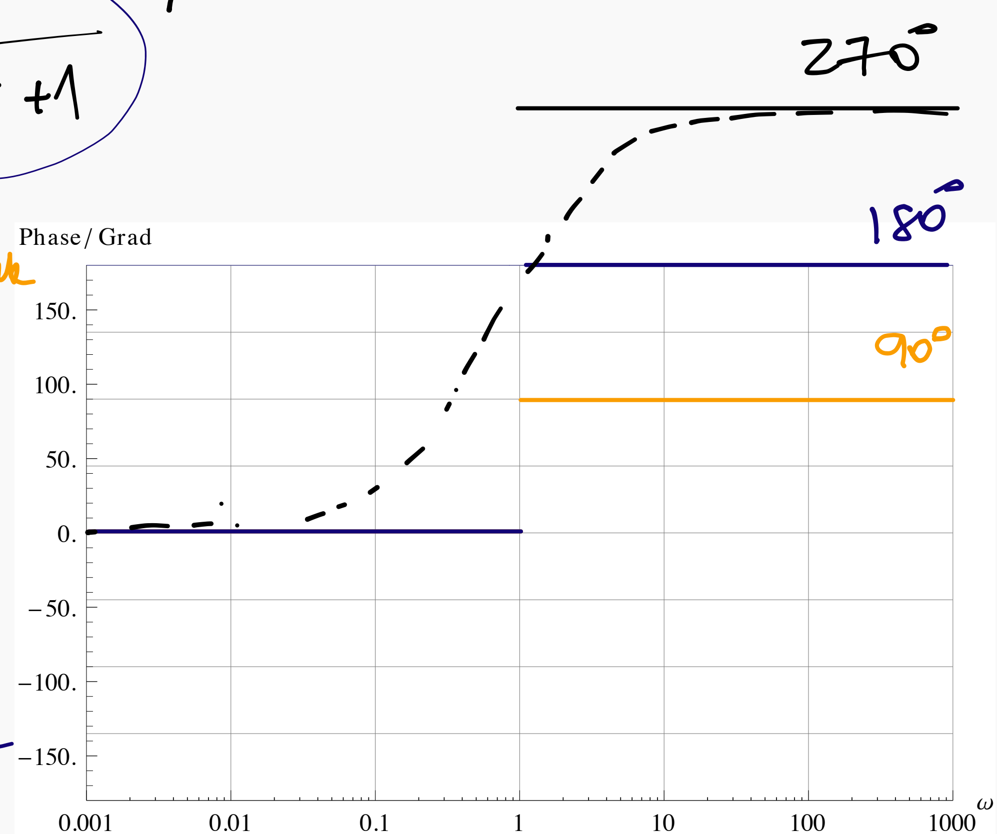
PD-Glied

PT2

$\omega_0 = 1\,\mathrm{rad/s}$

(vi) $G(s) = \frac{1+s}{1-s+s^2} = (1+s)\,\frac{1}{s^2-s+1}$





Aufgabe 2.1

PD $\Rightarrow$ $\omega_E = 3$ rad/s

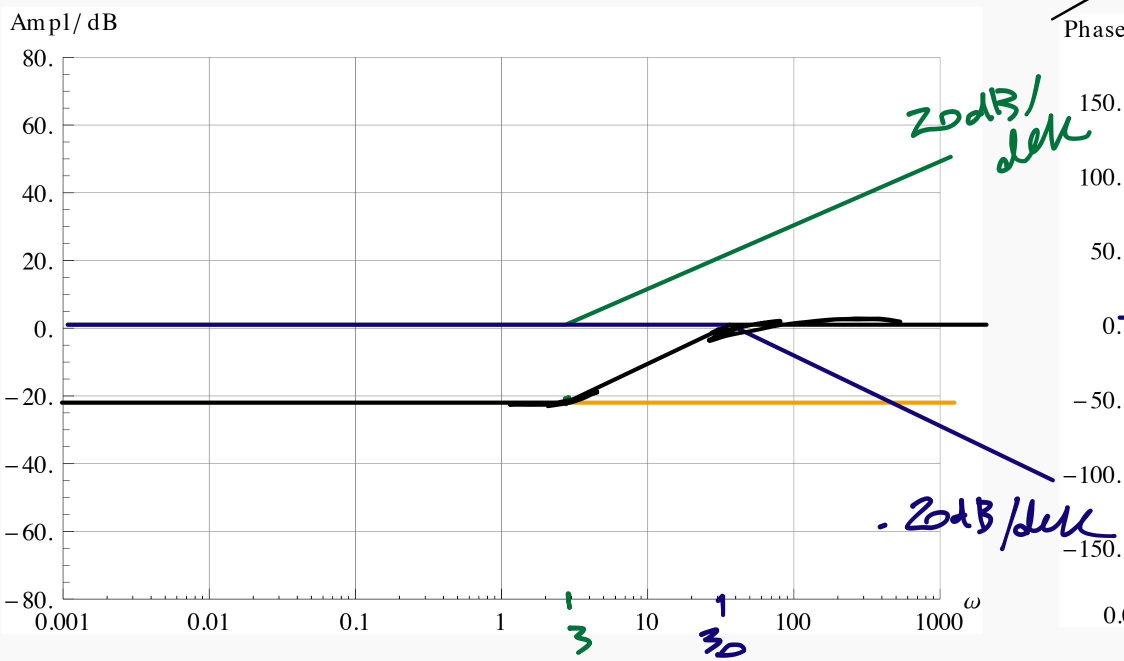
$0° \rightarrow 90°$

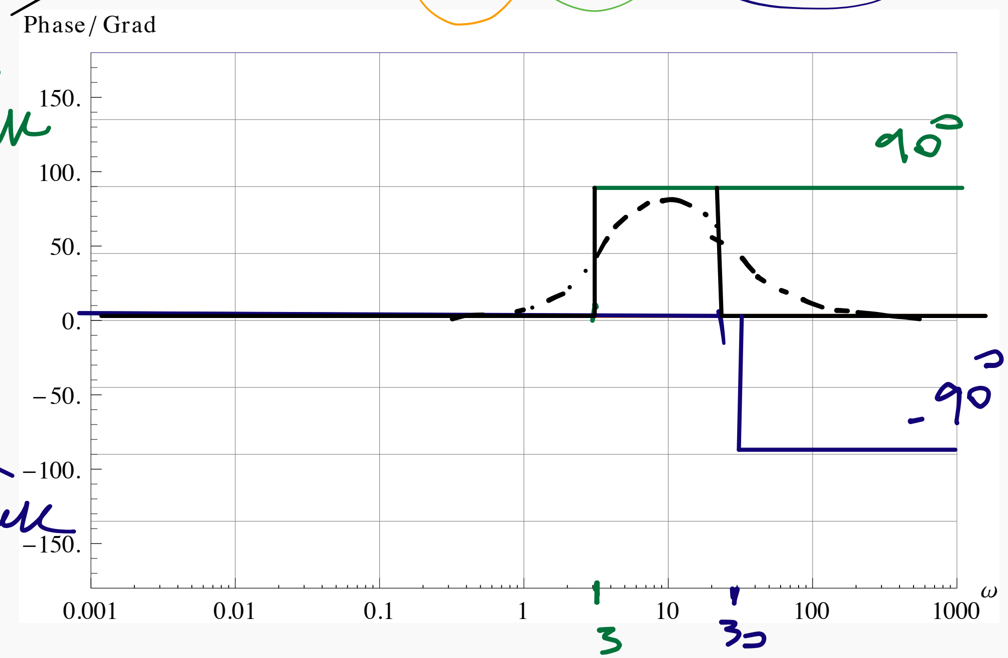
$12 + 7s + s^2 = 0 \rightarrow n_1 = -4,\ n_2 = -3$

$120 + 34s + s^2 = 0 \rightarrow p_1 = -4,\ p_2 = -30$

(vii) $G(s) = \frac{12+7s+s^2}{120+34s+s^2} \Rightarrow$

$$G(s) = \frac{12 + 7s + s^2}{120 + 34s + s^2} = \frac{\cancel{(s+4)}(s+3)}{\cancel{(s+4)}(s+30)} = \underbrace{\frac{1}{10}}_{\text{P glied}} \cdot \underbrace{\left(1 + \frac{s}{3}\right)}_{\text{PD}} \underbrace{\left(\frac{1}{1 + s/30}\right)}_{\text{PT1}}$$





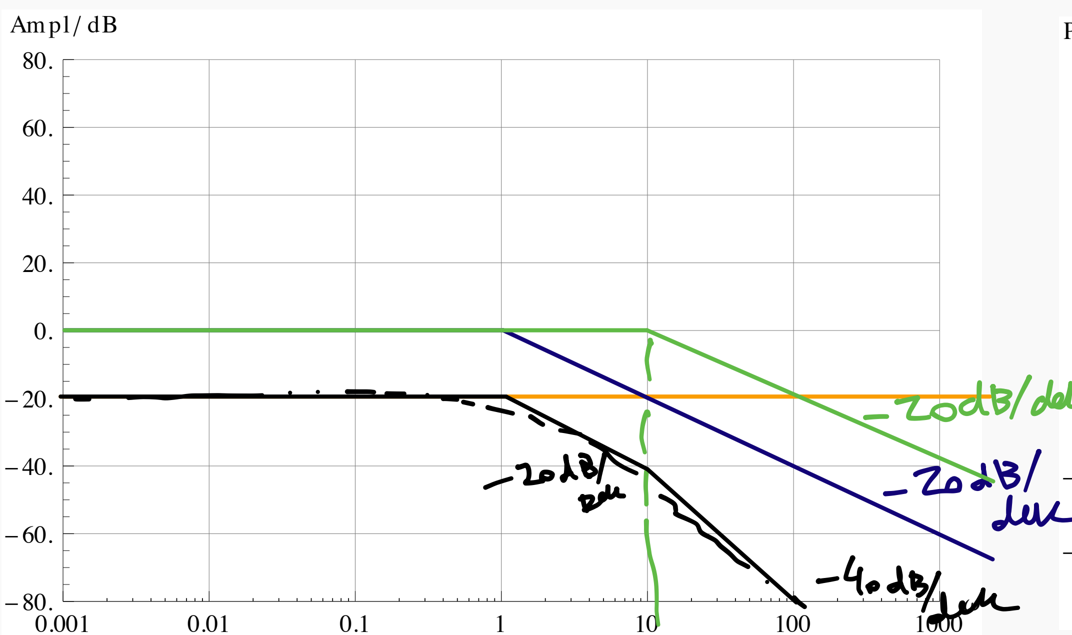
$20 \cdot \log(1/10) = -20$ dB

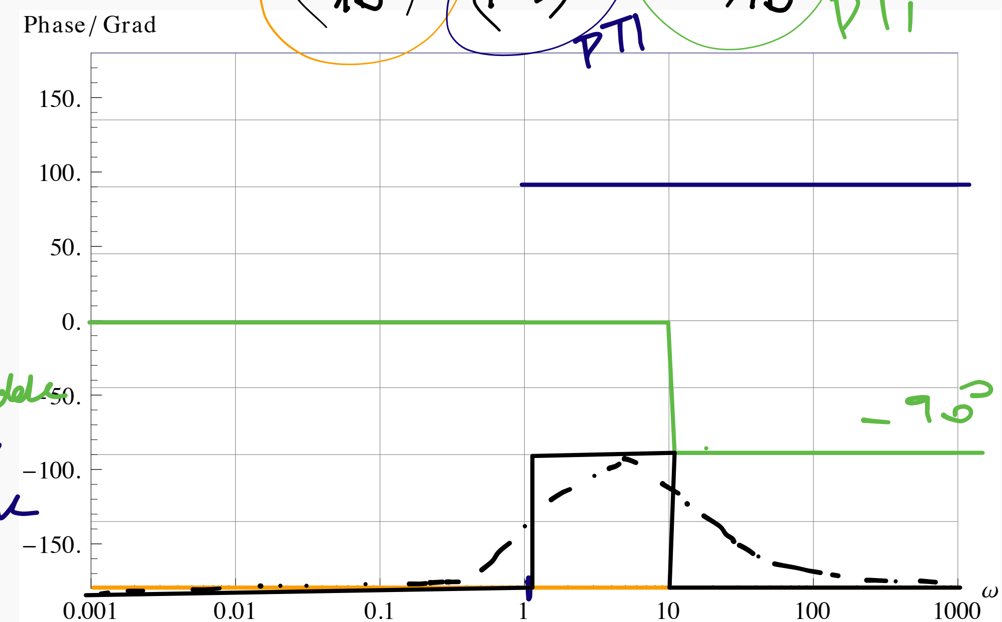
PT1 $\Rightarrow \omega_0 = 30$ rad/s

$0° \rightarrow -90°$

(viii) $G(s) = \frac{1}{s^2+9s-10}$ $\Rightarrow$ $P_1 = -10$, $P_2 = 1$ $\rightarrow$ $G(s) = \frac{1}{s^2+9s-10} = \frac{1}{(s+10)(s-1)} =$

$$= \underbrace{\left(-\frac{1}{10}\right)}_{P} \cdot \underbrace{\frac{1}{(1-s)}}_{PT_1} \cdot \underbrace{\frac{1}{1+s/10}}_{PT_1}$$





neg. Vorzeichen → Phase: −180° (oder 180°)

$20 \log(|-1/10|) = -20\,\text{dB}$

PT_1: $\omega_E = 1\,\text{rad/s}$
Phase: $0° \rightarrow 90°$ (instabil)

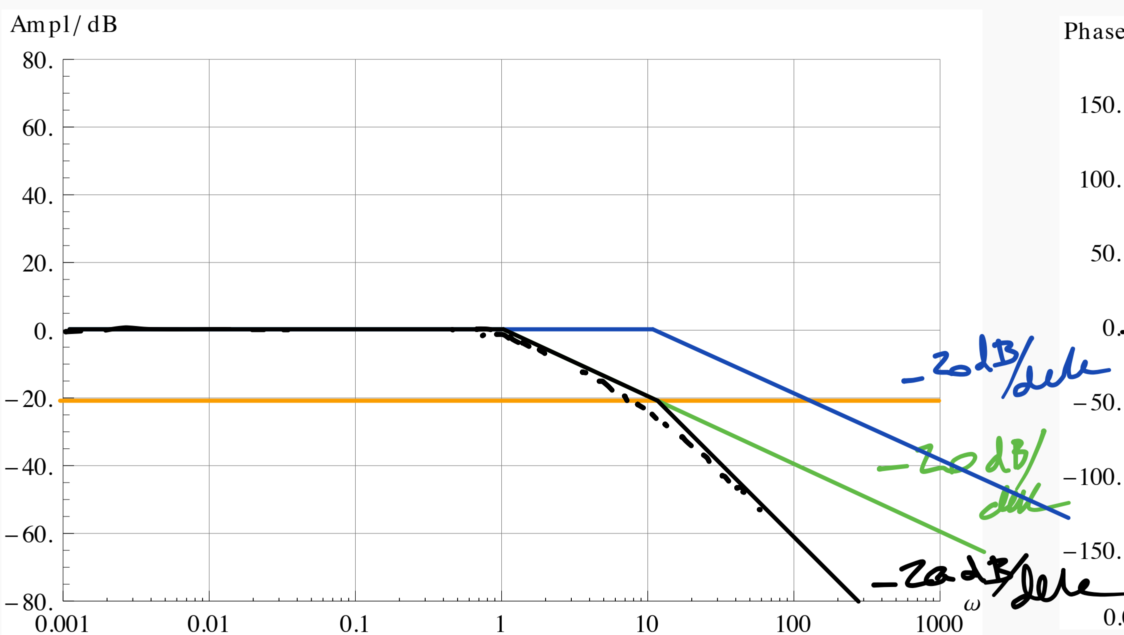
PT_1: $\omega_E = -10\,\text{rad/s}$
Phase $0° \rightarrow -90°$ (stabil)

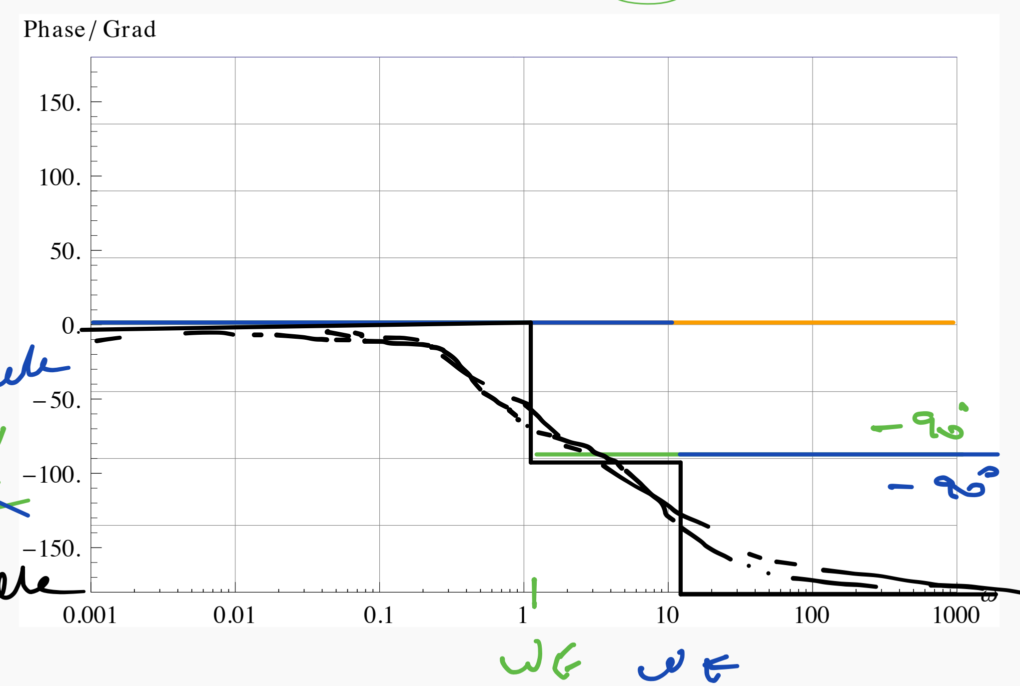
(ix) $G(s) = \frac{1}{s^2+11s+10}$ $\Rightarrow$

Pole: $p_1 = -10$, $p = -1$

$G(s) = \frac{1}{(s+1)} \cdot \frac{1}{(s+10)} = \frac{1}{10} \cdot \frac{1}{(s+1)} \cdot \frac{1}{(s+1/10)}$

P Glied PT1 PT1





$20\log(1/10) = 20dB$
Phase $= 0°$

$\omega_0 = 1$ rad/s
stabil $\rightarrow 0° - 90°$

$\omega_0 = 10$ rad/s
stabil $\rightarrow 0° \rightarrow -90°$