

Regelgröße

$$\begin{aligned}
 Y(s) &= G_0(s) \cdot E(s) \\
 &= G_0(s)[W(s) - Y(s)] \\
 &= G_0(s)W(s) - G_0(s)Y(s)
 \end{aligned}$$

Definition:

Die **Führungsübertragungsfunktion** $G_W(s)$ gibt die Wirkung der Führungsgröße $W(s)$ auf die Regelgröße $Y(s)$ an. Für den Standard-Regelkreis berechnet sie sich durch:

$$Y(s)[1 + G_0(s)] = G_0(s)W(s) \quad \Rightarrow \quad G_W(s) = \frac{Y(s)}{W(s)} = \frac{G_0(s)}{1 + G_0(s)}$$

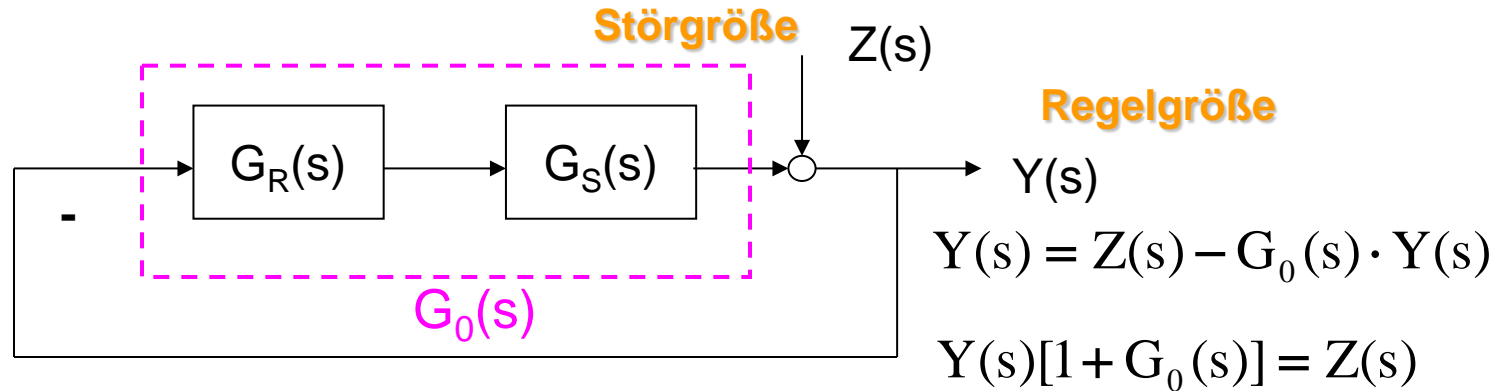
Führungsverhalten:

$$Y(s) = G_W(s) \cdot W(s)$$

Regelungsaufgabe:

$$Y(s) := W(s) \quad \Rightarrow \quad G_W(s) = 1$$





Definition:

Mit der **Störübertragungsfunktion $G_z(s)$** lassen sich die Wirkungen der externen Störungen $Z(s)$ auf die Regelgröße $Y(s)$ berechnen. Für den Standard-Regelkreis lautet sie:

$$G_z(s) = \frac{Y(s)}{Z(s)} = \frac{1}{1 + G_0(s)}$$

Störverhalten:

$$Y(s) = G_z(s) \cdot Z(s)$$

Regelungsaufgabe:

$$Y(s) := 0 \quad \Rightarrow \quad G_z(s) = 0$$

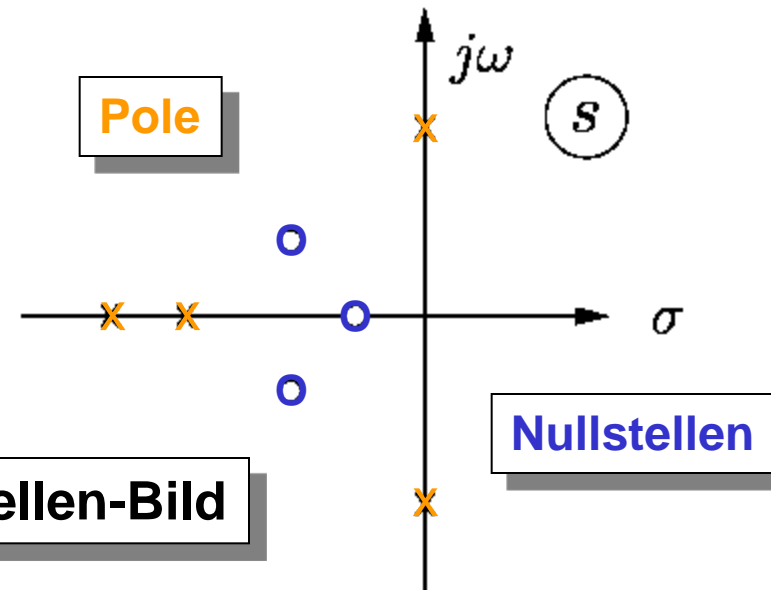


Darstellungsformen

$$G(s) = \frac{b_0 + b_1 \cdot s^1 + \dots + b_m \cdot s^m}{a_0 + a_1 \cdot s^1 + \dots + a_n \cdot s^n} \quad (\text{Polynomform})$$

$$G(s) = \frac{b_m}{a_n} \cdot \frac{\prod_{j=1}^m (s - n_j)}{\prod_{i=1}^n (s - p_i)} \quad (\text{Pol-Nullstellen-Form})$$

Das Pol-Nullstellen-Bild beschreibt eine Übertragungsfunktion bis auf den Faktor b_m/a_n vollständig !!



Pol- Nullstellen-Bild



Definition 2.4 Wurzelort und Wurzelortskurve

Der **Wurzelort** ist der geometrische Ort der Lösungen (**Wurzeln**) der **charakteristischen** Gleichung

$$1 + G_0(s) = 0$$

des **geschlossenen** Regelkreises in der komplexen Ebene.

Die **Wurzelortskurve (WOK)** stellt die Abhängigkeit der **Wurzelorte** von einem **Parameter** (vielfach der Verstärkung K_0) des **offenen** Regelkreises dar.

$$1 + \frac{Z_0(s)}{N_0(s)} = \frac{N_0(s) + Z_0(s)}{N_0(s)} = 0$$



Charakteristisches Polynom des **geschlossenen** Regelkreises:

$$C(s) = N_0(s) + Z_0(s)$$



Verfahrenstechnik

Rückführungen wurden eingesetzt um den Einfluß von **Störungen** zu reduzieren und Prozeßgrößen auf vorgegebene Werte zu halten.

Luft- und Schifffahrtstechnik

Rückführungen wurden eingesetzt um das System zu **stabilisieren** und den Einfluß von **Störungen** (Wind, Luftdruck) zu reduzieren.

Nachrichtentechnik

Rückführungen wurden eingesetzt um den Einfluß von **Parameterschwankungen** (Eigenschaften der elektronischen Röhren) und **Nichtlinearitäten** zu reduzieren.



Normierte Differentialgleichung

$$\frac{1}{\omega_0^2} \ddot{y}(t) + \frac{2D}{\omega_0} \dot{y}(t) + y(t) = u(t)$$

schwingungsfähiges PT₂-System

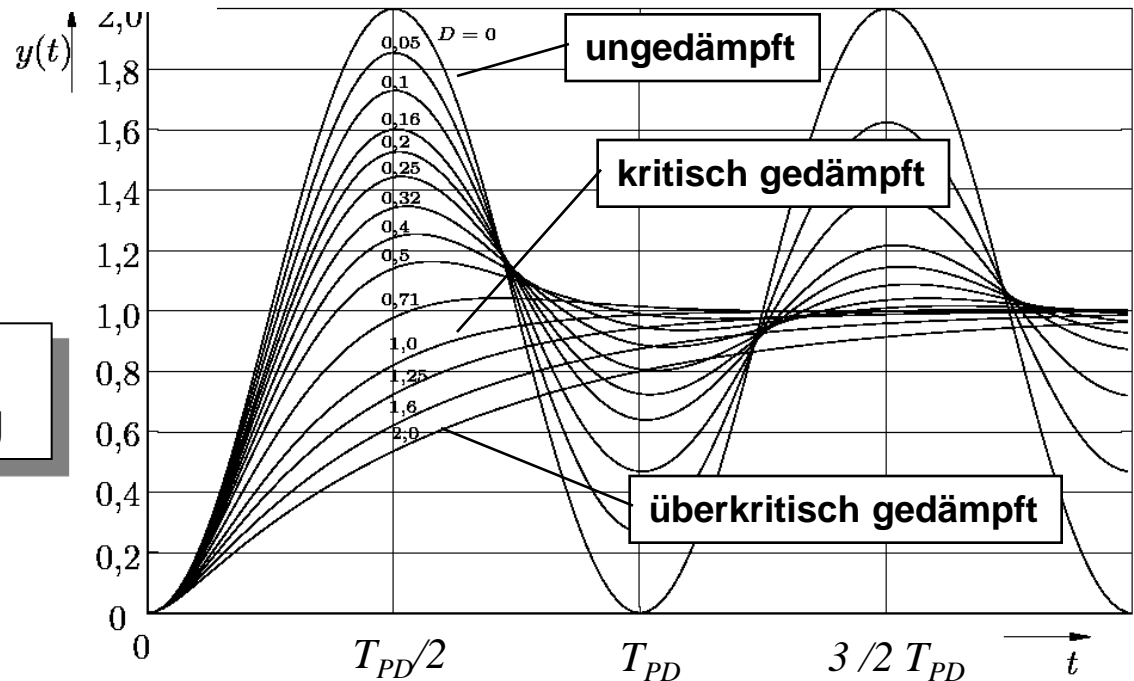
Dämpfungsgrad

$$D = \frac{d}{2} \sqrt{\frac{1}{mc}}$$

Eigen(kreis)frequenz der ungedämpften Schwingung

$$\omega_0 = \sqrt{\frac{c}{m}}$$

$$\omega_0 = 2\pi \cdot 1 / T_{PD}$$



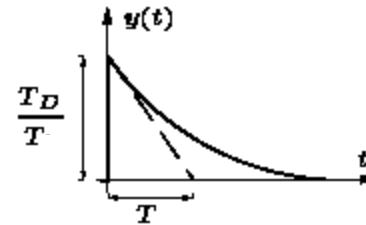
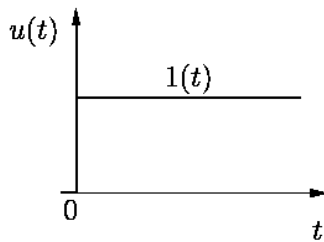
Sprungantwort



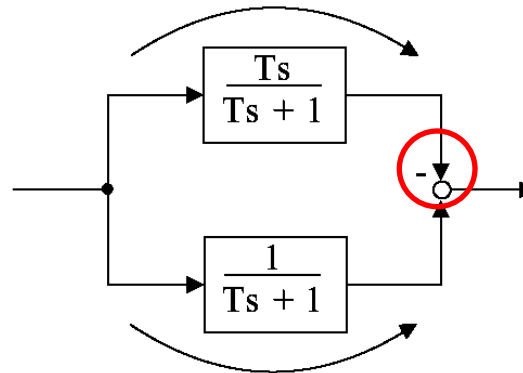
$$G(s) = \frac{1 - sT}{1 + sT}$$

$$= \frac{1}{1 + sT} - \frac{Ts}{1 + sT}$$

$\underbrace{\hspace{1.5cm}}_{PT_1}$
 $\underbrace{\hspace{1.5cm}}_{DT_1}$



schneller Wirkungsweg



langsamer Wirkungsweg

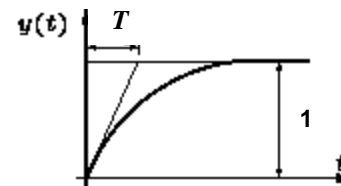
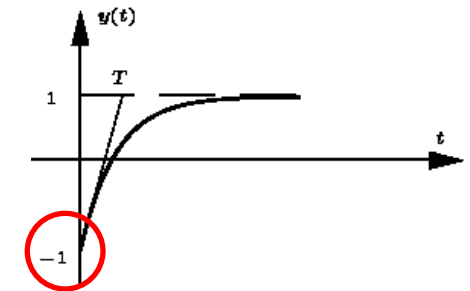


Tabelle 2.3: Verhalten der wichtigsten Regelkreisglieder

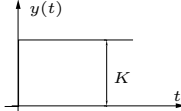
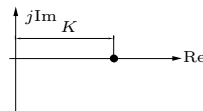
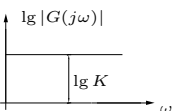
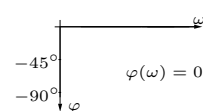
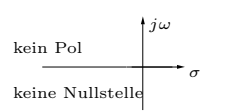
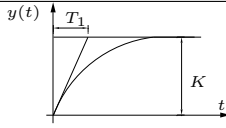
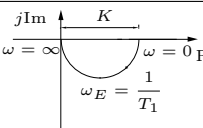
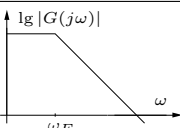
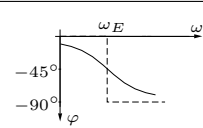
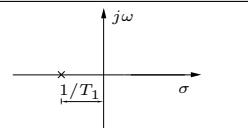
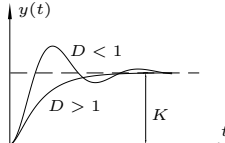
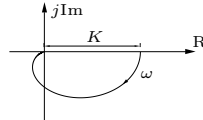
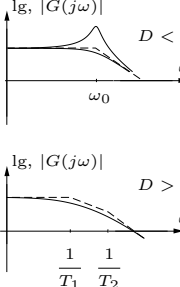
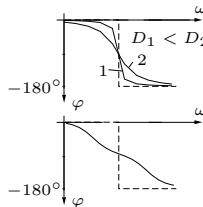
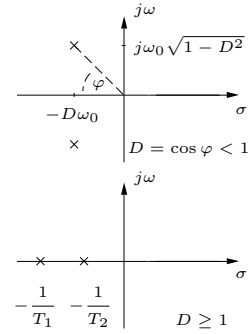
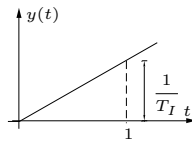
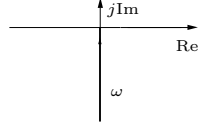
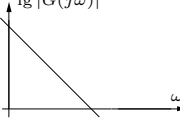
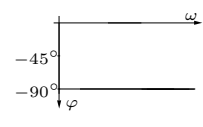
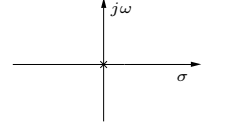
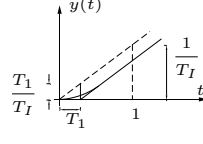
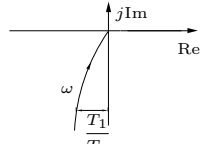
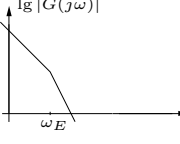
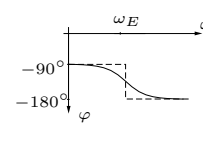
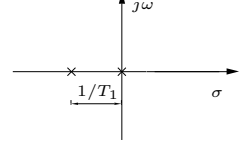
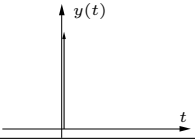
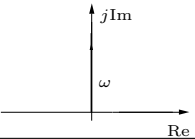
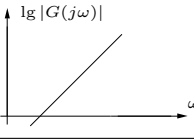
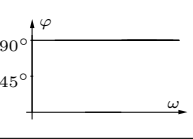
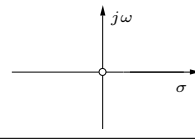
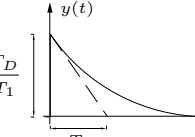
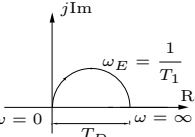
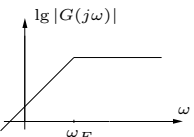
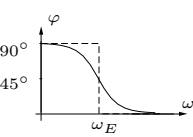
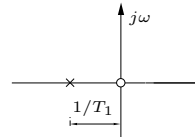
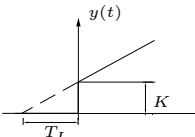
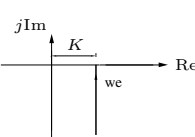
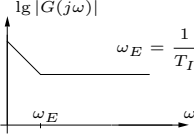
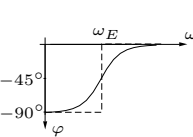
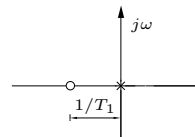
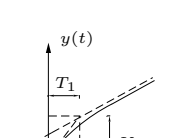
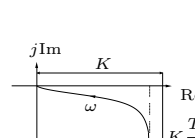
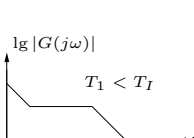
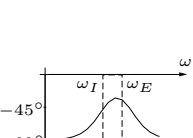
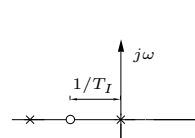
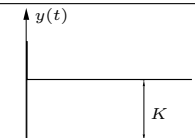
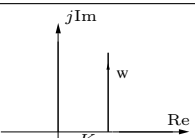
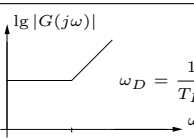
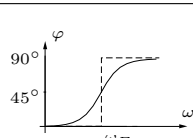
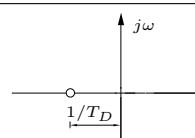
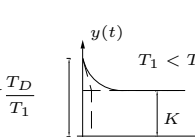
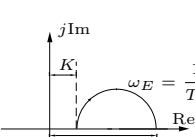
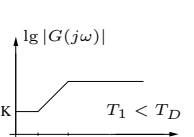
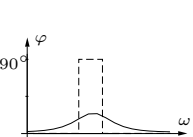
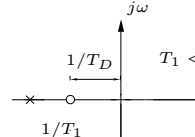
System	Zeitbereich Bildbereich	Übergangsfunktion	Ortskurve	Bode-Diagramm (Amplitudengang) (Phasengang)		s-Ebene × Pol ○ Nullstelle
P	$y(t) = K u(t)$ $G(s) = K$					
PT_1	$T_1 \dot{y}(t) + y(t) = K u(t)$ $G(s) = K \frac{1}{1 + T_1 s}$					
PT_2	$\frac{1}{\omega_0^2} \ddot{y}(t) + \frac{2D}{\omega_0} \dot{y}(t) + y(t) = K u(t)$ $G(s) = K \frac{1}{\frac{1}{\omega_0^2} s^2 + \frac{2D}{\omega_0} s + 1}$ $D < 1$: konjugiert komplexe Wurzeln der char. Gleichung $\lambda_{1,2} = -\omega_0(D \pm j\sqrt{1-D^2})$ $D \geq 1$: reelle Wurzeln der char. Gleichung $\lambda_{1,2} = -\omega_0(D \pm \sqrt{D^2-1}) = -1/T_{1,2}$					
I	$y(t) = \frac{1}{T_I} \int u dt$ $G(s) = \frac{1}{T_I s}$					
IT_1	$T_1 \dot{y}(t) + y(t) = \frac{1}{T_I} \int u(t) dt$ $G(s) = \frac{1}{T_I s(1 + T_1 s)}$					

Tabelle 2.3: Fortsetzung

System	Zeitbereich Bildbereich	Übergangsfunktion	Ortskurve	Bode-Diagramm		s-Ebene × Pol ○ Nullstelle
				(Amplitudengang)	(Phasengang)	
D	$y(t) = T_D \frac{du}{dt}$ $G(s) = T_D s$					
DT_1	$T_1 \dot{y}(t) + y(t) = T_D \frac{du}{dt}$ $G(s) = T_D \frac{s}{1 + T_1 s}$					
PI	$y(t) = K \left[u(t) + \frac{1}{T_I} \int u(t) dt \right]$ $G(s) = K \left[1 + \frac{1}{T_I s} \right]$					
PIT_1	$T_1 \dot{y}(t) + y(t) = K \left[u(t) + \frac{1}{T_I} \int u dt \right]$ $G(s) = K \frac{1 + \frac{1}{T_I} s}{1 + T_1 s}$					
PD	$y(t) = K [u(t) + T_D \dot{u}(t)]$ $G(s) = K [1 + T_D s]$					
PDT_1	$T_1 \dot{y}(t) + y(t) = K [u(t) + T_D \dot{u}(t)]$ $G(s) = K \frac{1 + T_D s}{1 + T_1 s}$					

Differentialgleichung:

$$\frac{1}{\omega_0^2} \ddot{y}(t) + \frac{2D}{\omega_0} \dot{y}(t) + y(t) = K u(t)$$

Übertragungsfunktion:

$$G(s) = \frac{K}{\frac{s^2}{\omega_0^2} + \frac{2D}{\omega_0} s + 1}$$

Pole: Aus $s^2 + 2D\omega_0 s + \omega_0^2 = 0$ folgt

$$s_{1,2} = -D\omega_0 \pm \sqrt{(D\omega_0)^2 - \omega_0^2}$$

$$|s_{1,2}| = \sqrt{(D\omega_0)^2 + \omega_0^2(1 - D^2)} = \omega_0$$

$$= -D\omega_0 \pm \omega_0 \sqrt{D^2 - 1}$$

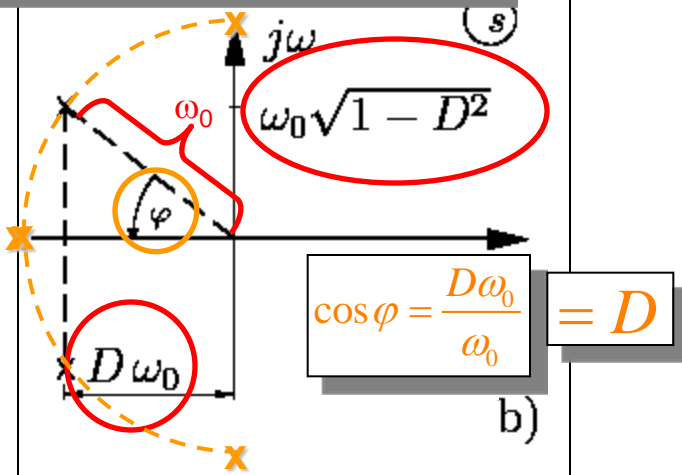
Für $D < 1$ ergibt sich:

$$s_{1,2} = -D\omega_0 \pm j\omega_0 \sqrt{1 - D^2}$$

Für $D = 1$ ergibt sich: $s_1 = s_2 = -\omega_0$

und für $D = 0$:

$$s_{1,2} = \pm j\omega_0$$

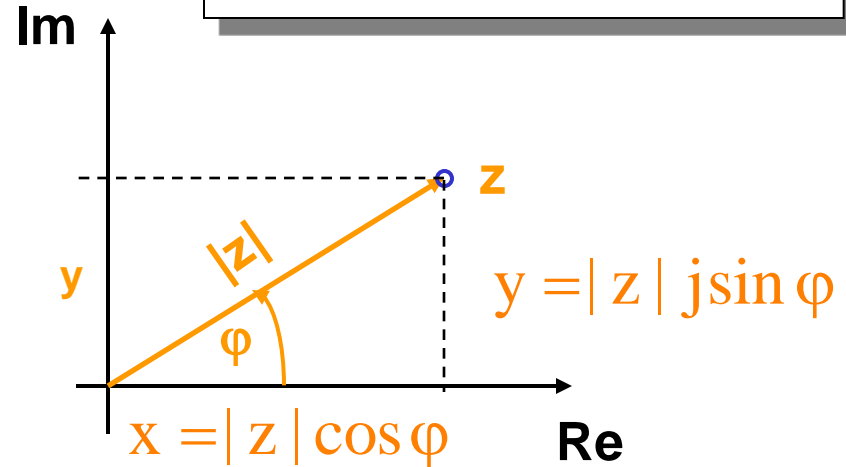


Kartesische Koordinaten

$$z = x + jy$$

Darstellung als **Zeiger** (Vektor) in der komplexen Zahlenebene (Gaußschen Zahlenebene).

Gaußsche Zahlenebene



Polarkoordinaten

$$z = |z| (\cos \varphi + j \sin \varphi)$$

Exponentialform

Aus der Eulersche Identität

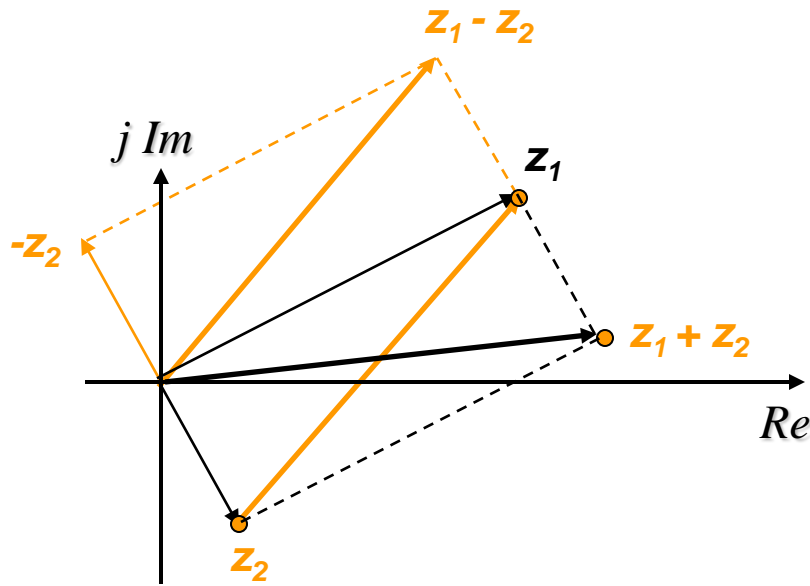
$$e^{j\varphi} = \cos \varphi + j \sin \varphi$$

folgt $z = |z| e^{j\varphi} = |z| e^{j\angle z}$



Addition und Subtraktion

$$\begin{aligned} z_1 + z_2 &= (x_1 + jy_1) + (x_2 + jy_2) \\ &= (x_1 + x_2) + j(y_1 + y_2) \end{aligned}$$



Multiplikation

$$\begin{aligned} z_1 \cdot z_2 &= (x_1 + jy_1) \cdot (x_2 + jy_2) \\ &= r_1 e^{j\varphi_1} \cdot r_2 e^{j\varphi_2} \\ &= r_1 \cdot r_2 \cdot e^{j(\varphi_1 + \varphi_2)} \end{aligned}$$

