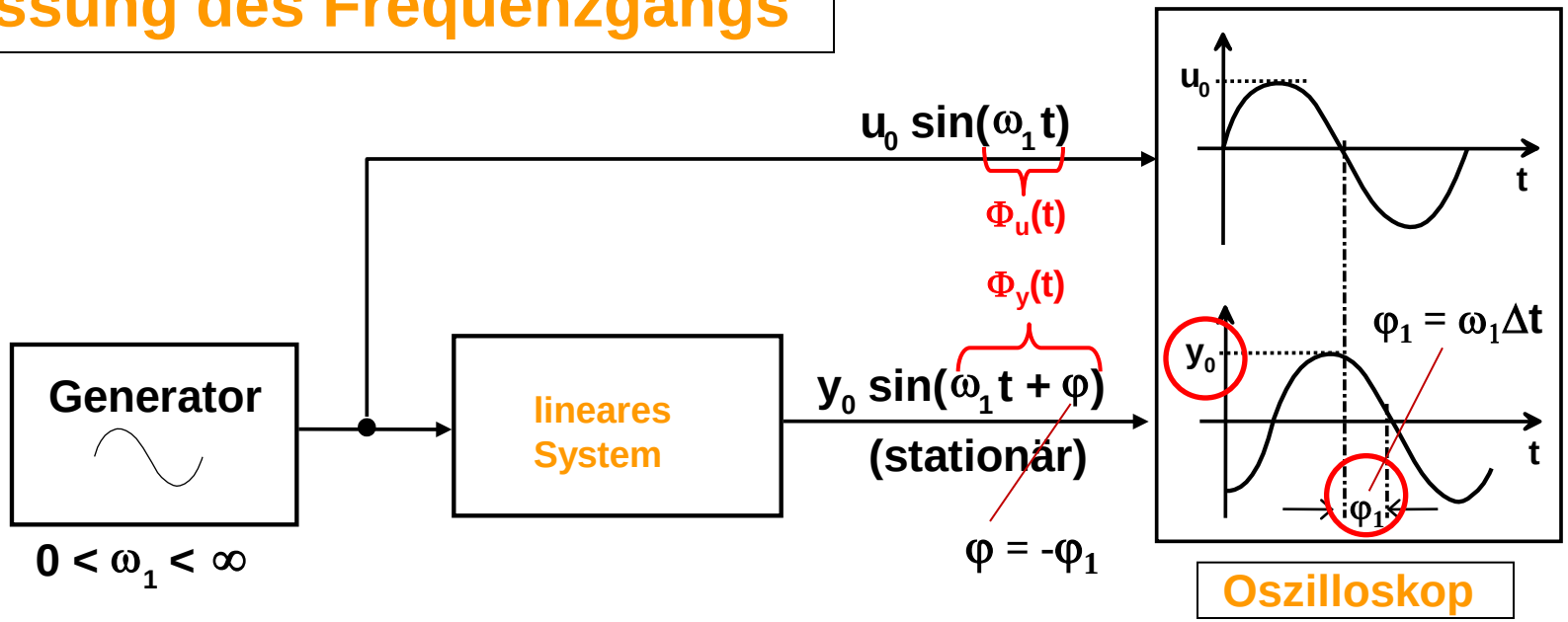
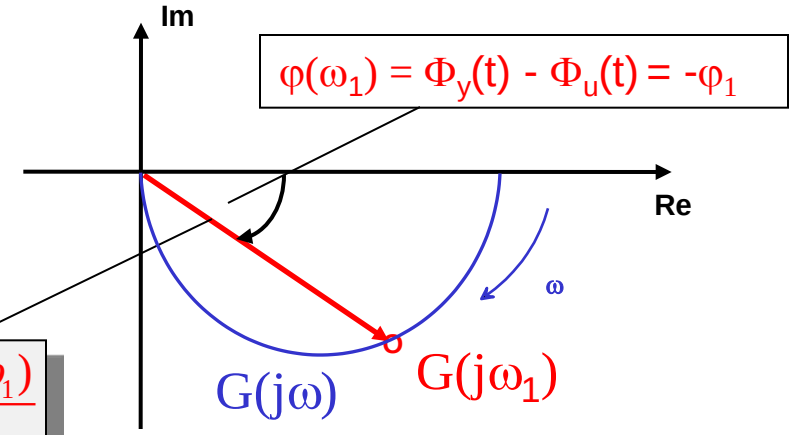
Messung des Frequenzgangs



Frequenzgang

$$G(j\omega_1) = \frac{y_0(\omega_1)}{u_0} e^{j\varphi(\omega_1)}$$

$$|G(\omega_1)| = \frac{y_0(\omega_1)}{u_0}$$



Frequenzgangdarstellung (2)

Frequenzgang:

$$G(j\omega) = \frac{y_0(\omega)}{u_0} e^{j\varphi(\omega)}$$

beschreibt wie ein dynamisches System eine sinus-förmige Eingangsgröße überträgt (stationäres Verhalten).

Amplitudengang:

$$|G(j\omega)| = \sqrt{\mathrm{Re}\,(G(j\omega))^2 + \mathrm{Im}\,(G(j\omega))^2}$$

ist ein Maß für die Amplitudenveränderung (frequenzabhängiger Verstärkungsfaktor).

Phasengang:

$$\varphi(\omega) = \arg G(j\omega) = \arctan \frac{\mathrm{Im}\, G(j\omega)^*}{\mathrm{Re}\, G(j\omega)}$$

gibt an mit welcher Verspätung das Ausgangssignal dem Eingangssignal folgt.

* Mehrdeutigkeit von arctan entsprechend der Vorzeichen von Im *G(jω)* und Re *G(jω)* muß berücksichtigt werden.

Frequenzgangdarstellung (3)

a) $G(j\omega)$ bestimmt die Signalübertragungseigenschaften eines Systems für harmonische Signale und kann beispielsweise experimentell bestimmt werden.

b) $G(j\omega)$ ist die Fouriertransformierte der Gewichtsfunktion $G(j\omega) = \mathcal{F}\{g(t)\}$

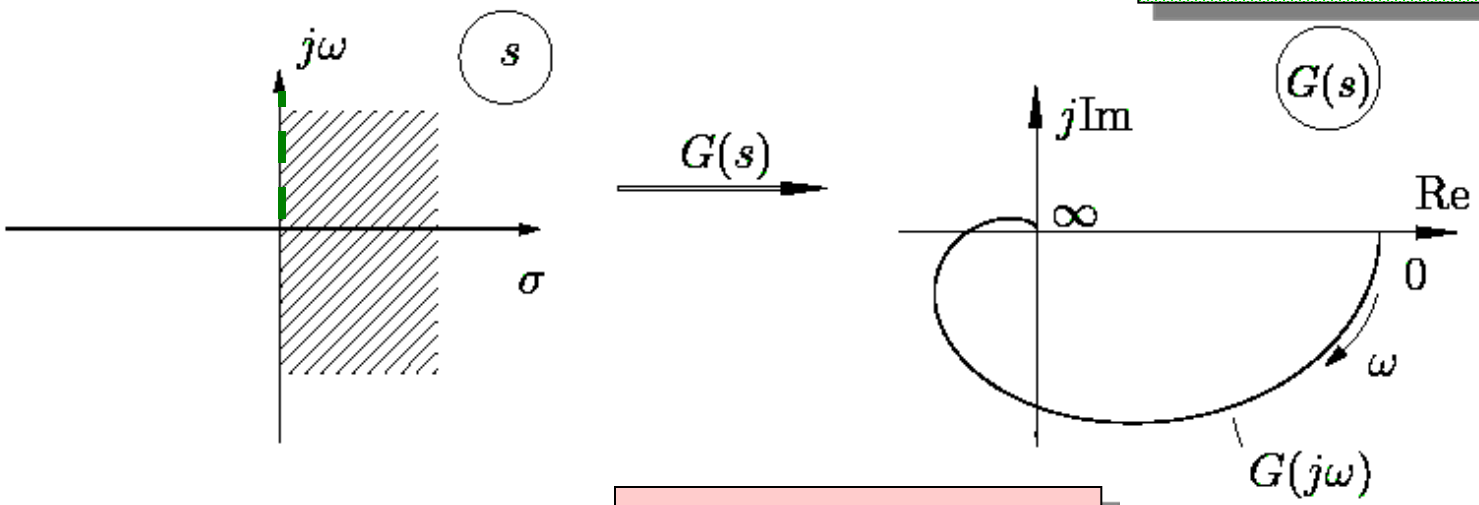
c) $G(j\omega)$ ist eine Randfunktion der komplexen Übertragungsfunktion $G(s)$:

$$G(j\omega) = G(s)|_{s=j\omega}$$

Laplacetransformation:

$s = \sigma + j\omega$

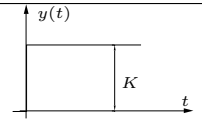
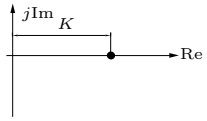
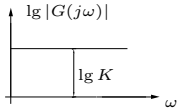
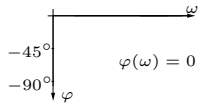
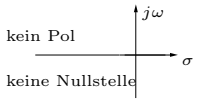
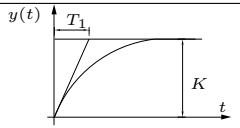
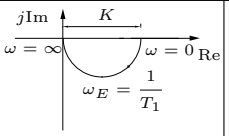
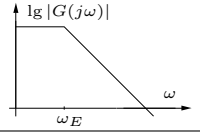
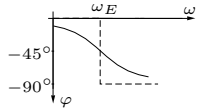
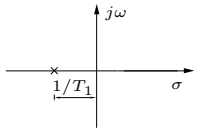
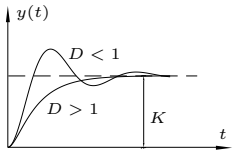
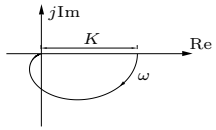
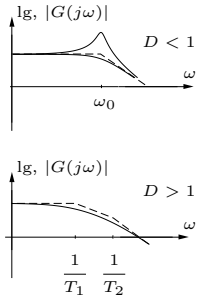
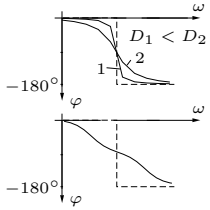
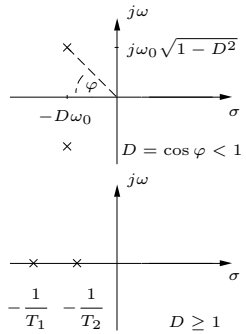
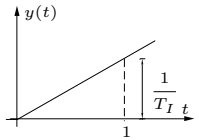
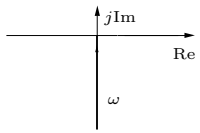
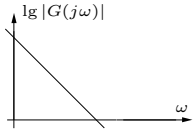
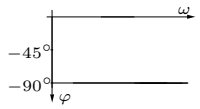
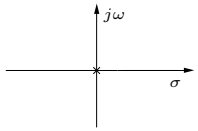
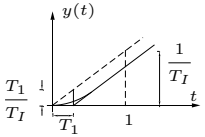
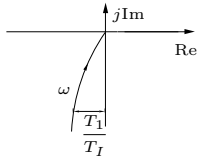
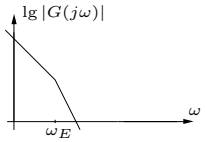
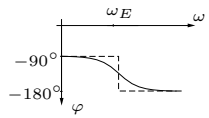
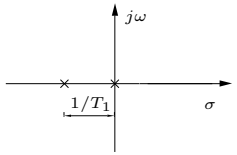
$$F(s) = \int_0^\infty f(t)e^{-st}dt$$



Fouriertransformation:

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

Tabelle 2.3: Verhalten der wichtigsten Regelkreisglieder

System	Zeitbereich Bildbereich	Übergangsfunktion	Ortskurve	Bode–Diagramm (Amplitudengang)	 (Phasengang)	s–Ebene × Pol ∘ Nullstelle
P	$y(t) = K\,u(t)$ $G(s) = K$					
PT_1	$T_1\,\dot{y}(t) + y(t) = K\,u(t)$ $G(s) = K\dfrac{1}{1+T_1\,s}$					
PT_2	$\dfrac{1}{\omega_0^2}\ddot{y}(t) + \dfrac{2\,D}{\omega_0}\dot{y}(t) + y(t)$ $= K\,u(t)$ $G(s) = K\dfrac{1}{\dfrac{1}{\omega_0^2}s^2 + \dfrac{2\,D}{\omega_0}s + 1}$ $D < 1$: konjugiert komplexe Wurzeln der char. Gleichung $\lambda_{1,2} = -\omega_0(D \pm j\sqrt{1-D^2})$ $D \geq 1$: reelle Wurzeln der char. Gleichung $\lambda_{1,2} = -\omega_0(D \pm \sqrt{D^2-1}) = -1/T_{1,2}$					
I	$y(t) = \dfrac{1}{T_I}\displaystyle\int u\mathrm{d}t$ $G(s) = \dfrac{1}{T_I\,s}$					
IT_1	$T_1\,\dot{y}(t) + y(t) =$ $\dfrac{1}{T_I}\displaystyle\int u(t)\mathrm{d}t$ $G(s) = \dfrac{1}{T_I\,s(1+T_1\,s)}$					

Frequenzgang: Ortskurvendarstellung

Darstellung des Frequenzgangs $G(j\omega)$ in der komplexen Ebene in Abhängigkeit von der Frequenz ω.

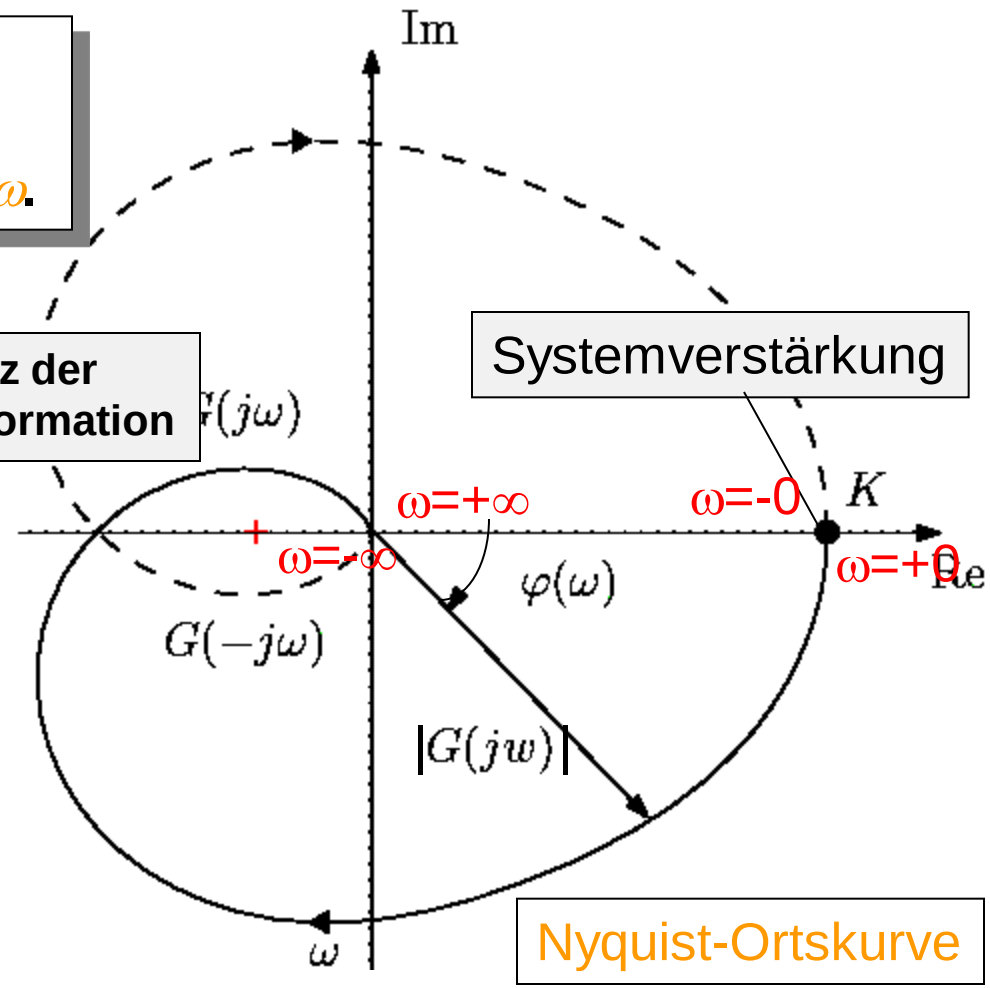
$$K = \lim_{t\to\infty} h(t)$$

Endwertsatz der Laplace-Transformation

$$= \lim_{s\to 0} s \cdot H(s)$$

$$= \lim_{s\to 0} \cancel{s} \cdot G(s) \cdot \frac{1}{\cancel{s}}$$

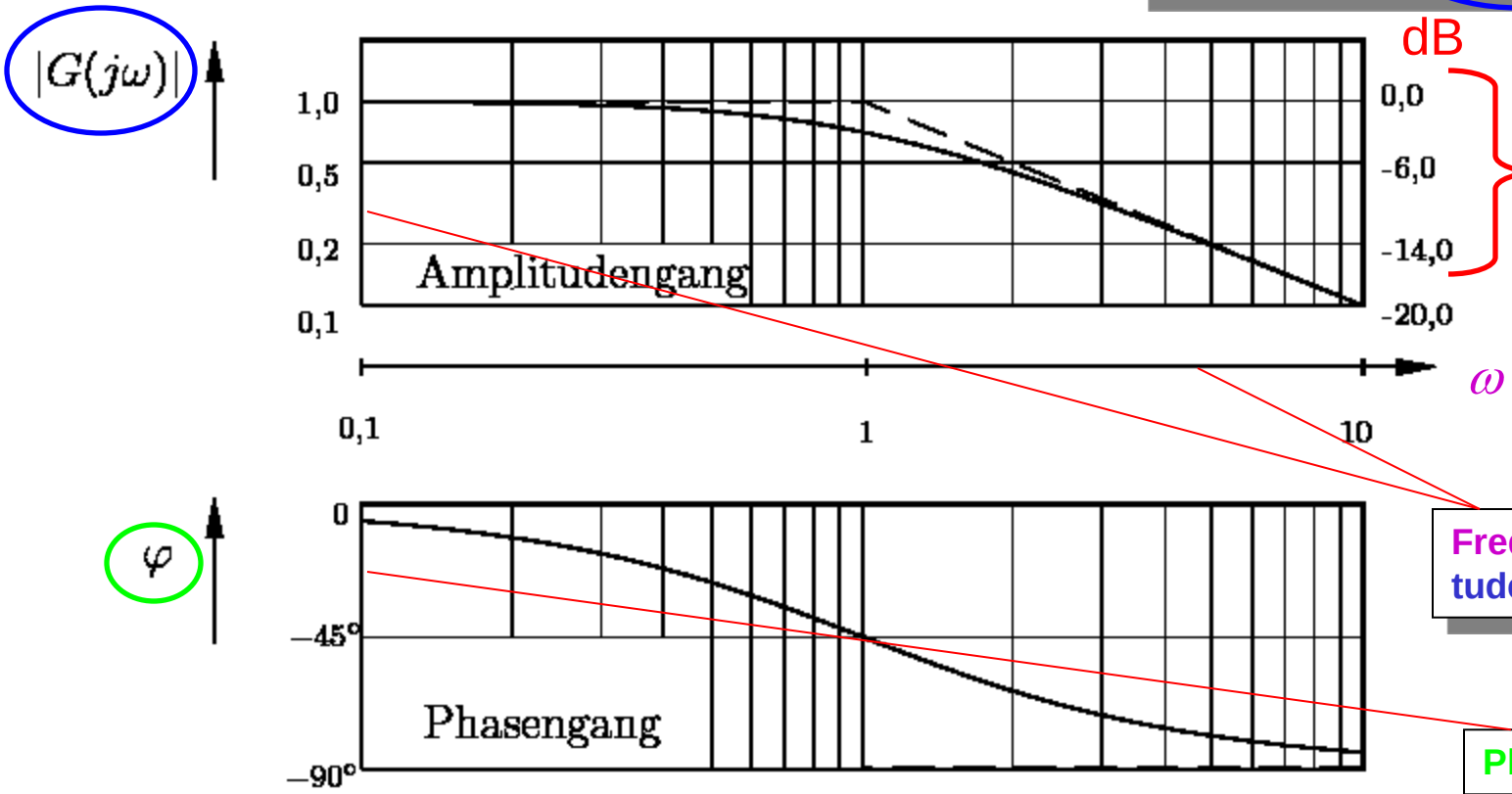
$$= \lim_{j\omega\to 0} G(j\omega)$$



Die vollständige Ortskurve von $\omega \in [-\infty, \infty]$ ist stets symmetrisch zur reellen Achse

Frequenzgang: Bode-Diagramm

$$G(j\omega) = |G(j\omega)|\, e^{j\varphi(\omega)}$$



20 lg |G(jω)| (Dezibel)

Frequenz und Amplitude logarithmisch

Phase linear in Grad

Darstellung nach Betrag (Amplitudengang) und Phase (Phasengang) als Funktion von ω.

Definition: Dezibel

Das Dezibel wird durch das logarithmierte Leistungs- oder Amplitudenverhältnis zweier Signale durch die Vorschriften

10lg(Leistungsverhältnis) [dB]

20lg(Amplitudenverhältnis) [dB]

berechnet.

dB	Leistungsverhältnis	dB	Amplitudenverhältnis
+20	100	+40	100
+10	10	+20	10
+3	2	+6	2
0	1	0	1
-3	1/2	-6	1/2
-10	1/10	-20	1/10
-20	1/100	-40	1/100

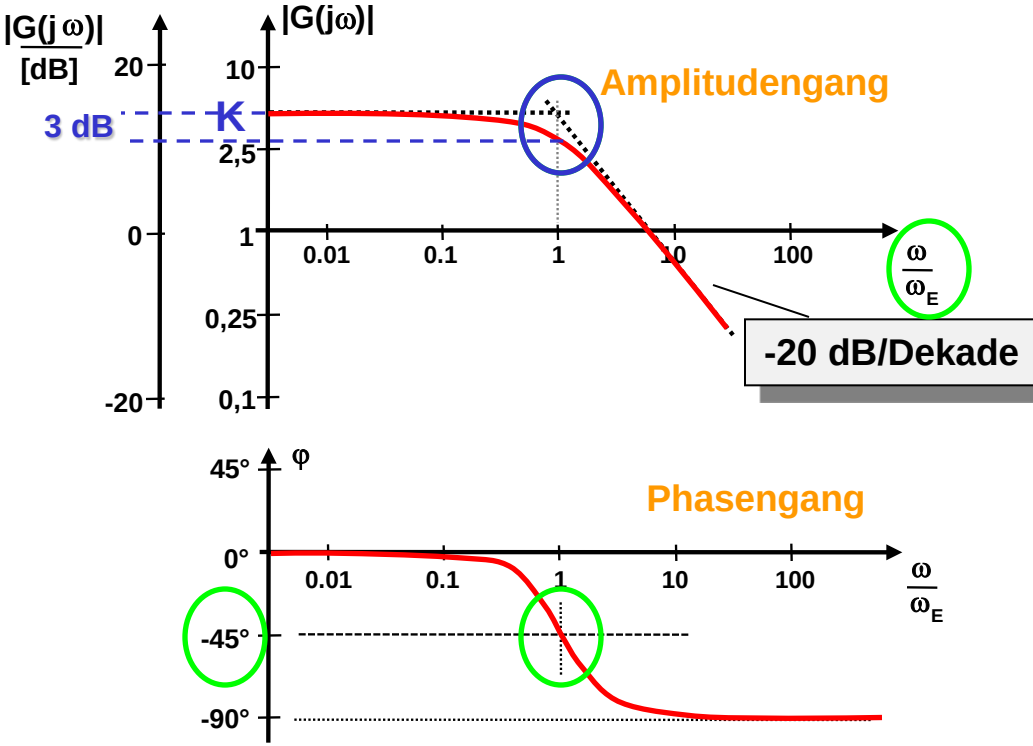
Frequenzgang PT_1-System (2)

Kenngrößen:

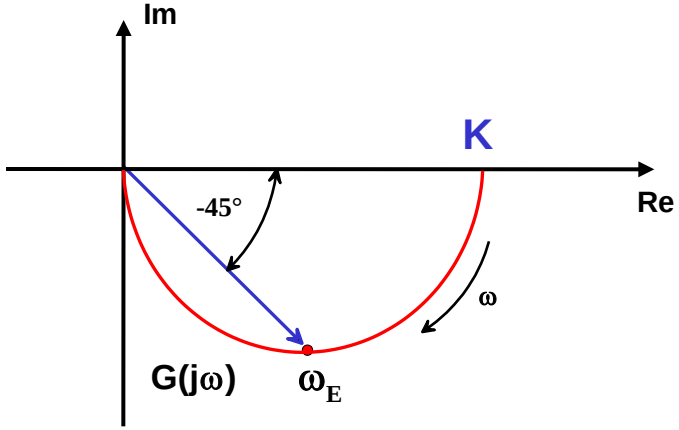
Verstärkung K

Eckfrequenz $\omega_E = 1/T_1$

3 dB-Abfall bei ω_E (Bandbreite).



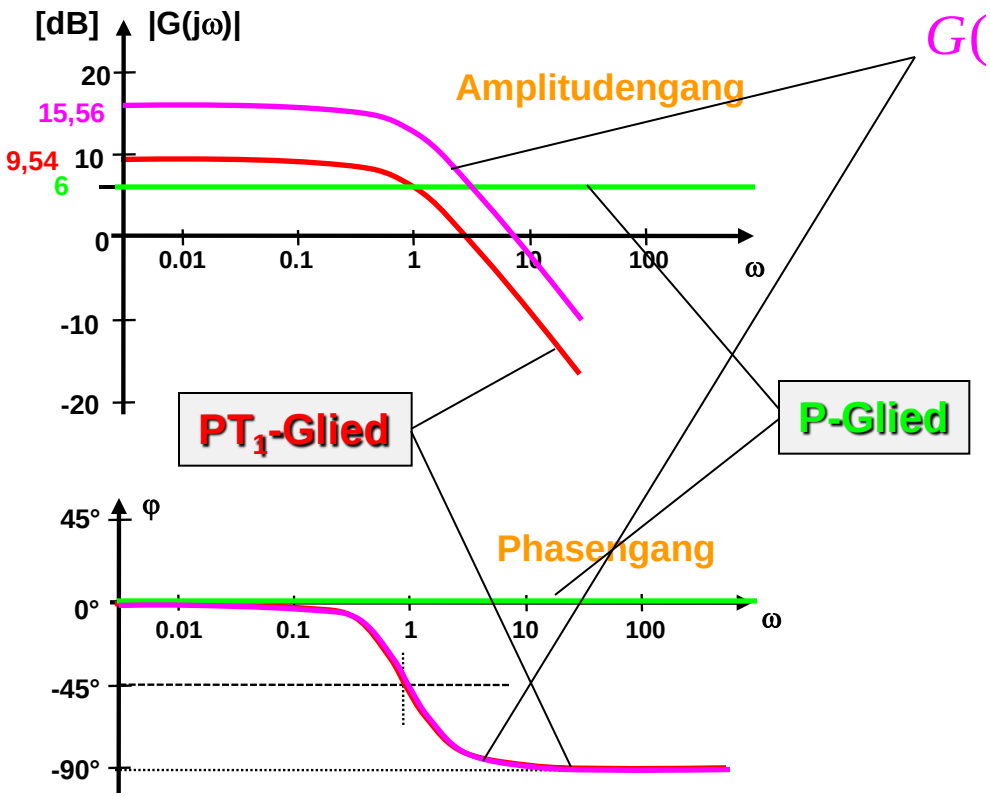
Bode-Diagramm



Ortskurve

Ein PT_1-System zeigt Tiefpassverhalten

Bode-Diagramm einer Reihenschaltung von PT_1- und P-Glied:



$$G(j\omega) = \frac{3}{1 + j\omega} \cdot 2$$

9,54 dB

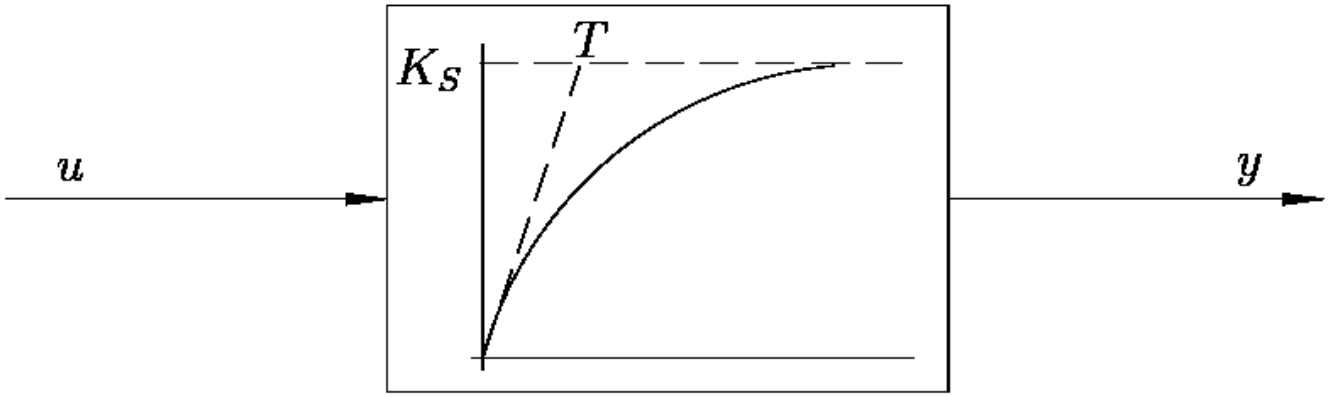
6,02 dB

$(3 \cdot 2)_{dB}$ = 9,54 dB + 6,02 dB
= 15,56 dB

PT1-System (System mit Ausgleich)

Differentialgleichung:

$$T\dot{y}(t) + y(t) = K_S u(t)$$



Blockschaltbild des Systems 1. Ordnung

Kenngrößen: Zeitkonstante T

Systemverstärkung $K_S = h(\infty)$

Beispiel: Mechanischer Schwinger (2)

Normierte Differentialgleichung

$$\frac{1}{\omega_0^2}\ddot{y}(t) + \frac{2D}{\omega_0}\dot{y}(t) + y(t) = u(t)$$

schwingungsfähiges PT_2-System

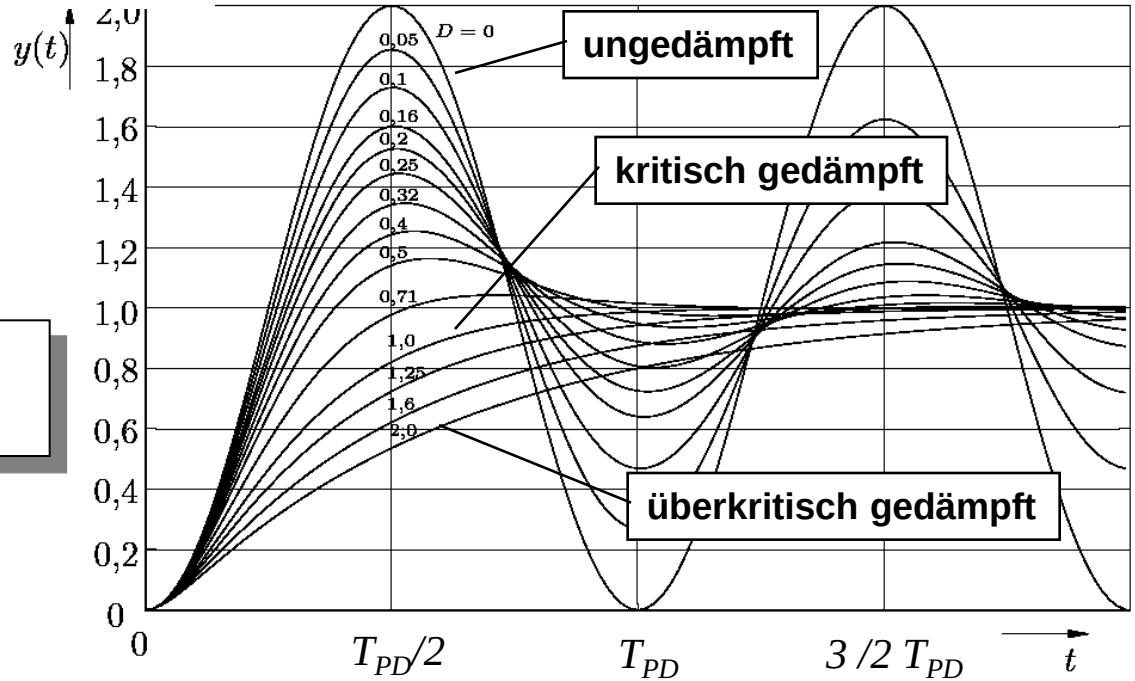
Dämpfungsgrad

$$D = \frac{d}{2}\sqrt{\frac{1}{mc}}$$

Eigen(kreis)frequenz der ungedämpften Schwingung

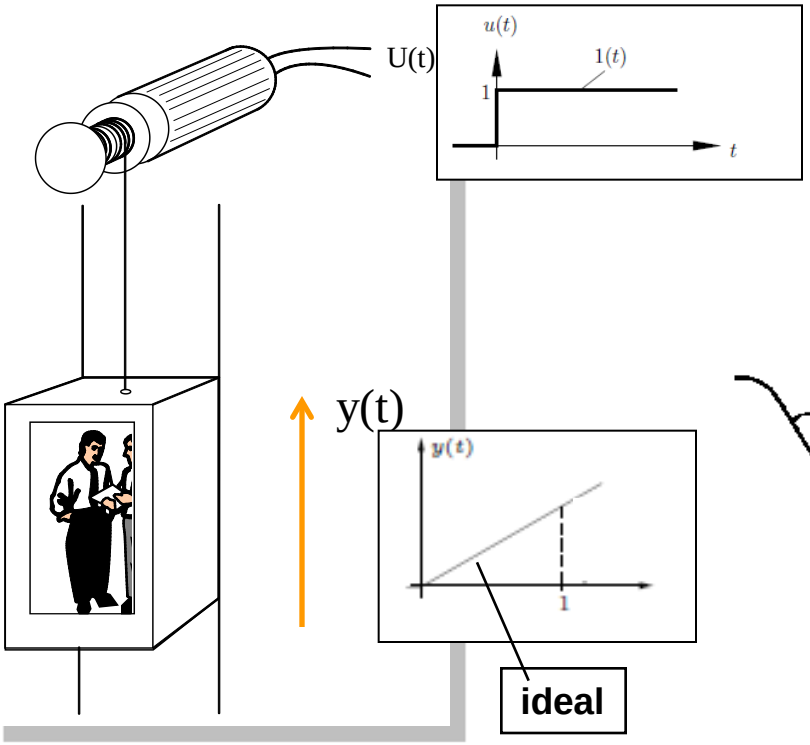
$$\omega_0 = \sqrt{\frac{c}{m}}$$

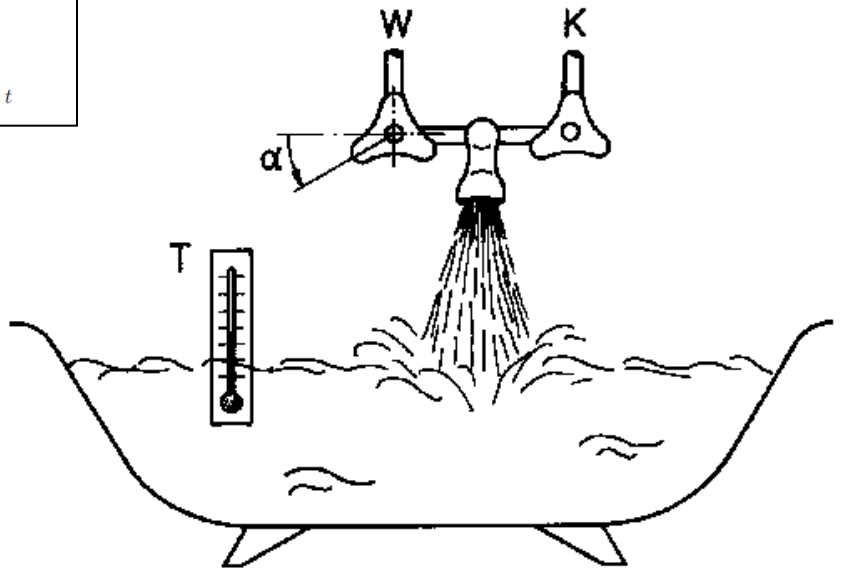
$$\omega_0 = 2\pi \cdot 1/T_{PD}$$



Sprungantwort

Systeme ohne Ausgleich (I-Systeme)

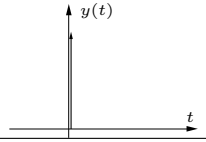
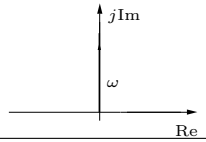
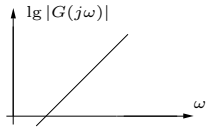
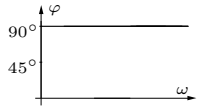
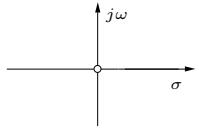
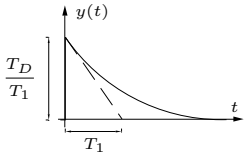
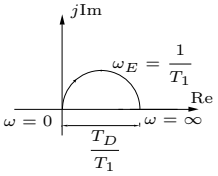
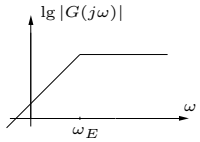
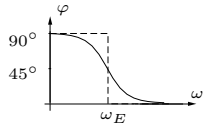
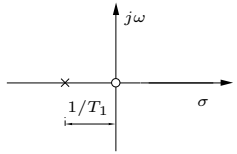
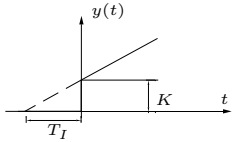
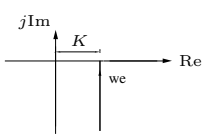
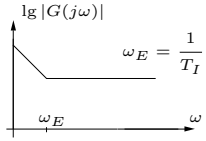
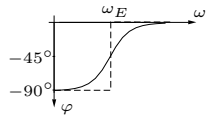
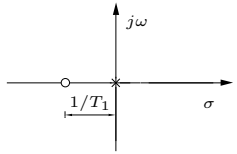
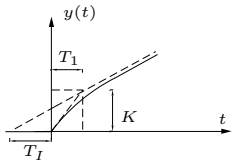
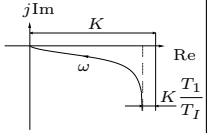
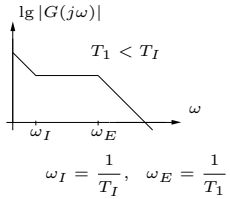
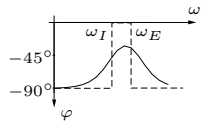
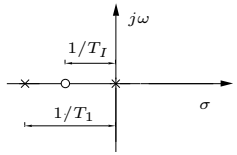
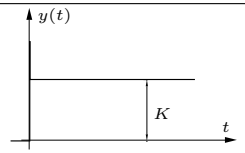
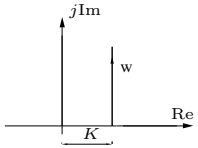
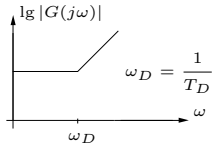
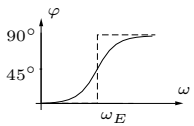
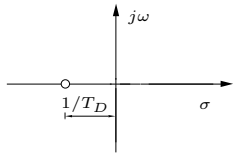
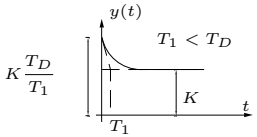
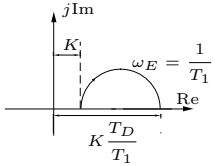
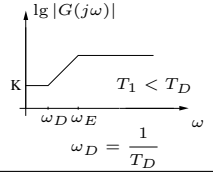
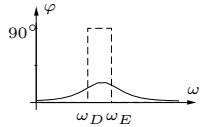
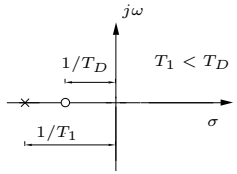




Füllstand einer Badewanne

Zurückgelegter Weg einer Aufzugskabine

Tabelle 2.3: Fortsetzung

System	Zeitbereich Bildbereich	Übergangsfunktion	Ortskurve	Bode–Diagramm (Amplitudengang)	 (Phasengang)	s–Ebene × Pol ∘ Nullstelle
D	$y(t) = T_D \frac{\mathrm{d}u}{\mathrm{d}t}$ $G(s) = T_D\, s$					
DT_1	$T_1\dot{y}(t) + y(t) = T_D \frac{\mathrm{d}u}{\mathrm{d}t}$ $G(s) = T_D \frac{s}{1+T_1\, s}$					
PI	$y(t) = K\left[u(t) + \frac{1}{T_I}\int u(t)\mathrm{d}t\right]$ $G(s) = K\left[1 + \frac{1}{T_I\, s}\right]$					
PIT_1	$T_1\,\dot{y}(t) + y(t) =$ $K\left[u(t) + \frac{1}{T_I}\int u\mathrm{d}t\right]$ $G(s) = K\frac{1+\frac{1}{T_I\, s}}{1+T_1\, s}$					
PD	$y(t) = K\left[u(t) + T_D\dot{u}(t)\right]$ $G(s) = K\left[1 + T_D\, s\right]$					
PDT_1	$T_1\dot{y}(t) + y(t) =$ $K\left[u(t) + T_D\dot{u}(t)\right]$ $G(s) = K\frac{1+T_D\, s}{1+T_1\, s}$					

Frequenzgang: I-System ohne Verzögerung

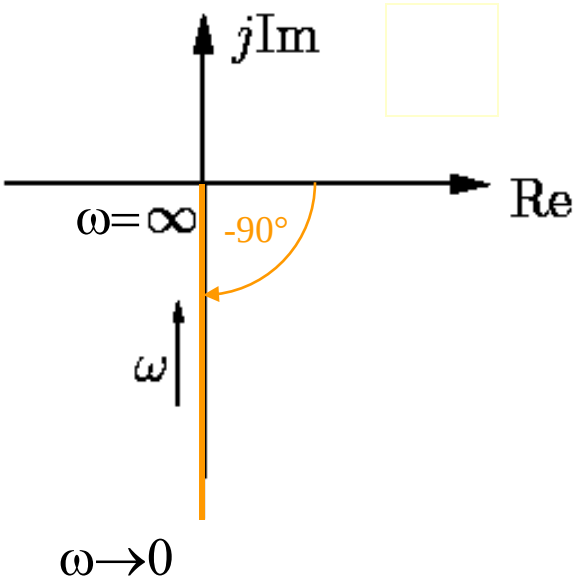
Frequenzgang:

$$G(j\omega) = \frac{K_I}{j\omega} \Rightarrow$$

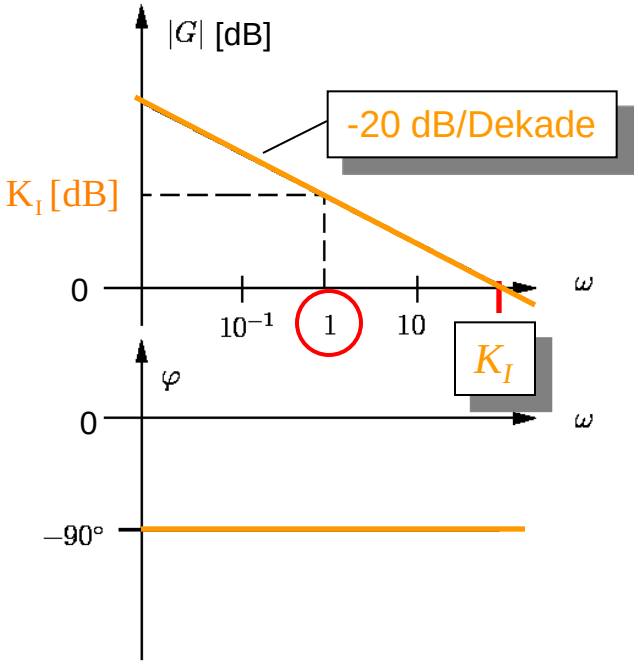
$$G(j\omega) = -j \cdot K_I/\omega$$

$$|G(j\omega)| = K_I/\omega$$

Ortskurve:



Bodediagramm:



Übertragungsfunktion: Realisierbarkeit

Für *m (Grad Z(s)) > n (Grad N(s))* **gilt**

$$G(s) = \frac{Z(s)}{N(s)} = \frac{Z_1(s)}{N(s)} + k_0 + k_1 s + \ldots + k_{m-n} s^{m-n}$$

mit *Grad $Z_1(s) = n - 1$*

Es treten ideal differenzierende Glieder auf !

$$u(t) = \sin \omega t \quad \Rightarrow \quad y(t) = \frac{du(t)}{dt} = \omega \cos \omega t$$

Ideales D-Glied ist technisch nicht realisierbar !!!

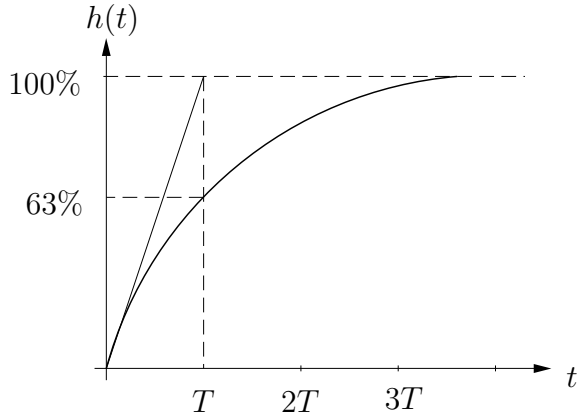


Bild 2.15: Zeitlicher Verlauf der Sprungantwort (2.42) des Systems aus Bild 2.14 mit $h(T) \approx 63\%$, $h(2T) \approx 86\%$ und $h(3T) \approx 95\%$

Bei sprungförmiger Erregung

$$u(t) = 1(t)$$

antwortet das System mit der Sprungantwort $y(t) := h(t)$:

$$h(t) = [1 - e^{-t/T}]\, 1(t) \quad , \tag{2.42}$$

die in Bild 2.15 dargestellt ist.

Mit (2.24) erhalten wir die Impulsantwort des Systems in Bild 2.14 zu (Bild 2.16):

$$g(t) = \frac{\mathrm{d}}{\mathrm{d}t} h(t) = \frac{1}{T} e^{-t/T}\, 1(t) \tag{2.43}$$

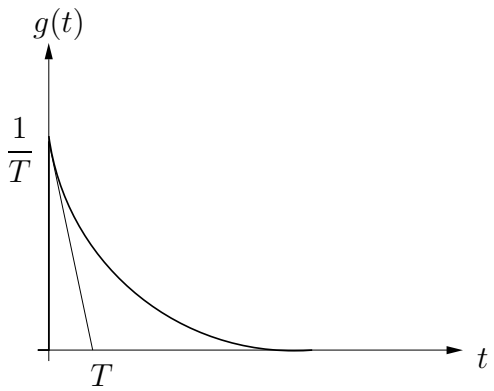


Bild 2.16: Gewichtsfunktion (Impulsantwort) des PT_1–Systems

Beispiel: RC-Glied (2)

$$[\Omega]\left[\frac{s}{\Omega}\right]$$

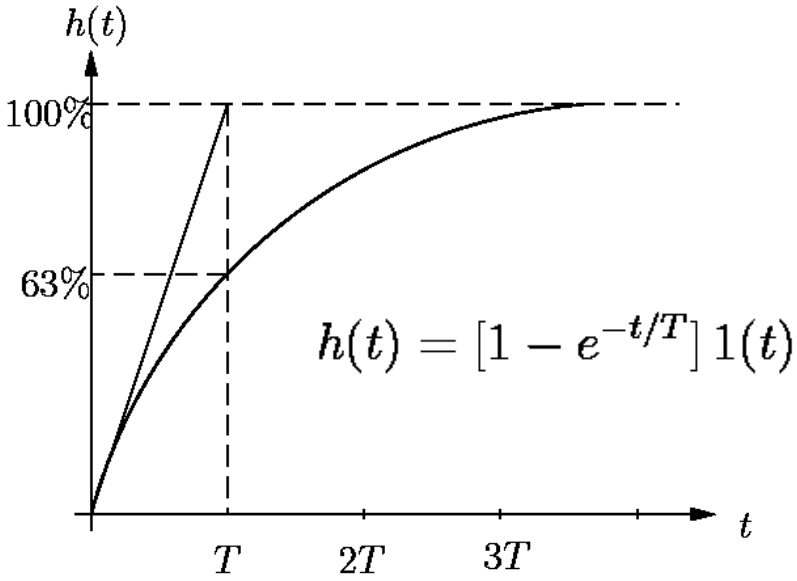
$$RC\,\dot{y}(t) + y(t) = u(t)$$

$$T = R \cdot C$$

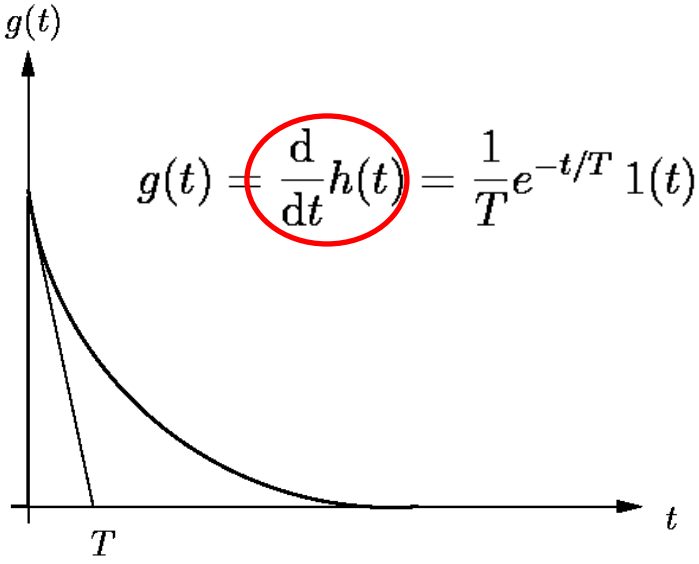
$$T\,\dot{y}(t) + y(t) = u(t)$$

$$y(t) = u_C(t)$$

PT_1-System



Sprungantwort



Gewichtsfunktion