## Aufgabe 9.1

(i) $$A = \begin{pmatrix} -\frac{1}{2u_{0,1}} & \frac{1}{2u_{0,1}} & 0 \\ \frac{1}{2u_{0,1}} & -\frac{1}{u_{0,1}} & \frac{1}{2 \cdot u_{0,1}} \\ 0 & \frac{1}{2u_{0,1}} & -\frac{2u_{0,1}+u_{0,2}}{2u_{0,1}(u_{0,1}+u_{0,2})} \end{pmatrix}$$

$u_0 = [0{,}5 \quad 0{,}5] \Rightarrow u_{0,1} = 0{,}5 \quad u_{0,2} = 0{,}5$

$$A = \begin{pmatrix} \frac{-1}{2 \cdot 0{,}5} & \frac{1}{2 \cdot 0{,}5} & 0 \\ \frac{1}{2 \cdot 0{,}5} & \frac{-1}{0{,}5} & \frac{1}{2 \cdot 0{,}5} \\ 0 & \frac{1}{2{,}0{,}5} & -\frac{(2 \cdot 0{,}5+0{,}5)}{2 \cdot 0{,}5(0{,}5+0{,}5)} \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -3/2 \end{pmatrix}$$

Zustandsrückführung: $u(t) = -k x(t)$

$k = [k_1 \quad k_2 \quad k_3]$

$$\dot{x} = A\,x(t) + B\,u(t)\,, \qquad u(t) = -k\,x(t)$$

$$\dot{x} = A\,x(t) - Bk\,x(t) = (A - Bk)\,x(t)$$

$$\dot{x} = \bar{A}.\,x(t)\,, \quad \text{wo} \quad \bar{A} = (A - Bk)$$

- Soll-polynom: $(1+s)(2+s)(3+s) = s^3+6s^2+11s+6$ //

- Charakteristisches Polynom von $(A-BK)$:

↳ $$BK = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} k_1 & k_2 & k_3 \end{pmatrix} = \begin{pmatrix} k_1 & k_2 & k_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\bar{A} = A - BK = \begin{pmatrix} -1-k_1 & 1-k_2 & -k_3 \\ 1 & -2 & 1 \\ 0 & 1 & -3/2 \end{pmatrix}$$

$$\det\left((A-BK) - s\,I_{3\times3}\right) = \det \begin{pmatrix} -1-k_1-s & 1-k_2 & -k_3 \\ 1 & -2-s & 1 \\ 0 & 1 & -3/2-s \end{pmatrix} =$$

$$= (-1-k_1-s)(-2-s)(-3/2-s) - k_3 - 1\,(-1-k_1-s) -$$

$$-(1-k_2)(-3/2-s) = (2 + s + k_1.2 + k_1.s + 2.s + s^2)(-3/2-s) -$$

$$-k_3 + 1 + 1.k_1 + 1.s + (3/2 + s - 3/2.k_2 - s.k_2)$$

$$= (s^2 + s(3+k_1) + (2+2k_1))(-3/2 - s) + s(2-k_2) +$$

$$+ 5/2 - k_3 + k_1 - 3/2\,k_2$$

$$= -s^3 - s^2(3+k_1) - s(2+2k_1) - \frac{3}{2}s^2 - \frac{3}{2}s(3+k_1) -$$

$$- \frac{3}{2}(2+2k_1) + s(2-k_2) + 5/2 - k_3 + k_1 - \frac{3}{2}k_2$$

$$= -s^3 - s^2(9/2+k_1) - s\left(k_2 + \frac{7}{2}k_1 + 9/2\right) - \left(\frac{1}{2} + 2k_1 + \frac{3}{2}k_2 + k_3\right)$$

$$= 0 \overset{\cdot(-1)}{\Longrightarrow} s^3 + s^2(9/2+k_1) + s\left(k_2 + \frac{7}{2}k_1 + \frac{9}{2}\right) + \left(1/2 + 2k_1 + \frac{3}{2}k_2 + k_3\right)$$

- Soll-Polynom: $s^3 + 6s^2 + 11s + 6$
- charac. Polynom: $s^3 + (9/2+k_1)\cdot s^2 + \left(k_2 + \frac{7}{2}k_1 + 9/2\right)\cdot s + \left(1/2 + 2k_1 + \frac{3}{2}k_2 + k_3\right)$

$$9/2 + k_1 = 6 \longrightarrow \boxed{k_1 = 3/2}$$

$$k_2 + 7/2\, k_1 + \frac{9}{2} = 11 \xrightarrow{k_1 = 3/2} \boxed{k_2 = 5/4}$$

$$\frac{1}{2} + 2k_1 + \frac{3}{2}k_2 + k_3 = 6 \xrightarrow[k_1 = 3/2]{k_2 = 5/4} \boxed{k_3 = 5/8}$$

$$\boxed{k = \begin{bmatrix} 3/2 & 5/4 & 5/8 \end{bmatrix}, \quad u(t) = -k\,x(t)}$$