Aufgabe 6.3:

(i) $G(s) = \dfrac{1}{(s+1)(s+3)(s^2+4s+13)}$

Pole: $p_1 = -1$, $p_2 = -3$, $p_{3,4} = -2 \pm 3i$

* Bedingung (1): $\sum_{i=1}^{n} \dfrac{1}{a-p_i} = \sum_{i=1}^{m} \dfrac{1}{a-n_i}$

$$\frac{1}{(a+1)} + \frac{1}{(a+3)} + \frac{1}{(a+2-3i)} + \frac{1}{(a+2+3i)} = 0$$

$$(a+3)(a+2-3i)(a+2+3i) + (a+1)(a+2-3i)(a+2+3i)$$
$$+ (a+1)(a+3)(a+2-3i) + (a+1)(a+3)(a+2+3i) = 0$$

für $a \neq \{-1, -3, -2 \pm 3i\}$.

$$(a+3)(a^2+4a+13) + (a+1)(a^2+4a+13) +$$
$$+ (a^2+4a+3)(2a+4) = 0$$

$$(a^3 + 4a^2 + 13a + 3a^2 + 12a + 39 + a^3 + 4a^2 + 13a + a^2 + 4a + 13 +$$
$$+ 2a^3 + 8a^2 + 6a + 4a^2 + 16a + 12 = 0$$

$$4a^3 + 24a^2 + 64a + 64 = 0 \Rightarrow \begin{cases} a_1 = -2 \\ a_{2,3} = -2 \pm 2i \end{cases}$$

(Taschenrechner)

Bedingung 2: $1 + G_0(s) = 0 \Rightarrow k > 0$

$$1 + k \cdot \frac{1}{(s+1)(s+3)(s^2+4s+13)} = 0$$

$$k = -1 \cdot (s+1) \cdot (s+3) \cdot (s^2+4s+13)$$

- $k_1 \Rightarrow a_1 = -2 \Rightarrow k_1 = -(-2+1)(-2+3)(4-8+13)$

$k_1 = -(-1)(1)(9) = 9 > 0$ ✓

- $k_2 \Rightarrow a_2 = -2+2i \Rightarrow k_2 = -(-2+2i+1)(-2+2i+3) \cdot$
$\cdot ((\cancel{4} - \cancel{8i} + \cancel{4i^2}) + 4(-2+\cancel{2i}) + 13)$

$k_2 = -(4 - \cancel{4i} - 6 - \cancel{4i} - 4 + \cancel{6i} - 2 + \cancel{2i} + 3) \cdot (5)$

$k_2 = 25 > 0$ ✓

- $k_3 \Rightarrow a_3 = -2-2i \Rightarrow k_3 = 25 > 0$ ✓

Verzweigungspunkte: $\boxed{\{-2, -2 \pm 2i\}}$ //

Aufgabe 6.3: (ii)

- Pole: $\{-1, -3, -2 \pm 3i\}$ $\rightarrow$ $n = 4$
- Nullstellen: $\{\ \}$ $\rightarrow$ $m = 0$
- Anzahl der Äste gegen ∞: (Regel 5) $n-m = 4$ //
- Winkel der Asymptoten: (Regel 7)

$$\varphi_k = \frac{(2k-1)\pi}{n-m} \rightarrow k = 1, 2, 3, 4$$

$$\varphi_1 = \frac{\pi}{4}, \quad \varphi_2 = \frac{3\pi}{4}, \quad \varphi_3 = \frac{5\pi}{4}, \quad \varphi_4 = \frac{7\pi}{4}$$

- Wurzelschwerpunkt (Regel 8):

$$\sigma_w = \frac{\sum_{i=1}^{n} p_i - \sum_{i=1}^{m} n_i}{n-m}, \quad \text{für } n-m \geq 2$$

$$\rightarrow \sigma_w = \frac{-1-3-2+\cancel{3i}-2-\cancel{3i}}{4} = -2 \,//$$