Improving computing time of the Dempster-Shafer theory through a Bootstrap method for applications in vehicle safety

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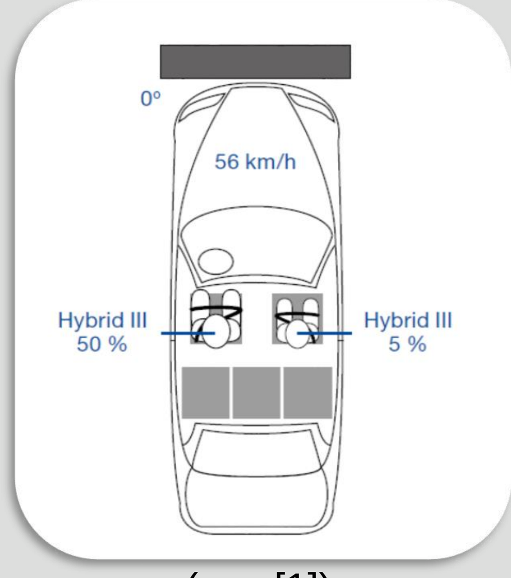
Why to consider uncertainties in the crash world?

Global aims for departments of vehicle safety:

- Preventing occupant injuries in the event of a crash.
- Assessing crashworthiness of the vehicles.
- Detecting car design concepts that might not achieve targets.

Ways to achieve these aims:

- Testing the vehicles by performing various crash tests in various construction phases.
- Evaluating the measured data.
- Finding methods to improve the crashworthiness.



(see [1])

(see [2])

Uncertainties in crash tests

Multiple hardware tests of the same vehicle in the same constellation lead to different measurement results.
Reasons are

- Changes in the impact angle and velocity,
- Positioning of the sensors,
- Differences in the instruments, e.g. the dummies.

⇒ Need to consider uncertainties to forecast possible results.

New Car Assesment Program (US NCAP)

What is US NCAP?

- Published by the National Highway Traffic Safety Administration.
- Program to rate the safety of vehicles by stars from one (worst) to five (best).
- Rating based on the performance of the car in frontal, side and rollover tests.
- Separate ratings for the driver and passenger dummy in a frontal crash.

Calculation of the rating for the Hybrid III 50% (driver dummy) in the Frontal-Impact against a Rigid Wall with 100% overlap

Filtered measurements – Injury Criteria

- HIC_{15}
- Chest deflection C_d $(= 21)$
- Neck criteria N_{ij} N_t, N_c
- Femur force F_f

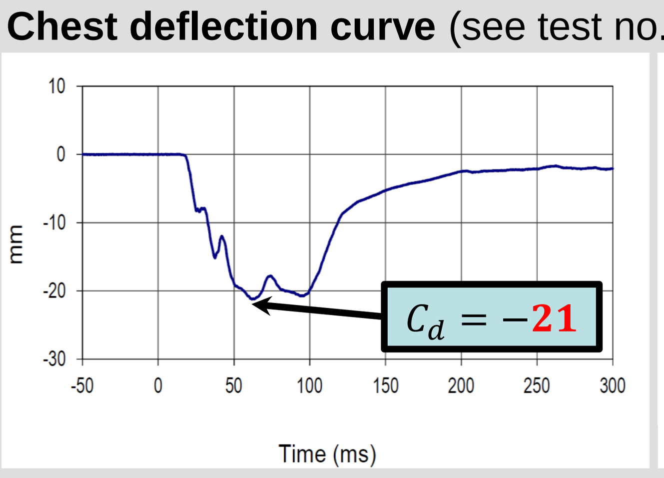
→ *Single injury probabilities*

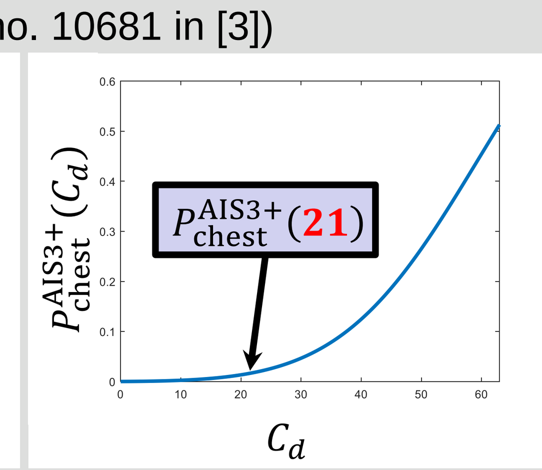
- $P_{\mathrm{head}}^{\mathrm{AIS3+}}$
- $P_{\mathrm{chest}}^{\mathrm{AIS3+}}(21) = 0.0154$
- $P_{\mathrm{femur}}^{\mathrm{AIS2+}}$
- $P_{\mathrm{neck}}^{\mathrm{AIS3+}}$

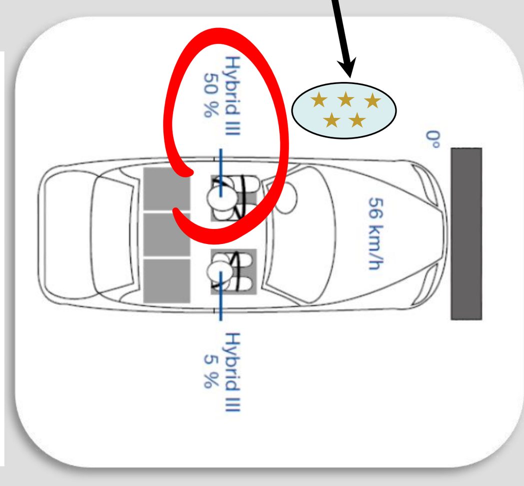
→ *Star Rating*

Joint injury probability P_{joint} $(= 0.0765)$
⇒
Relative Risk RR $(= 0.51)$

Chest deflection curve (see test no. 10681 in [3])







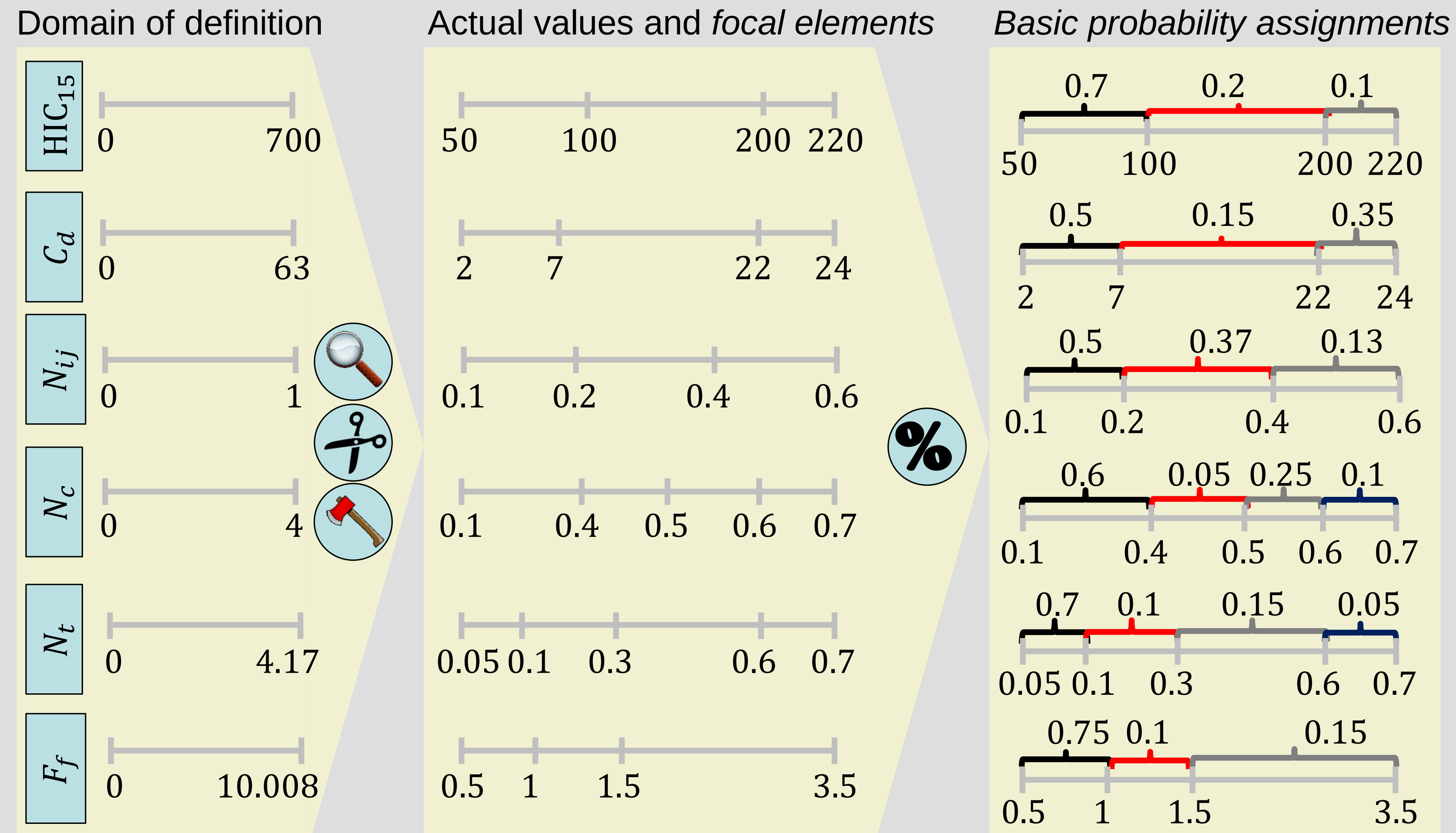
Remark: Ratings for other dummies or tests, e.g. side crashes, are determined similarly.

Dempster-Shafer theory (DST)

How to apply the DST and what to expect:

- Modeling of uncertainties in the crash test over intervals for the Injury Criteria.
- Propagating the uncertainties via DST to calculate possible outcomes of the rating with distributions represented by the *plausibilty* and *belief curve*.

Practical Procedure – shown by a fictive example

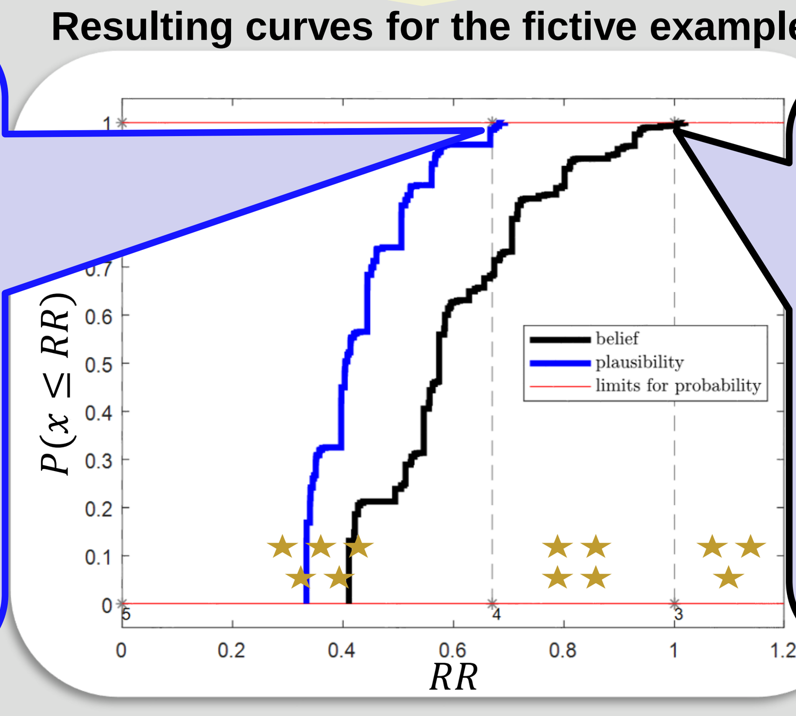


Steps of the DST after intervals were formed:

- Set up every subinterval combination called *interval cells* ($n = 3^4 \cdot 4^2$).
- Compute the probability for every interval cell.
- Solve $2n = 2592$ constrained optimization problems to determine the respective minimum and maximum of the system function (Output here: RR) for every interval cell.
- Sort the minima (plausibility values) and maxima (belief values), separately.
- Plot both with their corresponding cumulated probabilities.

Resulting curves for the fictive example

Plausibility curve
„It is 98% plausible that the Relative Risk RR is smaller than or equal to 0.67.“
or
„It is 98% plausible that the dummy will get rated with five stars.“
(Best case curve)



Belief curve
„It is 99% believable that the Relative Risk RR is smaller than or equal to 1.“
or
„It is 99% believable that the dummy will get rated with at least four stars.“
(Worst case curve)

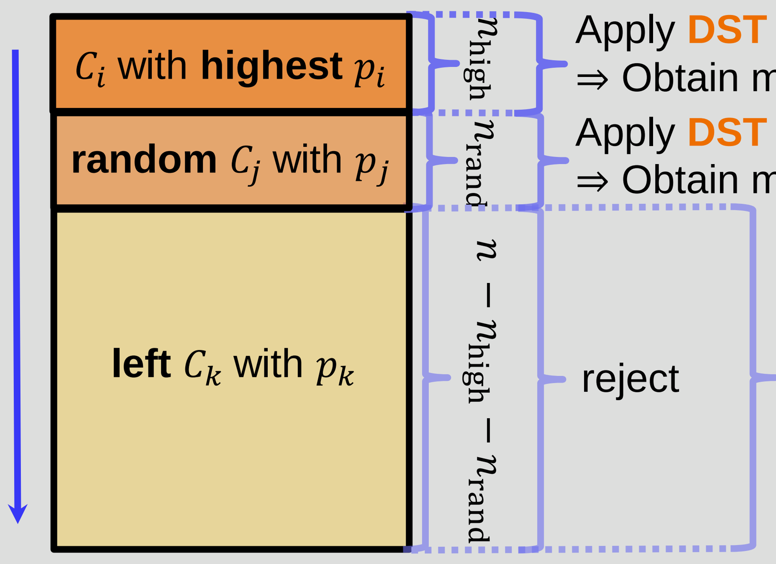
Emerging problems

- For a highly non-linear and/or discontinuous system function, optimization can be computationally costly – especially, for a large number of optimization problems.
- Increasing the input quantity as well as raising the numbers of subintervals leads to an even larger number of optimization problems.

Bootstrap method (BM) to reduce the number of optimizations

BM-DST approach

1. Build all interval cells $C_l, l \in \{1, \dots, n\}$, and compute their corresponding probabilities p_l.
2. Choose $n_{\mathrm{high}} + n_{\mathrm{rand}} \in \mathbb{N}$ interval cells and approximate the others in the way below.



Apply **DST optimization**.
⇒ Obtain min/max values L_i, U_i and save probability p_i.

Apply **DST optimization**.
⇒ Obtain min/max values L_j, U_j and save probability p_j.

Apply **BM** to approximate (L_k, U_k, p_k).

set $m = 0$
while $\sum p_i + \sum p_j + \sum_{k=1}^{m} p_k < 1$
- take a $v \sim N\left(0, \frac{1}{n_{\mathrm{rand}}}\right)$
- draw (L_j, U_j, p_j) by random from the random cells C_j
- $L_k = L_j + v, U_k = U_j + v, p_k = p_j + v$
- $m = m + 1$

end
save $\hat{m} = m$

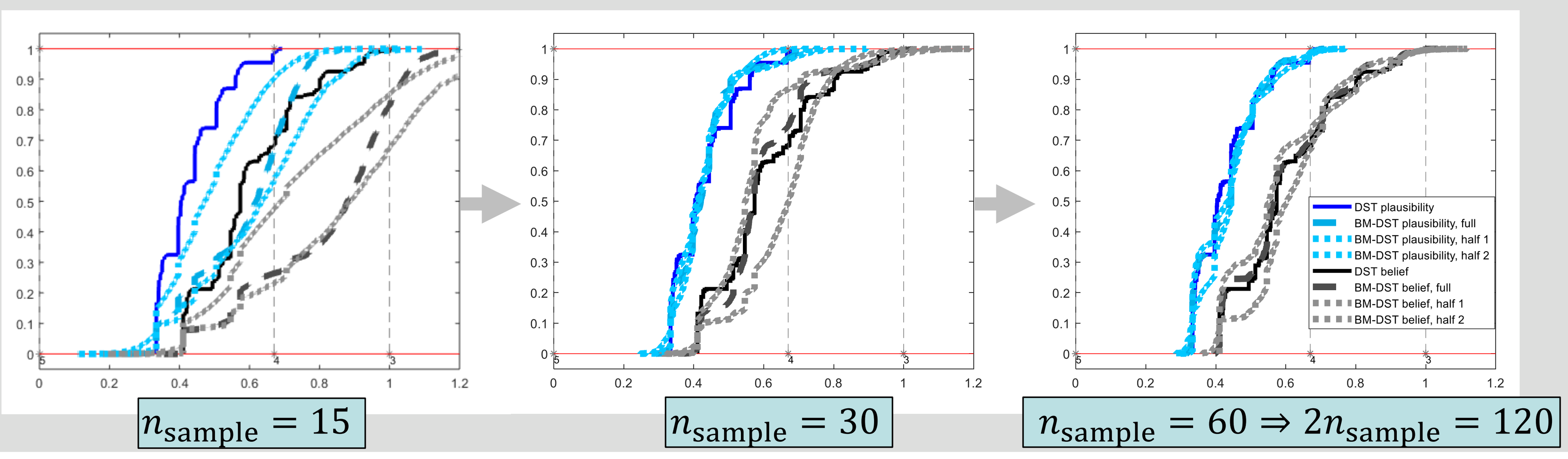
3. Collect the data, obtain $(L_{\hat{\imath}}, U_{\hat{\imath}}, p_{\hat{\imath}})$ for $\hat{\imath} \in \{1, \dots, \hat{n}\}$ with $\hat{n} = n_{\mathrm{high}} + n_{\mathrm{rand}} + \hat{m}$ and plot an approximated plausibility and belief curve with it.

(Often it holds $\hat{m} \neq n - n_{\mathrm{best}} - n_{\mathrm{rand}}$)

Algorithm to find a sufficient sample size $n_{\mathrm{sample}} = n_{\mathrm{high}} + n_{\mathrm{rand}}$

- Iterative approach (start with $n_{\mathrm{sample}} = n_{\mathrm{sample}}^0 \in \mathbb{N}$).
- After every loop, errors (areas between plausibility and belief curves, respectively) are calculated between the BM-DST with the full sample size n_{sample} (dashed curves), the BM-DST with one half of the sample size and the BM-DST with the other half of the sample size (all plotted as dotted curves).
- If errors do not fulfil the stopping criterion, start new loop with increased n_{sample}.
- If errors fulfil the stopping criterion, take the BM-DST with the last calculated sample size n_{sample} as the approximation for the exact DST.

Resulting curves for exact and approximated DST optimization



Conclusion and Outlook

- DST forecasts all possible star ratings under uncertain Injury Criteria originating from uncertainties in the crash test.
- For this application, BM-DST approximates the usual DST very precise while significantly decreasing the number of optimization problems from 2592 to 120. However, optimizations are not computationally costly here.
- For other problems like crash simulations, this procedure saves an enormous amount of computing time.

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