CORRIGENDUM: COMPUTATION OF 3D VERTEX SINGULARITIES FOR LINEAR ELASTICITY: ERROR ESTIMATES FOR A FINITE ELEMENT METHOD ON GRADED MESHES

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Abstract. Minor discrepancies in the assumptions in [1] have caused incorrect conclusions concerning the constants in some estimates presented in [1]. The assertions will be rectified in this corrigendum. The main results of [1] remain untouched nevertheless.

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Hooke's law states that the strain tensor σ is related to the stress tensor ε by $\sigma = A : \varepsilon$, where A is a fourth-order tensor describing the material under consideration. The components of A in the Cartesian basis are denoted by a_{ijln}. In the theory of linear elasticity it is common to assume that the elasticity tensor A enjoys the classical symmetry and positivity assumptions. In [1] they were formulated in the relations (5) and (6) as follows:

$$a_{ijln} = a_{lnij} = a_{jiln} = a_{ijnl} \tag{5}$$

and

$$M_1 \sum_{i,j=1}^{3} |\xi_{ij}|^2 \leq \sum_{i,j,l,n=1}^{3} a_{ijln}\xi_{ij}\xi_{ln} \leq M_2 \sum_{i,j=1}^{3} |\xi_{ij}|^2 \qquad \forall \xi_{ij} \in \mathbb{R},\ i,j = 1,2,3. \tag{6}$$

Relation (5) implies that skew-symmetric tensors ($\xi_{ij} = -\xi_{ji}$) are annulled by A so that the constant M_1 in (6) is zero. In combination with the symmetry (5), the positivity property (6) makes sense only if A is solely applied to symmetric tensors ($\xi_{ij} = \xi_{ji}$) as it is common in the material law. Then we get $M_1 > 0$.

In this context it is tidier (but equivalent) to define the sesquilinear forms $k : V \times V \to \mathbb{C}$ and $m : H \times H \to \mathbb{C}$ via

$$\begin{aligned}
k(u,v) &:= \frac{1}{4} \sum_{i,j,l,n=1}^{3} \int_\Omega a_{ijln}(e_j(u_i) + e_i(u_j))(e_n(\bar{v}_l) + e_l(\bar{v}_n))\,\mathrm{d}\omega \\
m(u,v) &:= \frac{1}{4} \sum_{i,j,l,n=1}^{3} \int_\Omega a_{ijln}(s_j(u_i) + s_i(u_j))(s_n(\bar{v}_l) + s_l(\bar{v}_n))\,\mathrm{d}\omega
\end{aligned}$$

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instead of exploiting the symmetry (5) and writing $k(u,v) = \sum_{i,j,l,n=1}^{3} \int_\Omega a_{ijln} e_j(u_i) e_n(\bar{v}_l)\,\mathrm{d}\omega$ etc., compare [1, page 1048]. The terms $e_j(u_i)$ and $s_j(u_i)$ are abbreviations for

$$e_j(u_i) := -\frac{1}{2} A_j u_i + B_j \partial_\theta u_i + C_j \partial_\varphi u_i, \quad s_j(u_i) := A_j u_i, \quad i,j = 1,2,3,$$

where

$$\begin{array}{lll} A_1 := \cos\varphi \sin\theta, & B_1 := \cos\varphi\cos\theta, & C_1 := -\sin\varphi/\sin\theta, \\ A_2 := \sin\varphi\sin\theta, & B_2 := \sin\varphi\cos\theta, & C_2 := \cos\varphi/\sin\theta, \\ A_3 := \cos\theta, & B_3 := -\sin\theta, & C_3 := 0, \end{array}$$

and (φ,θ) are the spherical angles. The spaces H and V are L^2- and H_0^1-Sobolev spaces adapted for spherical coordinates with the norms introduced below.

Lemma 2.1 of [1] states fundamental properties of the aforementioned sesquilinear forms. Bounding $m(u,u)$ and $k(u,u)$ from below by a material-independent norm with the constant M_1 from (6), one obtains at first

$$\begin{aligned} m(u,u) &\geq M_1 \sum_{i,j=1}^{3} |[\tfrac{1}{2}(s_j(u_i) + s_i(u_j))]|_{0,\Omega}^2, \\ k(u,u) &\geq M_1 \sum_{i,j=1}^{3} |[\tfrac{1}{2}(e_j(u_i) + e_i(u_j))]|_{0,\Omega}^2 \end{aligned}$$

with $|[v]|_{0,\Omega}^2 = \int_\Omega |v|^2 \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$ for scalar functions v.

For vector functions v, the norms $\|v\|_H := (\sum_{j=0}^{3} |[v_j]|_{0,\Omega}^2)^{1/2}$ and $\|v\|_V := (\frac{1}{4}|[v]|_{0,\Omega}^2 + [v]_{1,\Omega}^2)^{1/2}$ were introduced, where $[\cdot]_{1,\Omega}$ denotes the H^1-seminorm in spherical coordinates. (The factor $\frac{1}{4}$ was accidentally placed in front of the $[v]_{1,\Omega}$-seminorm in [1].) It can be shown that the norms $\|u\|_H$ and $(\sum_{i,j=1}^{3} |[\frac{1}{2}(s_j(u_i) + s_i(u_j))]|_{0,\Omega}^2)^{1/2}$ as well as $\|u\|_V$ and $(\sum_{i,j=1}^{3} |[\frac{1}{2}(e_j(u_i) + e_i(u_j))]|_{0,\Omega}^2)^{1/2}$ are equivalent, in particular,

$$\begin{aligned} \|u\|_H &\geq \Big(\sum_{i,j=1}^{3} |[\frac{1}{2}(s_j(u_i) + s_i(u_j))]|_{0,\Omega}^2\Big)^{1/2} &\geq c_0\|u\|_H, \\ \|u\|_V &\geq \Big(\sum_{i,j=1}^{3} |[\frac{1}{2}(e_j(u_i) + e_i(u_j))]|_{0,\Omega}^2\Big)^{1/2} &\geq c_1\|u\|_V, \end{aligned}$$

where $c_0 = 1/2$ and c_1 depends on the constant in Korn's inequality, see [2, Theorem 4.4], [3, Theorem 4.11].

As a consequence, the constant in [1, Lemma 4.1] has to be modified so that the corresponding estimate reads correctly

$$\|u - \mathcal{P}_h u\|_V \leq \sqrt{\frac{M_2}{M_1 c_1}}\varepsilon_h(u) \quad \forall u \in V.$$

The further constants used in [1] are generic so that the main result in Corollary 4.16 remains valid without change.

References

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