Solving Forward and Inverse Problems of Contact Mechanics using Physics-Informed Neural Networks

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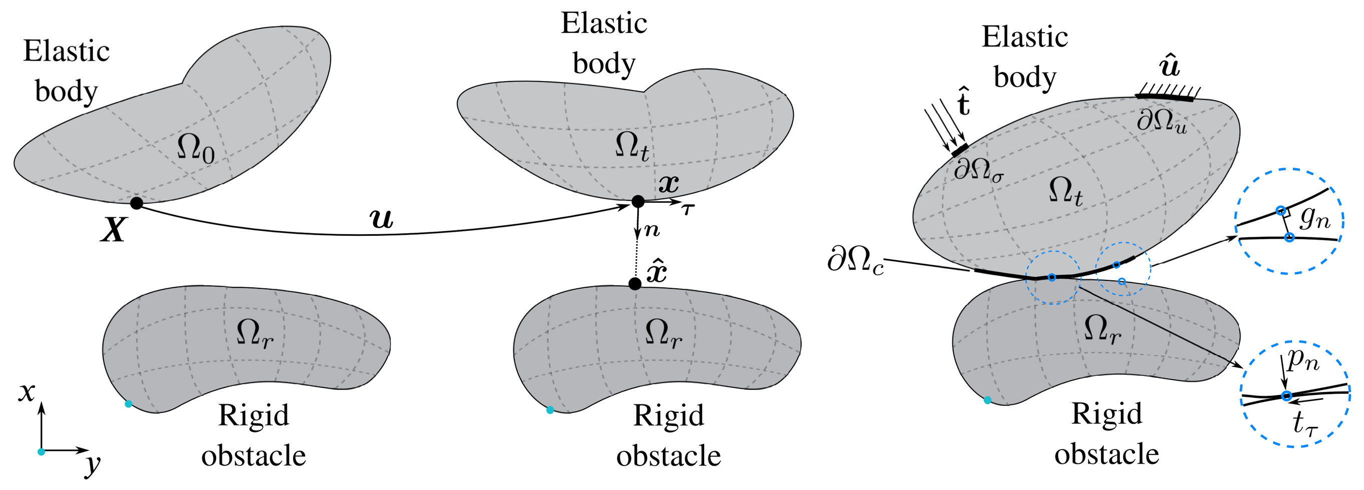
Motivation

Machine learning models often require large, high-quality datasets, which are difficult and expensive to obtain for complex physical systems. Physics-Informed Neural Networks (PINNs) address this challenge by embedding physical laws directly into the learning process, enabling accurate solutions even with scarce or noisy data. However, their application to contact mechanics remains largely unexplored—this study fills that gap.

Problem Formulation



Let us consider the boundary value problem (BVP) of small deformation elasticity

$$\begin{aligned} \nabla \cdot \boldsymbol{\sigma} + \hat{\mathrm{b}} &= 0 \quad \text{in } \Omega, \\ \boldsymbol{u} &= \hat{\boldsymbol{u}} \quad \text{on } \partial\Omega_u, \\ \boldsymbol{\sigma} \cdot \boldsymbol{n} &= \hat{\boldsymbol{t}} \quad \text{on } \partial\Omega_\sigma, \end{aligned}$$

with the following kinematic and constitutive equations

$$\begin{aligned} \boldsymbol{\varepsilon} &= \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T), \\ \boldsymbol{\sigma} &= \mathbb{C} : \boldsymbol{\varepsilon}, \end{aligned}$$

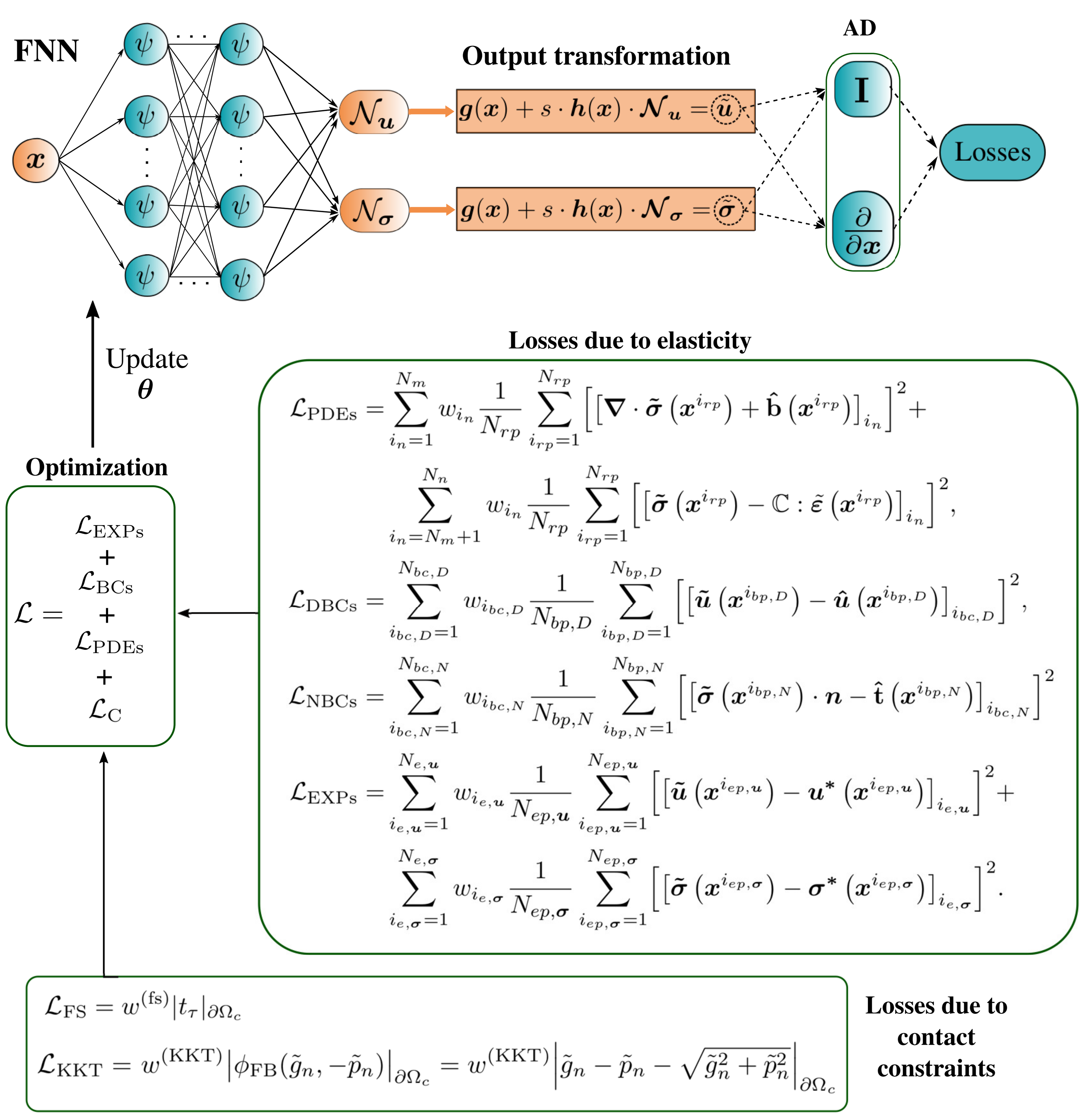
and the following contact Karush-Kuhn-Tucker (KKT) constraints

$$\begin{aligned} g_n \geqslant 0, \quad p_n \leqslant 0, \quad p_n g_n = 0, \\ t_\tau = 0. \end{aligned}$$

PINNs for Solid and Contact Mechanics

In the context of mixed-variable formulation, a fully-connected neural network (FNN) maps the given spatial coordinates $\boldsymbol{x}$ to the displacement vector and stress tensor as

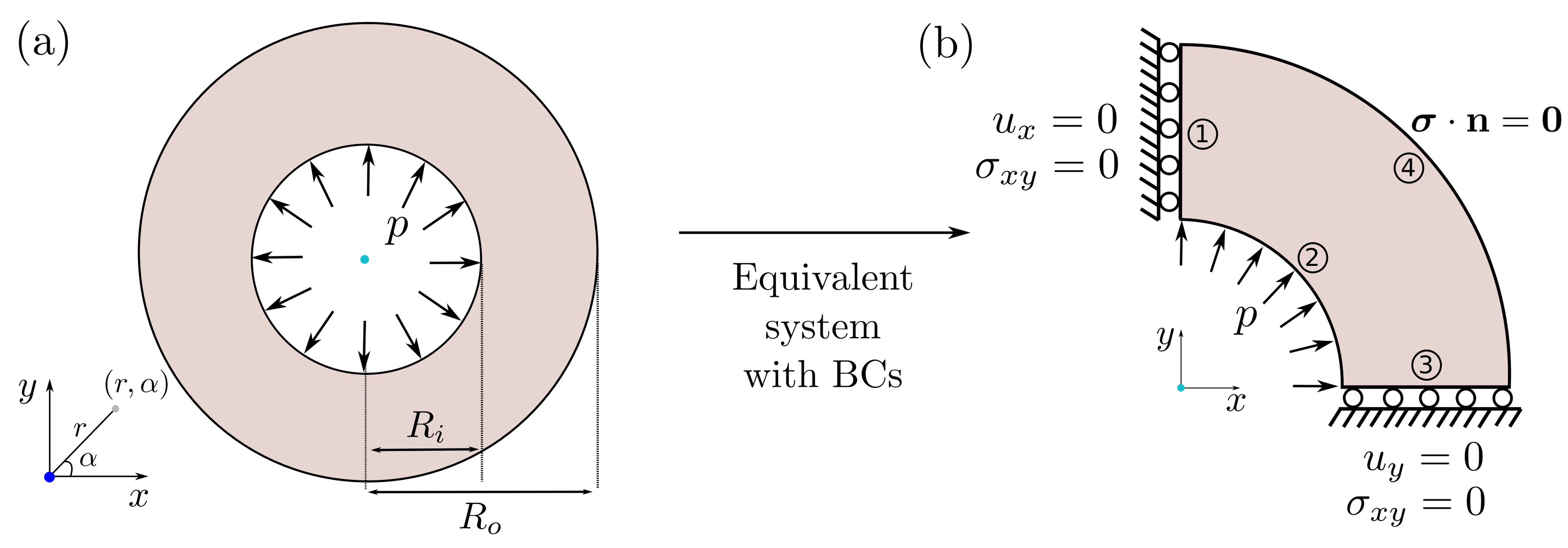
$$\tilde{\boldsymbol{u}} := (\boldsymbol{\mathcal{N}}_u(\boldsymbol{x};\boldsymbol{\theta}))' \text{ and } \tilde{\boldsymbol{\sigma}} := (\boldsymbol{\mathcal{N}}_\sigma(\boldsymbol{x};\boldsymbol{\theta}))'.$$



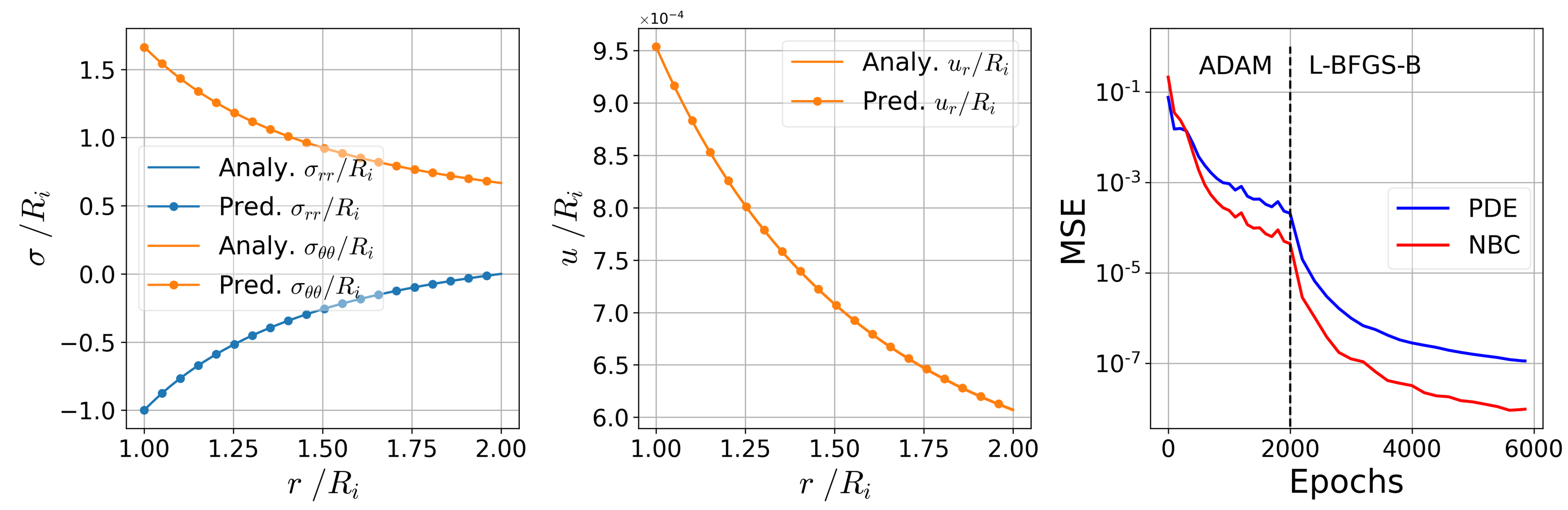
Numerical Examples

Lamé problem of elasticity

Problem setup



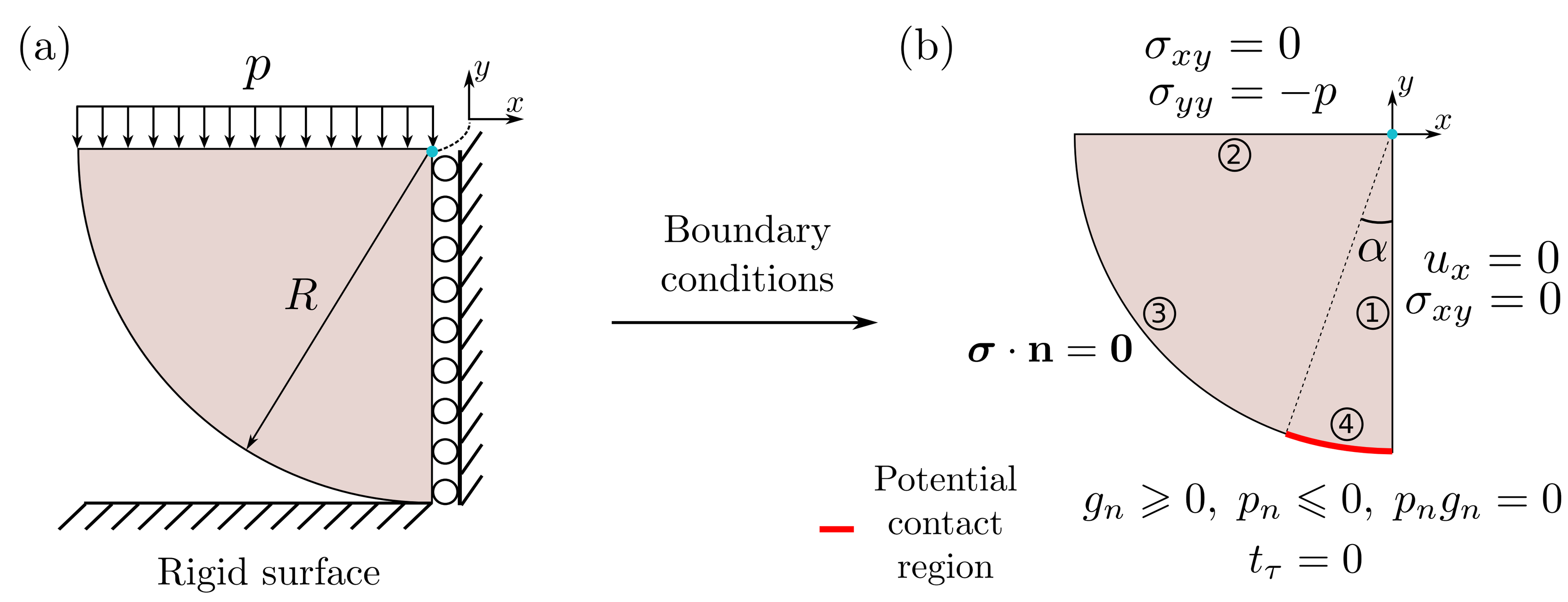
Results



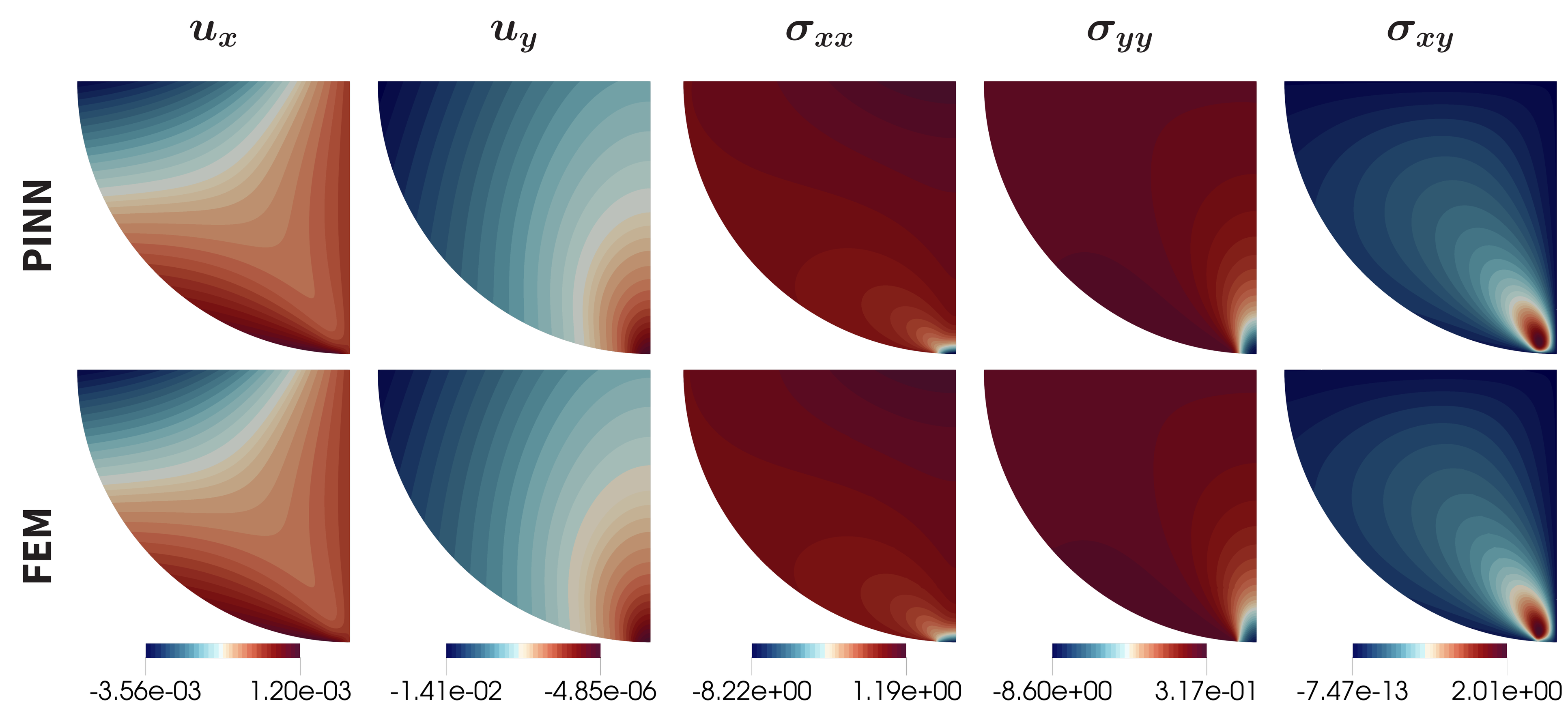
Comparison of normalized stresses obtained from the analytical solution and the predicted values and loss evolutions.

Hertzian contact problem

Problem setup



Results



Comparison of stress and displacement components obtained by PINN and FEM.

References

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