A Multi-Scale FEM-BEM Formulation for Contact Mechanics between Rough Surfaces

Jacopo Bonari[2], Maria R. Marulli[2], Nora Hagmeyer[1], Matthias Mayr[1], Alexander Popp[1], Marco Paggi[2]

[1]Institute for Mathematics and Computer-Based Simulation, University of the Bundeswehr Munich, Germany
[2]IMT School for Advanced Studies Lucca, Italy

der Bundeswehr Universität München

Motivation

At micro- and nano-scales, surface related phenomena become predominant over bulk properties. Waviness, roughness and general surface texturing play a crucial role in electrical transfer, optical properties, fluid-structure interactions and tribology. In industrial applications, where the size of the components is much larger than the scale of roughness, a multi-scale approach is necessary to represent such effects accurately.
The proposed FEM-BEM model [1] addresses the contact problem of a nominally smooth surface, taking into account micro-scale roughness data from profilometry measurements or numerical models [2].

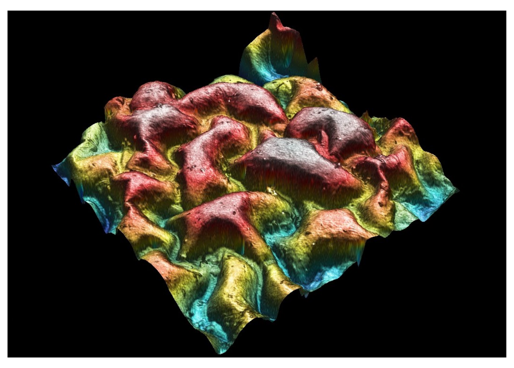


Figure 1: Roughness of a raspberry leaf

Macro-Scale Model

- Continuum mechanics model with contact constraints
$$g_n \geq 0,\ p_n \geq 0 \quad \text{on } \Gamma$$
- In contrast to classical Karush-Kuhn-Tucker (KKT) conditions, here the relation between the gap g_n and the contact pressure p_n is described by the micro-model to include effects due to roughness
- Discretized with an interface finite element (FEM) using a Gauss-point-to-segment approach
- The macro-scale finite element formulation can easily incorporate nonlinear material laws for the bulk as well as adhesion phenomena at negative contact tractions

Multi-Scale Coupling

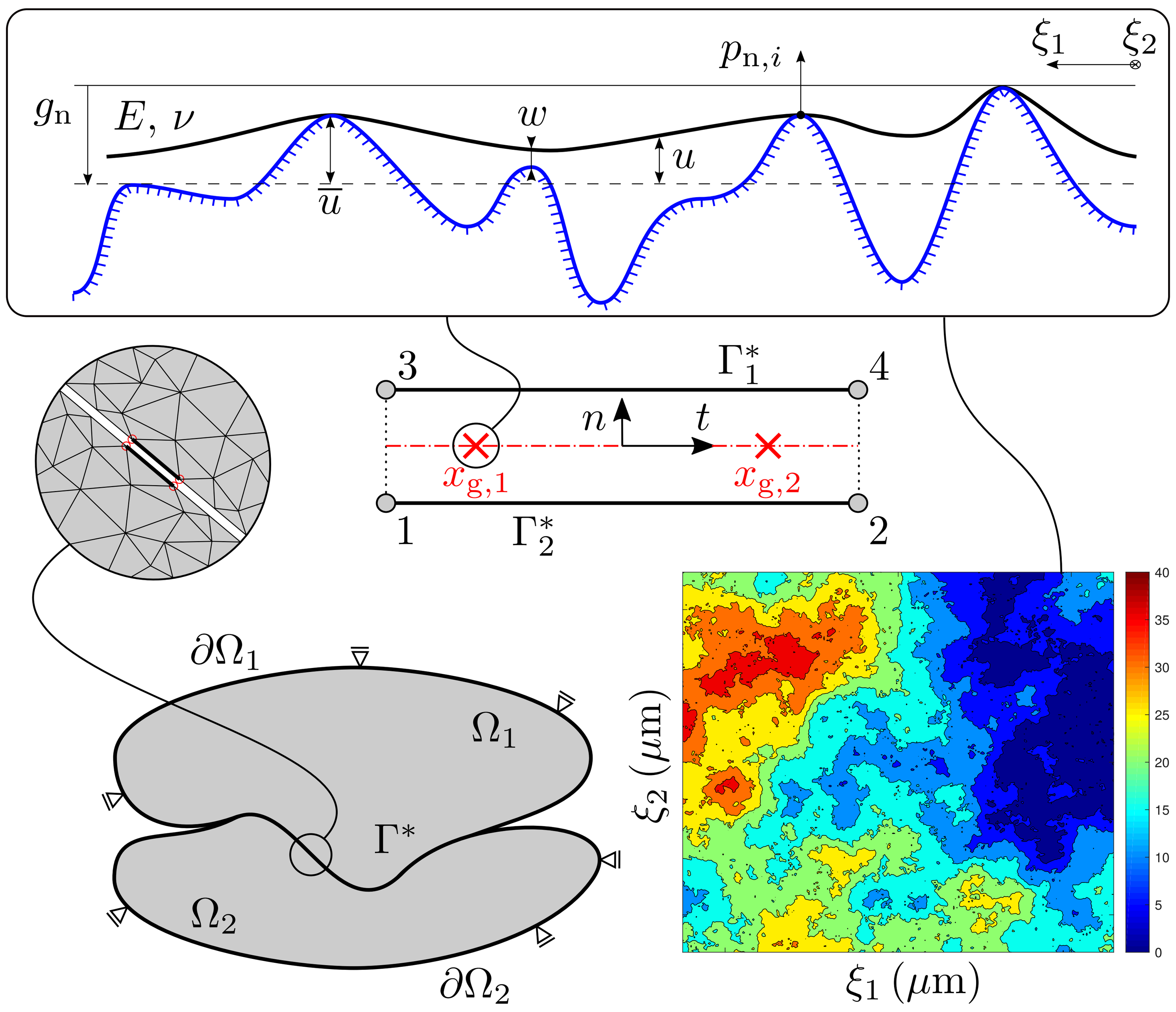


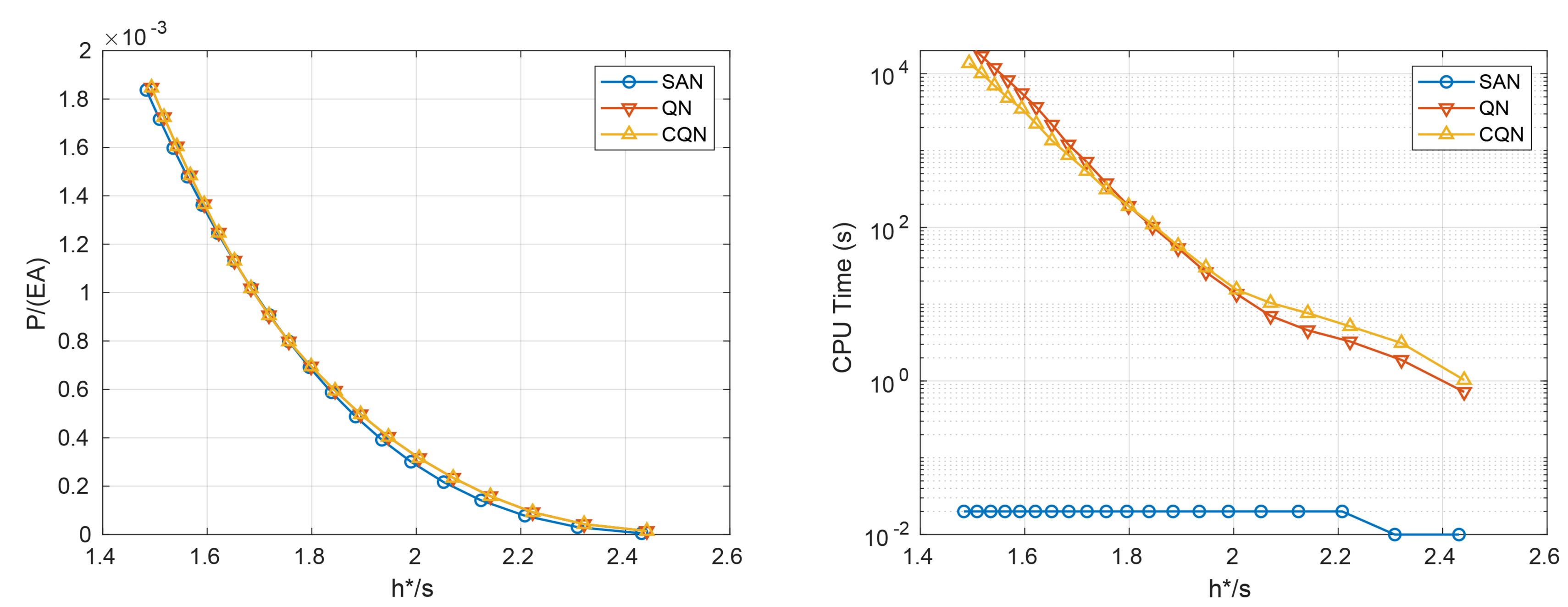
Figure 2: Multi-scale coupling procedure

Micro-Scale Model

- Classical KKT contact conditions
$$w(\xi, g_n) \geq 0,$$
$$p(\xi) \geq 0,$$
$$w(\xi, g_n)p(\xi) = 0,$$
with the displacement correction $w(\xi, g_n) = u(\xi) - \overline{u}(\xi, g_n)$, the indentation of the halfspace $\overline{u}(\xi, g_n)$, the normal displacement $u(\xi)$, and the pressure distribution $p(\xi)$
- Linear material constitutive law
- Discretized with the boundary element method (BEM) using piece-wise constant shape functions
- Based on numerically generated roughness profiles [2]
- Returns pressure distribution at the micro level, which is averaged to describe the contact pressure p_n for a given gap g_n at the macro-scale

Coupling Algorithms

1. Quasi-Newton approach (FBEM-QN): Fully embedded FEM-BEM coupling in conjunction with a Quasi-Newton solver approximating the Jacobian by means of a finite difference approach
2. Cheap Quasi-Newton (FBEM-CQN): In comparison to the FBEM-QN approach, additional solutions of the micro-scale system are avoided by using the solution of the last converged time step for the finite difference approximation
$$\frac{\partial p_n}{\partial g_n} \simeq \frac{p_{n,k}^t - p_n^{t-1}}{g_{n,k}^t - g_n^{t-1}},$$
3. Semi-analytical approach (FBEM-SAN): Description of the pressure-gap relationship via analytical functions, obtained by a least-squares fitting of micro-scale pressure solutions off-line computed with a simple power law
$$p_n = a g_n^b,$$
in conjunction with a full Newton solver



(a) Comparison of the pressure solution (b) Comparison of the CPU time

Figure 3: Comparison of the three coupling approaches

Conclusion

A comparison of the coupling strategies for the proposed multi-scale contact mechanics formulation leads to the following insights:

1. The QN and CQN approaches are computationally more expensive than the SAN approach, but allow to deal with any kind of topology in any regime of separation
2. The SAN approach with the simple power law fit introduces inaccuracies, particularly in the high and low separation zones, but its simplicity makes it better suited for practically relevant applications

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References

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