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#### Abstract

After a sketch of history of gyroscope technology, the dynamic law of a gyroscope and its specialization to the suspended gyrocompass are outlined. The equations of motion of a north-seeking gyro are reviewed forming the base for observation techniques. Following a state-of-the-art report, the relations between the observables and the final azimuth are fully discussed and actions and precautions required for precise results are proposed. With modern instruments and observation methods it is presently possible to determine an azimuth with a standard deviation of $3^{\prime \prime}$ to $6^{\prime \prime}$ within less than one hour. The principles of non-rotating optical gyroscopes are presented and their potential as alternatives to conventional instruments is discussed.


## 1. INTRODUCTION

Gyroscopes as instruments for measuring azimuths have always had their place on the fringe of geodesy. In the early days of this technology it really was a venture to carry out observations, since the equipment was voluminous and heavy, the procedure time consuming and the accuracy of the results unsatisfactory. In addition to that, the average surveyor did not understand the way the gyroscope works. Not that the theory is difficult, but it is beyond every day experience. By reason of this situation gyro technology acquired the reputation of being an exotic subject hardly of use for surveying applications. This still exists although the scene has changed completely.

Modern mechanical gyros are handy and provide the azimuth with an accuracy of a few seconds of arc in less than one hour. The dynamics of gyroscopes has been treated at length in numerous papers, admittedly sometimes more confusing than enlightening, but the interested surveyor can easily find suitable presentations of the theory. In spite of these significant advances the acceptance has remained on a low level.

Presently optical gyroscopes have attracted widespread attention as an alternative to mechanical gyros in navigation platforms. Their performance has already reached a level which makes them rate as potential candidates for a completely new generation of northseeking instruments.

## 2. HISTORICAL BACKGROUND

The dynamic behaviour of a spinning wheel had been known for a long time, when the French physicist L. FOUCAULT made his famous experiment in 1852. He suspended a gimballed rotor from an earth-fixed support using a thread, such that the axis of the rotor was forced to remain in a horizontal plane. The objective was to prove that the earth rotates and that therefore; caused by the torque due to the precession in the earth's gravitational field, the spin axis would turn towards true north. Unfortunately the experiment failed because it was impossible at that time to maintain the required angular velocity (LAUF 19631)). Nevertheless, this was the birth of a north-seeking instrument for which FOUCAULT coined the name gyroscope and which has become increasingly important for navigation and geodesy.

FOUCAULT's experiment turned the attention of many famous scientists of the last century to the theory of the gyroscope. Authors like CAYLEY, EULER, KELVIN, KLEIN, SOMMERFELD and GRAY made valuable contributions to the dynamics of this instrument. But it took until 1908 before working gyrocompasses were constructed by ANSCHUUTZ-KÄMPFE in Germany and SPERRY in the USA. These single-gyro compasses were designed for the navigation of vessels, but it turned out that they were too sensitive to the ship's rolling. Further research and experiments led to two- and three-gyro compasses showing much better performance. The final breakthrough was achieved by ANSCHOTZ in 1922 with a floated spherical gyroscope meeting all needs for use at sea.

Paralleling this development first studies and experiments were conducted aiming at the use of gyroscopes for bore-hole and tunnel measurements. These efforts were supported and advocated by the mining surveying authorities in Germany. The first experimental instrument was constructed by SCHULER in 1921 and tested in coal mines. Already in 1926 an improved gyroscope was built conjointly by ANSCHÜTZ and BREITHAUPT and in 1936/37 ANSCHÜTZ presented the third generation. All these instruments remained on the prototype level and were not adopted by the surveying community, because the accuracy was unsatisfactory, the weight was too heavy and the observational procedures were too complicated.

In 1948 a new start was made at the Mining University Clausthal by RELLENSMANN. The following period is noted by a steady progress and by noteworthy contributions of different companies, having led to the current arsenal of gyroscopic instruments for azimuth determination. There are basically two types of instruments: the gyroscope attachments and the gyrotheodolites. In Western Europe mainly three companies offer gyrocompasses for precision observations on the civil market which are suitable for geodetic applications:

Institute of Mining Surveying (WBK), Bochum, FRG
Hungarian Optical Company (MOM), Budapest
Wild, Heerbrugg, Switzerland
The instruments which are best documented and probably of most widespread usage are the attachment GAK 1 of Wild, the gyrotheodolites Gi-Bl and Gi-Bll of MOM and the MN 50 (MW 50 a) and Gyromat of WBK.

## THE FREE GYROSCOPE

The free gyroscope consists of a fast spinning rotor which is mounted in a set of mbals such that it is decoupled from all rotations of the support. Under the assumpon that the gimbal bearings are free from friction and that the instrument is balanced d strictly symmetrical so that the three axes coincide with the principle axes of inera and intersect in the centrepoint of the rotor, the spin axis will indefinitely mainin its orientation in inertial space. To an observer on the earth the spin axis appartly moves on a cone with an axis parallel to the rotation axis of the earth. Figure 1 lows a gimbal mounted elementary (free) gyro.


1 Outer Gimbal Axis
2 Inner Gimbal Axis
3 Spin Axis
a Rotor
b Inner Gimbal (Float)
c Outer Gimbal
d Frame (Base)

Figure 1 Schematic of a gimbal mounted free gyroscope (FABECK 1980)

If a torque is applied in the direction of the 2-axis then the angular momentum of the rotor changes its direction and the 3 -axis begins a motion about the 1-axis which is known as precession of the gyro. The free gyro tries to align the spin axis with the direction of the applied torque.

In the English literature this type of gyroscope is usually denoted as a two degrees of freedom (TDF) gyro. It represents the basic concept of the gyros used to control the attitude of inertial platforms.

The suppression of one degree of freedom of the rotor relative to the frame yields the single degree of freedom (SDF) gyro. A special version of this type is the northseeking gyroscope or gyrocompass. The gyroscope of Figure 1 could be converted into a
gyrocompass if the 1 -axis is kept in the direction of the local vertical and the 3 -axis in the horizontal plane, i.e. the rotational degree of freedom about the 2 -axis is suppressed.

## 4. THE SUSPENDED GYROCOMPASS

If a gyroscope is suspended from an earth fixed support such that the spin axis is constrained to remain in a horizontal plane (Figure 2), then a torque caused by the force of gravity acts upon the instrument. The force changes its direction relative to an inertial coordinate system with the angular velocity of the rotating earth. The rotor reacts with an angular motion (precession) towards the direction of the earth rate vector. The inertia of the system will prevent the spin axis from stopping when it reaches the meridian. Instead it will move further until the directional force exceeds the inertia and causes the system to swing back. Thus an oscillation


Figure 2 Suspended meridianseeking gyroscope


Figure 4 The horizontal component of Figure 3 resolved in directions of the 3 - and 2-axis of the gyro
is built up, which would last indefinitely if the system was free of friction. Since the motion is constrained to the horizontal plane, only the component $\omega \cos \varphi$ of Figure 3 is effective, where $\omega$ is the earth rate and $\varphi$ the latitude. Figure 4 shows the view from space at the horizontal plane. The horizontal component of the earth rate is decomposed in two components, the one pointing in the direction of the 3-axis (spin axis) is $\omega \cos \varphi \cos \alpha$ and the other one pointing opposite to the direction of the 2-axis is $\omega \cos \varphi \sin \alpha$ with $\alpha$ being the momentary angle between spin axis and local meridian. The exact equation of the precession of the gyroscope is rather involved. It expresses the resultant effect of the inertia of the rotor, the rotating gravity vector and the torsion of the suspension band. Since it is well documented in several text books e.g. DEIMEL (1950) ${ }^{2}$ )


Observation window showing two positions of the moving mark

Figure 5 Cross-section of a gyro attachment (Wild GAK 1)
The moving mark (4) oscillates with the rotor (3) about the vertical axis. The centre line of the oscillation is the direction of the meridian except for some corrections explained later. The observation aims at the determination of this centre line, which is achieved by taking time and/or position information of the moving mark with respect to reading scale.
and FABECK (1980) ${ }^{3}$ ) in this paper no derivation is given. The discussion of the equation in the next section follows basically the approach selected by VANICK (1972) ${ }^{4}$.

For a better understanding at first, an actual gyroscope attachment is presented in Figure 5, showing some constructive details and introducing the relevant terms.
5. EQUATIONS OF MOTION

The motion of a spinning suspended gyroscope can be described by a pair of coupled differential equations. The 3 -axis (spin axis) follows the curve of an elliptic spiral of the kind depicted in Figure 6. In reality the proportion $\beta$ : $a$


Figure 6 Plot of the motion of the spin axis, $\alpha$ horizontal, $B$ vertical

$$
\begin{equation*}
\left(J+\frac{N^{2}}{G}\right) \ddot{\alpha}+k \dot{\alpha}+(F+B) \alpha-B \alpha_{t}=0 \tag{1}
\end{equation*}
$$

with

| $J\left(g ~ c m^{2}\right)$ | moment of inertia toward 1 -axis |
| :--- | :--- |
| $N\left(\mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-1}\right)$ | gyroscopic moment torque of the rotor |
| $G\left(\mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-2}\right)$ | moment of gravity |
| $k\left(\mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-1}\right)$ | damping constant |
| $F\left(\mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-2}\right)$ | directive moment torque of the earth's rotation |
| $B\left(\mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-2}\right)$ | directive moment torque of the suspension |
| $\alpha t$ | zero-torque direction of the suspension tape |

Equation 1 can be decomposed yielding

$$
\begin{align*}
J \ddot{\alpha}+k \dot{\alpha}+B\left(\alpha-\alpha_{t}\right) & =0  \tag{1a}\\
\frac{N^{2}}{G} \ddot{\alpha}+F \alpha & =0 \tag{1b}
\end{align*}
$$

where Eq. la describes the torsional pendular motion of the non-spinning gyro about the 1-axis and Eq. 16 expresses the motion of the torsion-free suspended spinning gyroscope.

The solution of Eq. 1 results in the expression for the horizontal pendular motion of the real spinning gyroscope

$$
\begin{equation*}
\alpha=\alpha_{0}+C e^{-\lambda t} \cos \left(\omega_{p} t+c\right) \tag{2}
\end{equation*}
$$

where $C$ and $c$ are integration constants depending on the initial conditions. The equilibrium position deviates from true north due to the twist of the suspension tape by

$$
\begin{equation*}
\alpha_{0}=\frac{B}{B+F} \alpha_{t} \tag{3}
\end{equation*}
$$

The damping factor $\lambda$ is given by

$$
\begin{equation*}
\lambda=k / 2\left(J+\frac{N^{2}}{G}\right) \tag{4}
\end{equation*}
$$

and the freouency $\omega$ of the swing is

$$
\begin{equation*}
\omega_{p}^{2}=\frac{B+F}{J+\frac{N^{2}}{G}}-\lambda^{2} . \tag{5}
\end{equation*}
$$

These equations describe the horizontal component of the oscillation of the suspended gyroscope with an accuracy better than required for the determination of azimuths, but they have to be modified for practical use, since $\alpha, \alpha_{0}$ and $\alpha_{t}$ refer to true north, while the observations are carried out with respect to zero of the reading scale of the gyro or of the theodolite.

## 6. OBSERVATION METHODS

There are six methods of finding north using a suspended gyroscope (THOMAS 1982). ${ }^{5}$ ). The oscillation curve of the moving mark as observed through the collimator is depicted in Figure 7 as a function of time.

The equation of this damped simple harmonic motion is given by

$$
\begin{equation*}
\alpha=\Delta N^{4}+A e^{-\lambda\left(t-t_{0}\right)} \sin \left(\left(t-t_{0}\right) \frac{2 \pi}{T}\right) \tag{6}
\end{equation*}
$$

where $\Delta N^{\prime}$ is the required angle between the zero line $N^{\prime}$ of the observation scale and the centre line $M$ of the oscillation. A is the amplitude and $T$ the period of the motion. Since the damping factor $A$ is usually neglectably small, three independent observations suffice to uniquely determine $\Delta N^{\prime}$, $A$ and $T$. The classical method is the observation of three consecutive reversal points, $a_{1}, a_{2}$, and $a_{3}$ for example. The wellknown SCHULER Mean is then employed to calculate the required scale value of the centre line of oscillation


Figure 7 0scillation of the moving mark plotted against time

$$
\begin{equation*}
\Delta N^{\prime}=\frac{a_{1}+2 a_{2}+a_{3}}{4} \tag{7}
\end{equation*}
$$

The observation of transit times was first proposed by SCHWENDENER (1964) ${ }^{6}$ ). If three consecutive transits through a selected graduation line $\bar{\alpha}$ are timed using a stop watch with lap timing facilities, then the difference $\Delta \bar{\alpha}$ between this line and the centre line of oscillation is computed from

$$
\begin{gather*}
t_{12}=t_{2}-t_{1}, \quad t_{23}+t_{12}=T \\
t_{23}=t_{3}-t_{2}, \quad t_{23}-t_{12}=\Delta t \\
\Delta \bar{\alpha}=A \frac{\Delta t}{T} \tag{8}
\end{gather*}
$$

where again linear damping is assumed. The amplitude $A$ is read at the turning point.

A combination of two transit times and one reversal point reading can be used to determine $\Delta N^{\prime}$ as has been shown by THOMAS (1982) ${ }^{5}$ ). Let $t_{1}$ be the first transit time, next the turning point $a_{1}$ has to be read and finally the second transit time $t_{2}$ at the same selected graduation line $\bar{\alpha}$ is taken. If the period $T$ is known then

$$
\begin{align*}
& t_{12}=t_{2}-t_{1}, p=2 \sin ^{2} \frac{\pi}{2 T} t_{12} \\
& \Delta \bar{\alpha}=\left(\bar{\alpha}+a_{1}[p-1]\right) / p \tag{9}
\end{align*}
$$

can be used to localize the centre line of oscillation on the reading scale, where linear damping has been assumed.

All three basic methods can be used in the clamped and in the tracking mode.

The tracking mode requires a special theodolite with a double tangent screw, so that the observer can rotate the theodolite continuously such that the gyro mark remains in coincidence with the zero line of the reading scale. The reversing point readings and/or the transit times are taken at the horizontal circle of the theodolite. This observation method has the advantage that no torsion is applied to the suspension tape, so that the motion is governed by Eq. 1b. On the other hand the tracking requires much skill and patience and leads to systematic errors if not performed perfectly. The firm MOM has developed an automatic tracking system which is incorporated in the gyroscope series Gi-B2. In spite of the reportedly good results the manufacturing of these instruments has been given up.

It seems that generally the observations in the clamped mode are favoured. The disadvantage of having to consider the torsional oscillation of the system is outweighed by the absence of any interference of the observer with the oscillating system. To improve the accuracy of the results the basic methods as outlined above have been further developed. The reversal point method is usually employed taking more than the minimum of three turning point readings. The accuracy of the readings can be upgraded by using special optical micrometers and the redundant observations are evaluated in a strict least squares adjus tment.

The same trend can be observed for the transit method. Precise recording watches are used which make it possible to take the times of transit at all graduation lines, so that as many as 60 observations become available for each swing. The times are taken to one hundredth of a second. A rigorous adjustment on the basis of Eq. 6 yields the required results (CASPARY/SCHWINTZER 1981) ${ }^{7}$ ).

## 7. AUTOMATED OBSERVATION

Since the observation of transit times and turning points is wearisome, time-consuming and requiring experienced observers, efforts were soon made to automate the measuring procedure. First experiments were made in Canada with a MOM Gi-B1 and a WILD GAK 1 and similarly at CERN with the WILD attachment. The results were very encouraging as a considerable gain in accuracy and a reduction of observation time could be achieved. Today two automated gyroscopes of high precision are on the market.

MOM offers the Gi-B11 gyrotheodolite, which is equipped with two photodiodes. The transit of the moving mark through these diodes is automatically timed by a quarz-stabilized impulse counter. The results appear on a display and can be recorded or directly transferred to the computer (HP 41 C ) if connected. The position of the centre line of the oscillation is computed from Eq. 10, requiring the observation of one complete period.

The observation has to be repeated at least once. Figure 8 explains the terms of Eq. 10.

$$
\begin{equation*}
\Delta N^{\prime}=\alpha_{0} \frac{\cos \tau_{1}-\cos \tau_{3}}{\cos \tau_{1}+\cos \tau_{3}}, \quad \tau_{i}=\frac{t_{i} \pi}{T} \quad, \quad i=1,3 \tag{10}
\end{equation*}
$$



Figure 8 Automated time reading, MOM Gi-B11

WBK has automated the gyrotheodolite MW 77 and sells it under the name Gyromat. For the determination of the centre line of oscillation a novel integration method is used. The swing is continuously sensed by a photo-diode equipped with an opto-electronic pickoff. The signal of one period only is used. It is filtered and integrated, the result is a measure for the required angle $\Delta N^{\prime}$


Figure 9 Integration method of signal sensing in the Gyromat (EICHHOLZ 1978) as can be seen from Figure 9. The observation process is automatically temperature corrected and controlled by certain functions.

On a lower level of accuracy, furnishing military requirements mainly, BODENSEEWERK GERAETETECHNIK (BGT) has developed an automatic gyro attachment. The directive moment of the spinning gyro is sensed and converted into a proportional current which generates a counter moment suppressing the precession. The same current drives a torquer which directs a reference mark towards true north to be picked off by the theodolite.

## 8. CALCULATION OF THE AZIMUTH

The astronomic azimuth $A^{\prime}$ relative to the true meridian is computed from Eq. 11. Figure 10 shows the meaning of the terms which are discussed in due detail later.

$$
\begin{equation*}
A^{\prime}=\alpha_{0}+\Delta N^{\prime}+E+\beta_{p}+\beta_{r} \tag{11}
\end{equation*}
$$


M Centre Line of Oscillation
TZP Tape Zero Position
$N^{\prime} \quad$ Zero of the Reading Scale
PO Pre-orientation Reading
ZHC Zero of the Horizontal Circle

- Angle on the Horizontal
$=$ Circle
$=$ Angle on the Reading Scale
$\equiv$ Computed Angle

Figure 10 Graph of the quantities required to compute the azimuth

The zero-torque direction of the tape according to Eq. 1 is given by

$$
\begin{equation*}
\alpha_{t}=\alpha_{0}+\Delta N^{\prime}-\varepsilon \tag{12}
\end{equation*}
$$

substituting $\alpha_{0}$ of Eq. 3 yields

$$
\begin{aligned}
\alpha_{t}-\frac{B}{B+F} \alpha_{t} & =\Delta N^{\prime}-\varepsilon \\
\alpha_{t} & =\frac{B+F}{F}\left(\Delta N^{\prime}-\varepsilon\right)
\end{aligned}
$$

which finally results in

$$
\begin{equation*}
\alpha_{0}=\frac{B}{F}\left(\Delta N^{\prime}-\varepsilon\right) . \tag{13}
\end{equation*}
$$

Hence the computation requires the knowledge of $E$ and of the ratio $c=B / F$.
$\varepsilon$ is the zero-torque direction of the tape on the reading scale. It can be determined by any of the methods outlined in sections 5 and 6 observing the oscillation of the non-spinning gyro. Since the tape zero position is not very stable it is usually measured before and after the observation of the spinning gyro. If the difference is small, then the mean is used for the correction of Eq. 13 otherwise the last value should be preferred. The ratio $c$ is a constant which varies with latitude. The manufacturers provide a table of $c$ as the result of a calibration for each individual instrument. As SCHWENDENER $(1964)^{6}$ ) has shown, the factor $c$ is a direct function of the periods of the oscillations of the gyro in the tracking mode $T_{t}$ and in the clamped mode $T_{C}$.

$$
\begin{equation*}
\frac{B}{F}=c=\frac{T_{t}^{2}-T_{c}^{2}}{T_{c}^{2}} \tag{14}
\end{equation*}
$$

Other methods of determination are discussed in VANICEK (1972 ${ }^{4}$ ).

The angle $E$ in Eq. 11 is an instrument constant which relates the zero line of the reading scale to the horizontal circle of the theodolite. This constant is usually determined on an astronomical reference line. The determination has to be repeated at regular intervals if accurate results are required. Modern instruments e.g. the MOM Gi-B11 and the Gyromat, possess an instrument fixed reference mirror enabling the measurement of changes of $E$ together with each azimuth observation.

The last two terms of Eq. 11 are theodolites readings, $\beta_{r}$ after sighting the reference object, and $\beta_{p}$ after the pre-orientation of the gyroscope.

Since $A^{\prime}$ refers to the momentary direction of the earth's rotation axis, all standard reductions known from astronomic azimuth determination have to be applied to convert $A^{\prime}$ into a bearing defined in a selected coordinate system.

## 9. ACCURACY CONSIDERATIONS

On assessing the performance of gyroscopes it is particularly important to distinguish precision and accuracy and to consider the environmental circumstances at the observation site. Measurements in a laboratory or a tunnel yield more consistent results than those in the field. Most of the standard deviations being reported on in the literature are computed from repeat measurement and hence measures of precision. Figure 10 shows the number of elements, all affected by random and systematic errors, which are put together to make up the azimuth. It is obvious that the result cannot have an accuracy of one second of arc or better. But it is possible to create favourable conditions, to apply well designed procedures and to exercise care in order to get optimal results.

Two theodolite readings ( $\beta_{p}, \beta_{r}$ ) belong to each azimuth, they should be taken twice at the beginning and twice at the end of each azimuth determination. Differences indicate
instabilities of the set-up. Optimal results require observation pillars, tripods enable only second best results.

No azimuth can be more accurate than the instrument constant E . The methods of determination and of checking $E$ have been outlined already in section 8 . Experiments have shown that $E$ changes with temperature, therefore a calibration in a temperature controlled laboratory should be carried out if field measurements are considered.

The determination of the centre line in the non-spin mode ( $\varepsilon$ ) should be carried out before and after the observation of the equilibrium position of the spinning gyro ( $\Delta N^{\prime}$ ). If a transit method is employed, then three full swings for $e$ and two for $\Delta N^{\prime}$ are adequate.

The constant $c=B / F$ of Eq. 13 should be determined for each observation site. If a chronometric method is applied, then the period $T_{C}$ of Eq. 14 is available without extra measurements, only $T_{t}$ has to be measured, which can be done by timing four to five reversal points.

Some further rules to be exercised in order to get good results are:
Swinging of the gyro without spinning for half an hour eliminates torsions and deformations of the tape being imposed by clamping for transport.

After spin-up of the gyro a certain time is needed for the instrument to attain temperature equilibrium. Dependent on the air temperature this can require up to one hour for MOM and WILD instruments. The Gyromat needs less time since it only slightly warms up due to the low spinning rate.

The tape zero position ( $\varepsilon$ ) should be regulated near to zero to minimize systematic effects on the oscillation. For the same reason the pre-orientation should be performed as well as possible to get a small value of $\Delta N^{\prime}$.

Some authors recommend measuring with two different pre-orientations symmetrical to the zero line.

After these actions the actual observations can start. Regular checks and corrections of the vertical axis of the instrument contribute to the quality of the results.

A protection from certain environmental factors like magnetism, movement of air by ventilation or wind, and vibrations caused by heavy machinery or traffic is desireable. This can be achieved by carefully selecting the site and the time of measurements and by shielding the instrument.

If these rules are obeyed and the best observation procedures are employed then the observed azimuths have typically the following standard deviations. The WILD GAK 1 mounted on a single second theodolite using extended chronometric methods or special devices for reading the scale provides the azimuth with $5^{\prime \prime}$ to $6^{\prime \prime}$ standard deviation. The automated gyrotheodolites Gyromat and Gi-Bl1 perform slightly better, namely with an accuracy of $3^{\prime \prime}$ to 4 ".

The precision of results for measurements in a lab are usually better by a factor of two.

## 10. NON-CONVENTIONAL GYROSCOPES

Since the early 1960s optical gyroscopes have been under development. These instruments do not have moving parts, are of apparently somple design and seem to have the potential of better performance in respect to resolution and dynamic range, as compared with the conventional mechanical gyroscopes. But the first prognoses that these new instruments would soon completely replace the rotating machines had to be revised, since numerous problems were encountered in the development of prototypes. Today the laser gyro is on an operational level. It is used in strap-down platforms for aircraft navigation (Boeing 757, 767 and Airbus 310), where a wide measuring range is more important than a high precision.

The basic idea of optical gyroscopes is due to the French physicist SAGNAC, who built in 1913 the first ring interferometer to study the nature of light. If a ring interferometer is mounted on a turntable and rotated then a light beam travelling with the rotation will need more time to arrive back to the source than a beam travelling against the rotation. This is the so-called SAGNAC effect, which can easily be computed if minor corrections due to the general relativity theory are neglected. The difference $\Delta \mathrm{L}$ of the optical path length of the two beams is

$$
\begin{equation*}
\Delta L=\frac{4 F}{c} \omega \tag{15}
\end{equation*}
$$

where $F$ is the area enclosed by the path, $c$ the velocity of light and $\omega$ the angular velocity of the ring with respect to inertial space. The path difference can be measured indirectly by observing the fringe pattern produced by the interfering beams, see
Figure 11.


| 1 | Light Source (LASER) |
| :--- | :--- |
| 2 | Mirror |
| 3 | Beam Splitter |
| 4 | Fringe Pattern |

Figure 11 Schematic of a ring interferometer according to SAGNAC (RODLOFF 1982)8).

Since $c$ is constant, the path difference $\Delta L$ is directly proportional to the area $F$ and the angular velocity $\omega$ of the set-up. The technical problem is to generate a measureable effect, which is achieved by two different approaches.

The fiber-optic gyroscope attains the required sensitivity by use of a large area $F$. Optical fiber with a length up to 1000 m is wrapped into a coil of very low volume. The first rotation rate sensors of this type are now being readied for application. Figure 12 shows the concept of a fiber-optic gyro as developed by SEL. It has a dynamic range of $400^{\circ} / \mathrm{s}$, a repeatability of 50 ppm and a drift of less than $3 \% \mathrm{~h}$.

The laser gyro consists of a cavity as depicted in Figure 13. It converts the path length difference in a frequen-


Figure 12 Schematic of a fiber-optic gyroscope (SEL) cy signal, which can be observed more easily. The mirrors create a closed path calibrated to be an integral multiple of the wave length thus forming an optical resonator. Rotation of the ring laser induces a change of the eigenfrequencies of the two beams travelling in opposite directions. Their beat frequency is proportional to the rate of the resonator. It is measured by observing the fringe pattern produced by superimposing the beams. The earth's rate of $15^{\circ} / \mathrm{h}$ generates a beat frequency of 1 Hz in a laser gyro of
$F=100 \mathrm{~cm}^{2}$ and $\lambda=0.633 \mu \mathrm{~m}$, being easily measureable.

$\begin{array}{ll}1 & \text { Mirror } \\ 2 & \text { Dielectric Mirror } \\ 3 & \text { Anode } \\ 4 & \text { Cathode } \\ 5 & \text { Length Control } \\ 6 & \text { Cervit Block } \\ 7 & \text { Corner Prism } \\ 8 & \text { Readout Detector }\end{array}$
$\begin{array}{ll}\text { Figure } 13 & \begin{array}{l}\text { Schematic of a gyro block } \\ \text { assembly (RODLOFF 1982) }\end{array}\end{array}$

The main problem with the laser gyro is the so-called lock-in effect, which prevents the measurement of small angular velocities. The reason for this effect is the tendency of coupled oscillators to swing on a common frequency if the eigenfrequencies differ only a little. Thus input rates below a certain threshold produce a zero output. A remedy is to mechanically dither the laser block with a frequency above the lock-in rate.

As already mentioned, laser gyros are operational as attitude sensors for inertial platforms. They have an effective range of $10^{-2} 0 / \mathrm{h}$ to $1000^{\circ} / \mathrm{s}$. Prognoses of today are that the sensitivity to smaller rates will improve and will become comparable to the best mechanical gyroscopes. Since their production is less expensive, they may possibly one day compete with conventional suspended gyroscopes. Suitably mounted, so that rotations about a horizontal and a vertical axis are possible, optical gyros are capable of sensing the true north direction and the geographic latitude.
11. CONCLUSION

Over the last two decades significant advances have occured in the technology of conventional suspended gyroscopes. The accuracy of determining north has improved considerably due to the advent of new instruments, the introduction of more effective chronometric observation methods and to automating the observation of the centre line of oscillation. The same actions have simplified the measurements and reduced the time required, so that it has become possible to measure an azimuth within 15 to 60 minutes with a standard deviation of $3^{\prime \prime}$ to $6^{\prime \prime}$.

In future, optical gyroscopes may replace the conventional instruments. Within 25 years laser gyros have reached a level of performance rendering them suitable for application in strap-down platforms. Fiber-optic gyros have been under development for 10 years, seemingly meeting now the performance requirements of inertial navigation. Both realizations of the SAGNAC effect hold the promise of giving higher performance at lower cost.

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