Heft 5

München, September 1981

Accuracy Improvement in Close Range Photogrammetry

by
K. Torlegård

SCHRIFTENREIHE

Wissenschaftlicher Studiengang Vermessungswesen
Hochschule der Bundeswehr München

Heft 5

München, September 1981

Accuracy Improvement in Close Range Photogrammetry

by
K. Torlegård

SCHRIFTENREIHE

Wissenschaftlicher Studiengang Vermessungswesen
Hochschule der Bundeswehr München

Der Druck dieses Heftes wurde aus Haushaltsmitteln der Hochschule der Bundeswehr München gefördert.

Verantwortlich für die Herausgabe der Schriftenreihe: Prof. Dr. G. Neugebauer
Dipl.-Ing. J. Peipe

Bezugsnachweis:
Hochschule der Bundeswehr München
Fachbereich Bauingenieur- und Vermessungswesen
Studiengang Vermessungswesen
Werner-Heisenberg-Weg 39
8014 Neubiberg

ACCURACY IMPROVEMENT IN CLOSE RANGE PHOTOGRAMMETRY

PROF.DR.ING. KENNERT TORLEGÅRD

Hochschule der Bundeswehr München
Institut für Photogrammetrie und Kartographie

and

Kungliga Tekniska Högskolan Stockholm
Institutionen för Fotogrammetri

Munich, 1981

PREFACE

The following study is part of a report from a six months´ research stay as Visiting Professor for Engineering Photogrammetry at the Hochschule der Bundeswehr München. This study may be seen as a summary of the state of art in highly accurate close range photogrammetry with emphasis on detection, localization and elimination of blunders in observations. Several hitherto unsolved problems have been defined, and some new research plans have been outlined. It is my hope that the survey of available methods to achieve high accuracies in close range photogrammetry given in this report will be of value for engineers and surveyors using close range photogrammetry as a measuring tool, and that it will be a stimulance for scientists and researchers to set out new goals for the development of this mensuration technique.

ACKNOWLEDGEMENTS

I am very much indebted to Prof. Dr.-Ing. Egon Dorrer and his collaborators at the Division of Photogrammetry, Hochschule der Bundeswehr München, for the preparations for my stay in Munich, for a valuable cooperation, for stimulating discussions, for the initiation of contacts with other research organizations in the Federal Republic of Germany and for support to and interest in my research activities. I am also indebted to Deutsche Forschungsgemeinschaft (DFG) for the financial support, to the Hochschule der Bundeswehr München for providing the facilities, and to the Royal Institute of Technology (KTH), Stockholm, for the possibility to have a sabbatical leave.
Financial support to the printing of this publication was provided by Hochschule der Bundeswehr München through special funds.

CONTENTS

0 ABSTRACT

The state of art in highly accurate analytical photogrammetry is briefly overviewed. The concept of accuracy is described as precision, model fidelity, and reliability, where precision is related to random errors and their propagation, model fidelity to systematic discrepancies between reality and mathematical model, and reliability to detection, localization and elimination of blunders. The effect of blunders in least squares adjustments is outlined, and a procedure in order to localize blunders after adjustment is suggested. The possibility to localize and eliminate blunders in all phases of the photogrammetric process is discussed in detail, particularly for close-range applications. The principle of redundancy to achieve reliability is demonstrated, leading to the multi-concept in photogrammetry. It can be applied to all steps: coordinate readings, fiducial marks, targets, frames (new method of overlapping), stations, control points, object geometry, computer program options. Simultaneous adjustment of observations, of image coordinates, observations of exterior orientation elements, geometric conditions in object space, and approximations for the unknowns are reviewed. Several hitherto unsolved problems and open questions have been stated.

1 INTRODUCTION

Analogue methods for photogrammetric measurements have been practiced for a long time. Cameras, instruments and methods have reached a standardization that makes it possible top plan projects and predict costs and accuracies with narrow limits. This is certainly the case for aerial photogrammetry as applied to topographic mapping. Wide angle cameras with a 0.15 m camera constant are used for taking near vertical photographs with 60 % overlap within and 20 % between the strips. Line maps with contours and orthophotos are the typical products. As an intermediate step the aerial triangulation provides the necessary control points in a manner that even emphasizes the standardization.

In non topographic photogrammetry, however, the situation has been somewhat different. The measuring tasks have been very varied, and the requirements and specifications so different from one application to another that it has been difficult to develop standardized analogue methods for the restitution of photographs. A certain standardization has been reached in architectural photogrammetry with the normal case stereo photography often using stereometric cameras, and restitution in normal case stereoplotters yielding as result a line drawing on vertical or horizontal projection planes. This equipment and technique has been used also for other purposes, sometimes successfully, but very often certain aspects in the requirements could not be fulfilled, e.g. type of output, accuracy or completeness. The customer or end-user has sometimes difficulties in defining his requirements, and the photogrammetrist then recommends his well-established methods even though both parties have the feeling that something else would be more appropriate.

During the last decade analytical methods have demonstrated its capability as a very flexible tool for solving the most varied measuring problems in almost any field where geometric quantities are needed. The limitations due to the necessary standardization in the analogue methods are not relevant any longer, especially concerning the choice of cameras, their location and direction in relation to the object, type of object space control, overlap and number of photos, accuracy, computerized evaluation and presentation of results. This flexibility has opened new vistas for the application of close range photogrammetry. At the same time, however, planning the projects and predictive accuracies have become more uncertain, since the well-known standard procedures have been abandoned. The automatic checking on the ray intersection condition in the analogue stereomodel by the operator´s stereo vision is now lost when mono or stereo cooperators are used for measurements. On-line computation or use of analytical plotters eliminate this drawback. Object space control based on straight lines, horizontal of vertical planes, angles, etc. that easily were taken to advantage in the analogue stereomodel have now to be included in the adjustment computations by a series of program routines and extension of the basic mathematical formulations. The analogue approach is typical on-line in the sense that the result is readily available for checking, correction, deletion and amendment. In the analytical approach, on-line computer programs have to be developed. To implement this, an analytical stereo plotter is needed, and that is an expensive development and instrumentation. The photogrammetrist has now to develop his occupational skill and experience, and to learn how to use the new possibilities that are opened up by analytical methods. This comprises, among other things, extensions of mathematical models, correction of systematic errors, detection, localization and elimination of blunders, prediction of precision and reliability, planning photogrammetric projects, statistical methods for testing hypotheses in the evaluation process.

2 MATHEMATICAL MODEL

2.1 THE BUNDLE APPROACH

Notations, coordinate systems and rotations follow the conventions recommended by the International Society for Photogrammetry 1960 which are characterized by

a) a right-handed XYZ system in object space with Z positive upwards and X positive mainly in base direction (if this is appropriate in the close range case)

b) a free choice of the origin of the XYZ coordinate system

c) a right-handed coordinate system xyz in the image space, with positive directions as in the same sense as in the XYZ system

d) the choice of the X-axis as primary axis and the Y-axis as secondary axis; rotations around rotated axes

e) treating as positive: clockwise rotations about the positive direction of the X- Y- and Z-axes in conformity with the conventions of a right-handed system

Notations:

$x_k \ y_k$	comparator coordinates
$x' \ y' \ / \ x'' \ y''$	plane image coordinates in left/right photo
c	camera constant (calibrated focal length)
$x_1 \, y_1 \, z_1 \ / \ x_2 \, y_2 \, z_2$	three dimensional image coordinates in left/right photo with axes parallel to XYZ

$x\ y\ z$	model coordinates
$bx\ by\ bz\ /\ BX\ BY\ BZ$	base components in model/object
$\omega_i\ \varphi_i\ \kappa_i$	rotations around the $x_i\, y_i\, z_i$ axes for photo i
$\xi\ \ \eta\ \ \alpha$	rotations of the model system $x\,y\,z$ around the $X\,Y\,Z$ axes
$X_O\ Y_O\ Z_O$	origin of the model system´in the $X\,Y\,Z$ system
$X_g\ Y_g\ Z_g$	known control point coordinates
$O_1\ O_2\ \ldots\ O_i\ \ldots$	projection centres

See also Fig. 2.1 .

The basic relation is the collinearity condition for the imaging ray from the object point through the perspective centre to the image point, which is expressed by

$$\begin{bmatrix} X & - & X_O \\ Y & - & Y_O \\ Z & - & Z_O \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \lambda\, R \begin{bmatrix} x' & - & x'_O \\ y' & - & y'_O \\ & - & c \end{bmatrix} .$$

Here, λ is a scalar factor, and R is an orthonormal matrix with, e.g. the elements

$$\begin{aligned}
r_{11} &= \cos\varphi\ \cos\kappa \\
r_{12} &= -\cos\varphi\ \sin\kappa \\
r_{13} &= \sin\varphi \\[1ex]
r_{21} &= \cos\omega\ \sin\kappa + \sin\omega\ \sin\varphi\ \cos\kappa \\
r_{22} &= \cos\omega\ \cos\kappa - \sin\omega\ \sin\varphi\ \sin\kappa \\
r_{23} &= -\sin\omega\ \cos\varphi \\[1ex]
r_{31} &= \sin\omega\ \sin\kappa - \cos\omega\ \sin\varphi\ \cos\kappa \\
r_{32} &= \sin\omega\ \cos\kappa + \cos\omega\ \sin\varphi\ \sin\kappa \\
r_{33} &= \cos\omega\ \cos\varphi
\end{aligned}$$

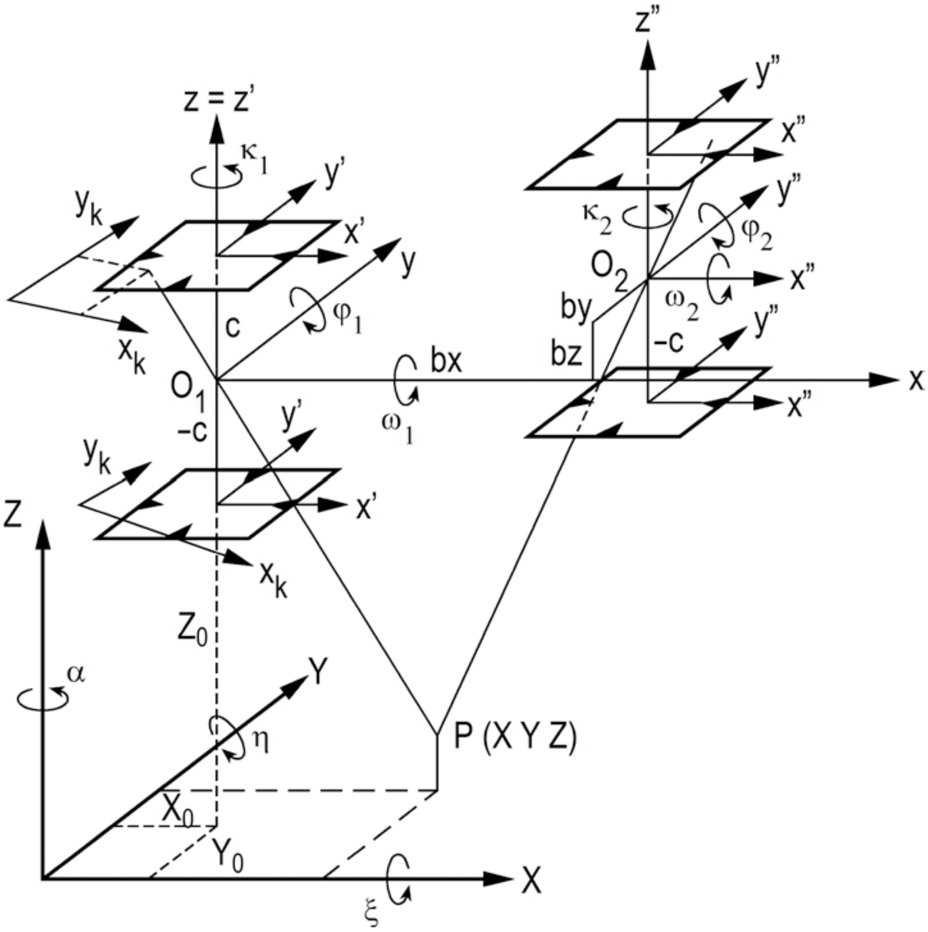


Fig. 2.1

Axes, rotations and notations recommended by ISP 1960

As $c \neq 0$, we divide the first two equations by the third, thus obtaining image coordinates as functions of the orientation elements and the object coordinates

$$x' = x'_O - c\,\frac{T_x}{N}$$

$$y' = y'_O - c\,\frac{T_y}{N}$$

Where

$$\begin{matrix} T_x \\ T_y \\ N \end{matrix} \quad = \quad R^T \cdot \begin{matrix} X - X_O \\ Y - Y_O \\ Z - Z_O \end{matrix}$$

For the determination of orientation elements from image coordinates $x'\,y'$ and known points $X_g\,Y_g\,Z_g$, we linearize the expressions either by a Taylor series expansion or by numerical differentiation from approximate values for the unknowns. The system of linear equations will, as a rule, be overdetermined and can be written in the form

$$A\,X = L + V$$

2.2 CONVERGENCE CRITERIA FOR ITERATIONS

The least squares solution of X represents corrections to the approximations, and the procedure has to be iterated until convergence in order to give the final values of the orientation elements. The iteration process can be terminated on different criteria, e.g. the following.

Assume A to have the dimensions $(n, p, n>p)$ and full rank, rank $(A) = p$.

The least squares estimate of X will be

$$\hat{X} = (A^T A)^{-1} A^T L \quad .$$

The simultaneous magnitude of the unknowns is described by the quadratic form

$$L^T A\,(A^T A)^{-1} A^T L \quad .$$

This is the reduction of $L^T L$ to $V^T V$ by the adjustment, and

$$L^T L = V^T V + L^T A (A^T A)^{-1} A^T L \quad ,$$

where the three terms have the degrees of freedom $n, n-p$ and p, respectively. The two latter ones are independent. We form the test-variate

$$t = \frac{L^T A \; (A^T A)^{-1} A^T L}{p} \cdot \frac{n-p}{V^T V} \quad .$$

If $t < t_O$ the iterations may be stopped. Because of rounding errors, t will never be zero, and t_O may be chosen, e.g., in the interval [0.001, 0.1] .

The effect of the omitted higher order terms is smaller than that of the unknowns, and thus they have very little influence on $V^T V$. This test must not be regarded as an F-test used for testing the hypothesis that two variances are equal, because here we require one quadratic form to be zero as a measure for convergence, but we still can make the convergence criterion dependent on the measuring precision. We simply do not want to perform unnecessarily many iterations in relation to the precision of observations.

Experience has shown that for relative orientation and three dimensional conformal transformation a value of $t_O = 0.001$ is suitable. This means that the linear effect of the unknowns is in the image scale of the magnitude $0.001 \cdot s$ to $0.1 \cdot s$, where s is the standard error of unit weight in the adjustment

$$s = \sqrt{\frac{V^T V}{n-p}} \quad .$$

The convergence test is thus related to the estimated variance, which has the advantage that it works for various types of observations, various cameras, various project designs and iteration procedures. This is important for close range applications with their varied conditions. At the same time, however, it has the drawback that the estimated variance itself is a random variable, and for a small number of redundant observations, the iteration may be stopped too early or too late compared to what is needed for convergence.

2.3 LENS DISTORTION AND ADDITIONAL PARAMETERS

For some applications the basic expressions are sufficient, but in other cases the mathematical model shows systematic errors when it is compared to physical reality. We then have to extend the model to include parameters for these systematic errors in the expressions. Radial distortion is effectively covered by the following expressions

$$dx' = x' \cdot \{a_3 (r^2 - r_0^2) + a_5 (r^4 - r_0^4) + a_7 (r^6 - r_0^6) + \ldots\ldots\}$$

$$dy' = y' \cdot \{a_3 (r^2 - r_0^2) + a_5 (r^4 - r_0^4) + a_7 (r^6 - r_0^6) + \ldots\ldots\} \,.$$

The number of parameters depend on the type of radial distortion and how well we want to model it.

The decentering distortion based on the thin-prism model is described by

$$dx' = \{ (r^2 + 2\,x'^2)\, P_1 + 2\,x'y'P_2\} \{1 + P_3\, r^2 + P_4\, r^4 + \ldots\ldots\}$$

$$dy' = \{ 2\,x'y'P_1 + (r^2 + 2\,y'^2)\, P_2\} \{1 + P_3\, r^2 + P_4\, r^4 + \ldots\ldots\} \,,$$

where $r^2 = (x' - x'_0)^2 + (y' - y'_0)^2$ and r_0

is a constant for which the radial distortion is zero. The distortion may be known from calibrations and introduced as corrections to the image coordinates before adjustment, or it can be unknown and used as additional parameters to be determined in the adjustment.

In aerial photogrammetry the atmospheric refraction, as a rule, is taken into consideration, but this is not very often necessary in the close range case. Here, however, other systematic effects have to be considered, e.g. refraction in filters and port windows, departures from emulsion flatness, regular deformations of the image during photographic processing. Again we have two possibilities: calibration or additional parameters in the adjustment.

In the calibration case the systematic errors are determined in some representative points covering the image area. In the successive measurements the new image coordinates are corrected for systematic error by interpolation from the calibration points. Different interpolation methods can be used. Some popular ones in photogrammetry and surveying are

1. Polynomial approximations
2. Spline interpolations
3. Linear prediction with covariance functions
4. Multiquadratic interpolation.

See e.g. Hein - Lenze (1979), Shut (1976) and Rauhala (1974).

In the case of additional parameters in the adjustment one has to avoid over-parametrization and linear dependencies to already introduced variables. Ebner (1976) and Grün (1978a) show the efficiency of using orthogonal parameters, and statistical testing for the selection of the appropriate parameter set. Jacobsen (1980) has another set of additional parameters and he combines statistical tests with a test statistic that measures the remaining systematic effect after adjustment. The set of additional parameters has in these cases been designed to fit the geometric conditions of aerial photogrammetry, and if the close range case differs from this it is not longer certain whether the selected set is optimal.

2.4 ACCURACY OF CONTROL POINTS

In analogue photogrammetry the double point resection in space is solved in two steps, relative and absolute orientations. The given object control is, in the second step, regarded as free from noticeable error, but experienced stereo-operators often use their knowledge about the control, its accuracy, the targeting, the type of surveying behind the given coordinates, etc., in such a way that the residuals in the model after adjustment are not distributed in as it would be the case from a numerical least squares solution. The operator has taken into consideration information on the accuracy of the control that the numerical absolute orientation did not know when the orientation parameters were calculated from the discrepancies between the control point coordinates in the model and object system. This sort of observational skill, experience and feeling for weakness and strength in the control can be transferred to the mathematical model by a formulation which allows for corrections also to the given control. The control coordinates are treated as unknowns, and in order to compensate for the rank deficiency, ficticious direct observations on the control point coordinates are introduced with a weight that corresponds to the accuracy of the control in relation to the other equations of the system.

2.5 INTERPRETATION OF RESULTS

The mathematical model has to be developed to such a degree that it can be used for prediction without bias and with known precision. We thus want our model to have stochastic properties such that

a) $A\ X = E\ (L) = \lambda$

b) L normally distributed according to $N\,(\lambda,\ \sigma^2\,Q_{LL})$.

This gives us the a priori weight matrix $P = Q_{LL}^{-1}$.
Very often, one assumes that $Q_{LL} = I$, i.e. the observations are independent and of equal weight $P = Q_{LL}^{-1} = I$. In other cases $Q_{LL} = D$ is a diagonal matrix, which means that the observations have different weights but they are still independent or uncorrelated:

It is a main task for scientists to develope models having the above quoted properties. Another task is to give a physical explanation to the parameters X of the model, e.g. such as orientations elements, radial and tangential distortion parameters. If additional parameters such as purely mathematical parameters a_{ij} and b_{ij} in a general regression formulation yield the above properties to the model, the scientist should find the physical explanation to this in order to increase our knowledge. Such knowledge can stimulate to new instrument designs, and improve new possibilities to correct the model, based on methods and measurements that was not earlier applied in photogrammetry.

3 MEASURES OF PRECISION, FIDELITY, AND RELIABILITY

3.1 TERMINOLOGY

According to Baarda, accuracy comprises two parts, viz. precision and reliability (Baarda, 1977a). Here, we amend the theory with the concept of model fidelity (German: Modelltreue; French: fidelité de modèle; Swedish: modellriktighet). Precision is expressed as standard error computed by the law of error propagation through functions of the observations, and based on the assumptions, on their stochastic nature. The fidelity of the model is the goodness of fit of the mathematical model to the observations, and the model should be general enough to the valid under varying experimental conditions. Reliability is the possibility to detect, localize and correct or exclude blunders from the adjustment, and still having redundancy enough to check for blunders in the remaining system.

Hallert (1967) used to discriminate between accuracy and precision. In his sense precision was the internal closeness of data from repeated observations. Accuracy on the other hand was the closeness of observations and functions of these to given data with much higher accuracy than the observations, or closeness to mathematical conditions between the observations. E.G.: standard deviations from a series of repeated settings on y-parallaxes in a point of a stereomodel are, according to Hallert, measures of precision, while the standard error of unit weight after a least squares adjustment of the relative orientation from observations on y-parallaxes having unit weight is a measure of accuracy.

Here we will try to use the definitions in such a way that precision is related to the stochastic behaviour of variables and functions of observations, model fidelity being related to the absence of systematic errors of the mathematical model, and reliability being the possibility of detection, localization and elimination of blunders.

Accuracy is the term covering all the three quoted concepts. In recent publications by other authors (e.g. Baarda, Grün, Förstner) on reliability of photogrammetric and geodetic observations, the concept reliability covers blunders as well as systematic errors. Here we thus introduce the new concept of fidelity. It should be noted here that the term reliability has another meaning in mathematical statistics. There, "reliability has been formulated as the science of predicting, estimating, or optimizing the probability of survival, the mean life, or, more generally, the life distribution of components or systems" (Mann et al., 1974, see Preface). It is also started by Mann that the concept of reliability should be understood to mean "the probability of a device (or item or organism) performing its (or his or hers) defined purpose adequately for a specified period of time, under the operating conditions encounted" (Mann et al., 1974, chapt. 1.1). With statistical methods times to failure of devices or items are studied. In the theory of errors in geodesy and photogrammetry statistical methods are used for detection and localization of blunders in a set of observations. There are common features between the two sciences in the use of statistics, e.g. the importance of order statistics.

3.2 DEFINITIONS

We have the model

$$A\,X = \lambda\ .$$

We make a series of observations on λ, denoted by L

$$E\,(L) = \lambda\ , \qquad E\left((L_i - \lambda_i)\,(L_i - \lambda_i)\right) = \sigma^2$$

$$E\,\left((L_i - \lambda_i)\,(L_j - \lambda_j)\right) = Cov\,\lambda_i\lambda_j\ .$$

Then we can introduce the true errors e

$$A\,X = L + e\ ,\ E\,(e) = 0\ ,\ E\,(e_i e_j) = Cov\ e_i e_j\ .$$

The individual true errors e_i are unknown, and with redundant observations we have to introduce residuals V to obtain a consistent system,

$$A\,X = L + V\ ;\ P = \frac{1}{\sigma_O^2}\cdot\left(Cov\ e_i e_j\right)^{-1}\ ,$$

which we solve by the methods of least squares, and we get the estimates

$$\hat{X} = (A^T\,PA)^{-1}\,A^T\,PL\ ,$$

$$\hat{V} = (A\,(A^T\,PA)^{-1}\,A^T\,P - I)\,L\ ,$$

$$\hat{s}_O^2 = \hat{V}^T P \hat{V} / r$$, where r is the number of redundant observations.

The general law of error propagation then gives the following estimation of the standard error of a function $U = FL$ of the observations, viz.

$$Cov\,(UU^T) = \hat{s}_O^2\, F P^{-1} F^T \quad .$$

Let, e.g.

$$U = \hat{X} \text{ then } F = (A^T PA)^{-1} A^T P$$

and $$Cov\left(\hat{X}\hat{X}^T\right) = \hat{s}_O^2\, (A^T PA)^{-1} = \hat{s}_O^2\, Q \quad ;$$

or $$U = \hat{V} \text{ , then } Cov\left(\hat{V}\hat{V}^T\right) = \hat{s}_O^2\, (P^{-1} - A^O) \ , \ A^O = A\,(A^T PA)^{-1} A^T \quad .$$

For $P = I$ we get

$$Q_{\hat{V}\hat{V}} = I - A^O \quad .$$

Now, we mean with <u>precision</u> all measures of variation based on $\hat{s}_O^2$, e.g. standard errors of unknowns $\hat{x}_i$:

$$\hat{s}_{\hat{x}_i} = \hat{s}_O \sqrt{q_{ii}} \quad ,$$

or the standard error of any other function of the observations. A general measure of precision of an adjustment (a project, a design of an experiment) is

a) $tr(Q)/p$ which is the average variance of the estimated unknowns measured in units of σ^2 , and

b) the $\hat{s}_O$ which is an estimate of σ .

By model <u>fidelity</u> we mean the absence of bias of the estimates, and a lack of fidelity should measure the departure from the mathematical expectations in the model, which is the same as the systematic error D of the model

$$D = A\,X - E(L) \quad .$$

This means that our assumptions are not fulfilled. To check this, we have to design certain experiments to test if $D = 0$. This can be done with so-called controlled experiments, with testfields, etc.

By <u>reliability</u> we mean the possibility to detect, localize and eliminate blunders from the observations. If we suspect blunders in a group of observations, and if they form a submatrix

$\left(P_2^{-1} - A_2^O\right)$ in $(P^{-1} - A^O)$, then this submatrix must be non-singular.

As $(P^{-1} - A^O)$ is idempotent with rank $= r$, not all blunders can be detected.
The system can be said to be reliable if as many as $(b_O < r)$ blunders in any combination can be detected.

General measures of the reliability can be

a) $$\frac{tr(I - A^O)}{n} = \frac{r}{n}$$

or b) $$\left(\prod_{i}^{n}(I - A^O)_{ii}\right)^{1/n} \leq \frac{r}{n}$$

or c) $$0 \leq \sum_{i}^{n}\left[(I - A^O)_{ii} - \frac{r}{n}\right]^{\frac{2}{r}} \leq \frac{r}{n} \; .$$

The trace $tr(I - A^O)$ is often referred to as a measure of the global reliability. The individual diagonal elements $q_{V_iV_i}$ are measures of the local reliability.

The individual $q_{V_iV_i}$ measures the contribution of the observation l_i to the global reliability.

Baarda (1976) has introduced the concepts of internal and external reliability. (See also Förstner (1979) and Grün (1979b)). If all observations are equally well controlled (all $q_{V_iV_i}$ taking the same value), then the internal reliability is high.
If nondetected blunders influence the estimated variables to a very small extent in the adjustment, the external reliability is high.

3.3 EXPERIMENTAL DESIGN

Requirements and specifications for experiments and projects can be formulated in several ways and can have several parameters. Some of these parameters are contradictory, others are not. For the contradictory parameters an optimizing function has to be found. Very often the formulations are verbal, and as such they are of little help to the scientist or engineer for the design of the model. New formulations have then to be derived. As parameters in the new formulation we can have

--precision:	$s_O^2 \cdot Q$	and derivations thereof can be used;
	$s_O^2 \rightarrow \min$	can be obtained from including additional parameters;
	$tr\, Q \rightarrow \min$	more observations per photo, more photos, more photo stations in locations such that new pairs of rays do not lie in already established epipolar planes;

precision of unknowns and precision of functions of unknowns can be traced back to the above items, viz. s_0^2 and Q .

-- fidelity: no systematic errors in the model, i.e. $A\,X = E\,(L)$, or in functions of the estimate $\hat{X}$; this can be tested by controlled experiments. Departures between discrepancies in check points and the corresponding precision is a measure of the remaining systematic errors. Fidelity can also be checked by inspection of the residuals. The standardized residuals shall be independent and follow a normal distribution. It is more difficult to design experiments to guarantee just precision.

-- reliability: the design matrix A should be such that each observation is at least double checked, so that if one observation is deleted due to a blunder, there still is a possibility to detect (but perhaps not to localize) blunders in the remaining observations. The consequence will be that the redundancy should be rather high, e.g. $r = 2p$ or which is the same $n = 3\,p$ and the relative redundancy r/n thus 2/3 in the planning stage. The design matrix A influence the matrix $(I - A^O)$, and the aim should be to have the same magnitude on all diagonal elements of $(I - A^O)$.

-- economy: this can be expressed in terms of risks for the producer (photogrammetrist) or the consumer (client, end-user), or in cost-functions which consider the cost for the measuring operations and costs for damages caused by errors in the results due to lacking precision, fidelity and reliability. Here decision theory is of interest.

4 BLUNDERS IN PHOTOGRAMMETRIC DATA

4.1 BLUNDERS IN ADJUSTMENT

Up to now detection, localization and elimination of blunders from photogrammetric data has primarily been based upon the operational skill of the photogrammetrist. Methods have been designed to avoid blunders rather than accepting their presence in observations, which logically lead to blunder detection procedures. Theoretical studies and practical examples for blunder detection, localization and elimination in photogrammetric data have been published by a few authors during the last decade, e.g.

Förstner (1976), (1978) and (1979), Grün (1978a), (1978b) and (1979b), Bouloucos (1979), Molenaar (1976) and (1978), Stefanovic (1978a) and (1978b), Jacobsen (1980).

All these studies are based on a so-called "data-snooping" according to the theories developed by Baarda (1965), (1967), (1968) and (1976) for geodetic observations, just to mention a few important papers from the rich production by this author. Pelzer (1979) has demonstrated the method on some clear and simple examples in surveying. Recently a new theoretical approach to blunder detection in surveying and geodesy has been presented by Heindl and Reinhart (1979) based on linear programming and tolerance for the residuals. This linear formulation obviously reveals the blunders easier than the residuals after a least squares adjustment. The computational effort, however, seems to be larger with linear programming.

With matrix notations as in Bjerhammar (1973) the effect of blunders in least squares adjustment can be summarized as follows:

Functional model $$A\,X = L + e$$

$$E\,(L) = \lambda$$

$$E\,(e) = 0 \quad , \quad Cov\,(ee^T) = \sigma^2 \cdot I \quad .$$

Matrix A has dimensions (n, p) and rank $(A) = p$, $r = n - p$.

We now have blunders (denoted ∇ nabla) in the observations so that

$$A\,X = L + e + \nabla \quad .$$

The number of blunders must be less than $r = n - p$.

The problem is that we do not know in which observations these blunders occur. Thus, we proceed in the standard way of least squares adjustment by writing

$$A\,X = L + V$$

$$\left(V = e + \nabla + A\left(\hat{X} - X\right)\right) \quad ,$$

from which we estimate

$$\hat{X} = (A^T A)^{-1} A^T L \quad .$$

As we have blunders in the observations the expectation of X is

$$E\,(X) = (A^T A)^{-1} A^T E\,(L + \nabla) = X + (A^T A)^{-1} A^T V \quad .$$

We also have the influence from the blunders on the residuals

$$\widehat{V} = (A\,(A^T A)^{-1} A^T - I)\,(L + \nabla)\ , \qquad A^O = A\,(A^T A)^{-1} A^T$$

so that the blunders are distributed among the residuals by the operator $A^O - I$.
The weight coefficient matrix of the residuals is given by

$$Q_{\widehat{V}\widehat{V}} = I - A^O \quad .$$

If σ^2 is unknown, it is estimated by the estimator

$$s_O^2 = \widehat{V}^T\,\widehat{V}/r \quad ,$$

but since we have blunders,

$$\widehat{V}^T\,\widehat{V} = L^T\,(I - A^O)\,L + \nabla^T\,(I - A^O)\,\nabla + 2\,\nabla^T\,(I - A^O)\,L \quad .$$

Thus, the bias is given by the two last terms, which is the sum of the squared blunders times the corresponding diagonal elements of $(I - A^O)$, plus twice the sum of the product of the blunders and the correct observation, multiplied by the same elements. The sum is taken over the observations that contain blunders, the problem still being that we do not know where the blunders are.

Suppose for a moment that we have blunders in b of the n observations, forming group 2; the observations in group 1 are assumed without blunders.

$$A_1\,X = L_1 + V_1$$

$$A_2\,X = L_2 + V_2 + \nabla$$

$$Q_1 \quad = (A_1^T\,A_1)^{-1}$$

$$Q \quad = (A_1^T\,A_1 + A_2^T\,A_2)^{-1}$$

Rank $(A_1) = p$, rank $(A_2) = b$, $n - p = r$, $r \geq b$.

The estimate of X will be

$$\hat{X} = Q\,A_1^T\,L_1 + Q\,A_2^T\,L_2 + Q\,A_2^T\,\nabla$$

where $\qquad Q = (A^T\,A)^{-1} \qquad A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$

The residuals will be

$$\widehat{V}_1 = (A_1\,Q\,A_1^T - I)\,L_1 + A_1\,Q\,A_2^T\,L_2 + A_1\,Q\,A_2^T\,\nabla$$

$$\widehat{V}_2 = A_2\,Q\,A_1^T\,L_1 + (A_{21}\,Q\,A_2^T - I)\,L_2 + (A_2\,Q\,A_2^T - I)\,\nabla \quad .$$

Now we are interested to see what the result would be if we used only group 1 for the estimation. As we already have computed inverse $Q = (A^T\,A)^{-1}$ and estimate $\bar{X}$, we use the method of sequential adjustment (see Mikhail (1976), chapt. 13 and app 9) to find the inverse $Q_1 = (A_1^T\,A_1)^{-1}$, the estimate $\hat{X}_1$, and the residuals $\overline{V}_1$ from the adjustment of group 1.

The new inverse after deletion of group 2 will be

$$Q_1 = Q\,\{\,I + A_2^T\,(I - A_2\,Q\,A_2^T)^{-1}\,A_2\,Q\,\}\quad .$$

As we expect blunders to occur only in a limited number of observations, the inversion of the matrix $(I - A_2\,Q\,A_2^T)$ will be rather easy to be calculated, as the dimensions are small. However, the inverse must exist, a problem to that we will return to later. The new estimate of X will be

$$\bar{X}_1 = \hat{X} + Q\,A_2^T\,(I - A_2\,Q\,A_2^T)^{-1}\,\hat{V}_2\quad .$$

The new residuals of group 1 will be

$$\overline{V}_1 = A_1\,\bar{X}_1 - L_1\quad .$$

The old residuals were

$$\hat{V}_1 = A_1\,\hat{X} - L_1\quad ,$$

and thus the new ones can be calculated from the old ones by

$$\overline{V}_1 = \hat{V}_1 + A_1\big(\bar{X}_1 - \hat{X}\big) = \hat{V}_1 + A_1\,Q\,A_2^T\,(I - A_2\,Q\,A_2^T)^{-1}\,\hat{V}_2\quad .$$

It can be shown that the sum of the squared residuals will be reduced from $\hat{V}^T\,\hat{V}$ to $\overline{V}_1^T\,\overline{V}_1$ after excluding group 2 from the adjustment, viz.

$$\overline{V}_1^T\,\overline{V}_1 = \hat{V}^T\,\hat{V} - \hat{V}_2^T\,\{\,I + A_2\,Q\,A_2^T\,(\,I + A_2\,Q_1\,A_2^T\,)\,\}\,\hat{V}_2\quad .$$

The new matrix of weight coefficients, $Q_{\overline{V}\overline{V}}$, for the residuals $\overline{V}_1$ can be obtained directly from the old matrix $Q_{\hat{V}\hat{V}}$:

$$Q_{\overline{V}_1\overline{V}_1} = I - A_1\,Q\,A_1^T - A_1\,Q\,A_2^T\,(I - A_2\,Q\,A_2^T)^{-1}\,A_2\,Q\,A_1^T$$

where

$$I - A_1\, Q\, A_1^T = Q_{\hat{V}_1\hat{V}_1}$$

$$I - A_2\, Q\, A_2^T = Q_{\hat{V}_2\hat{V}_2}$$

$$A_1\, Q\, A_2^T = Q_{\hat{V}_1\hat{V}_2}$$

$$A_2\, Q\, A_1^T = Q_{V_2V_1}$$

$$\left[\begin{array}{c|c} Q_{\hat{V}_1\hat{V}_1} & Q_{\hat{V}_1\hat{V}_2} \\ \hline Q_{\hat{V}_2\hat{V}_1} & Q_{\hat{V}_2\hat{V}_2} \end{array}\right]$$

which are all submatrices of the old matrix $Q_{\hat{V}\hat{V}}$.

The new discrepancies for group 2 using the new estimate of X from group 1 is $\overline{V}_2 = A_2\, \overline{X}_1 - L_2$. This can directly be obtained from the old $\hat{V}_2$ and the matrix $(I - A_2\, Q\, A_2^T)^{-1}$ as $V = (I - A_2\, Q\, A_2^T)^{-1}\, \hat{V}_2$. Or of group 2 has only one observation $\bar{v}_{2i} = \hat{v}_{2i}/q_{2v_iv_i}$.

Data-snooping according to Baarda is based on a test, where the test statistic $w_i = v_i/s_O \sqrt{q_{v_iv_i}}$ is compared to a critical value that is found after fixing the risk and power for a simultaneous test of the hypothesis H_0 that there are no blunders in the observations, against the group of alternative hypotheses H_i that there is a blunder in observation number i . In this way, the blunders are localized to a limited number of observations.

The error propagation from group 1 to group 2 is given by

$$Cov\, \overline{V}_2 = s_O\, (I + A_2\, Q_1\, A_2^T) \ .$$

It can be shown that $I + A_2\, Q_1\, A_2^T = (I - A_2\, Q\, A_2^T)^{-1}$.

Like Baarda, most authors have based the test on the assumption that there is not more than one blunder in the data. Stefanovic (1978a) has suggested a strategy for dividing the observations in two groups, one with observations that are free from blunders and the other with observations contaminated by mistakes, blunders and outliers. He presupposes known critical values for the sum of squared residuals belonging to each group, and these critical values are based on a priori known variances for the observations that are dealt with.

4.2 A SUGGESTED TEST STATISTIC

In the following we will to try to find a method to group the observations without any a priori information on the variances. This is also often the most realistic starting point when dealing with close range photogrammetry, where the conditions determining the variances may vary considerably and thus are difficult to predict.

The new discrepancies $\bar{V}_2$ between the observations L_2 of group 2 and their prediction $A_2 \bar{X}_1$ based on the observations in group 1 also contain the blunders, and we now want to describe their magnitude. We form the test statistic

$$F^{max} = \frac{(\bar{V}_2 + \nabla)^T \, (I + A_2 \, Q_1 \, A_2^T)^{-1} \, (\bar{V}_2 + \nabla)}{\bar{V}_1^T \, (I - A_1 \, Q_1 \, A_1^T)^{-1} \, \bar{V}_1} \quad .$$

If the blunder $\nabla \neq 0$, the nominator increases with increasing ∇ . Thus, if $F^{max} > F^{max}_{crit}$, where F^{max}_{crit} is a critical value for the test, we regard the observations in group 2 as being contaminated by blunders. The F^{max} is a ration of two quadratic forms, yet in this case they are not independent because of two reasons: firstly, they originate from the same adjustment, in which case it is necessary that $A_1 \, Q \, A_2^T = 0$ for the two quadratic forms to be independent (see Graybill (1961), Theorem 4.21); secondly, we select the observations in such a way that we have the largest standardized residuals in the nominator, and the smallest in the denominator. If these two restrictions would not be at hand, we easily could find the critical values from the F-distribution (Snedecor´s variance ratio). As this is not possible, we try to derive the distribution of F^{max} under the conditions mentioned. An algebraic derivation is extremely difficult, and the possible alternative method seems to be computer simulation. However, also here the linear dependence through $A_1 \, Q \, A_2^T$ may be difficult to formulate in a general way. The grouping of the residuals, on the other hand, is rather straight forward.

4.3 THE Q_{VV} - MATRIX

The idempotent properties of Q_{VV} mean that

$$Q_{VV} \cdot Q_{VV} = Q_{VV}$$

$$q_{ii} = \sum_{j}^{n} q_{ij}^2 = \sum_{k}^{n} q_{ki}^2$$

$$0 \leq q_{ij} \leq 1 \quad .$$

Thus, q_{ii} is always the absolute largest element in its row or column. If $q_{ii} = 1$ all other elements are zero. If $q_{ii} = 0$ all other elements of row i are 0 which happens when no other equations in A check the observation l_i belonging to q_{ii} . A blunder in observation l_i influences the unknowns X directly. This, e.g., is the case for the image coordinates in the epipolar planes of a single stereopair. This becomes directly evident when the epipolar plane is parallel to one image coordinate axis. The partial correlation coefficients of the residuals V_2 can be calculated from the submatrix $(I - A_2\, Q\, A_2^T)$ by dividing rows and columns by the square roots of the diagonal elements

$$r_{ij} = q_{v_i v_j} \Big/ \sqrt{q_{v_i v_i} \cdot q_{v_j v_j}} \ .$$

If $r_{ij} = 1$, the observations i and j fully compensate each other. An error in one of the observations will contribute to the residuals in i and j with the same amount when we chose

$\sqrt{q_{v_i v_i}}$ and $\sqrt{q_{v_j v_j}}$, respectively, as units.

The localization of a blunder and its designation to one of the observations i or j is not possible. It can be either of the observations. The inverse of the submatrix $(I - A_2\, Q\, A_2^T)$ containing observations i and j does not exist as its determinant is zero. Thus, the sequential adjustment routine is not applicable for both observations i and j . A least one of them necessarily lies in A_1 , in order to maintain the rank $(A_1) = p$. The observations i and j are the only ones that check each other. Such design matrices A yielding these effects should be avoided in order to maintain the reliability of the system. To be able to localize blunders we must have "double" redundancy of all observations. If we want to be able to localize b blunders in an adjustment of n observations with r degrees of freedom $(r > b)$, this requirement can be transferred to the condition that all $\binom{n}{b}$ combinations of observations leading to group 2 must give a non-singular submatrix $(I - A_2\, Q\, A_2^T)$ of the total matrix Q_{vv} , so thatr the inverse of the submatrix exists. For direct measurements of the same unknown (mean of repeated settings) this means that the diagonal elements of Q_{vv} should be larger than 2/3 . Single redundancy yields $q_{vv} = 1/2$. Further, the correlation coefficients between observations must not be equal to ± 1, their magnitude should preferably be smaller than 0.5 .

4.4 DISTRIBUTION FUNCTION OF F^{max}

Methods for the exact derivation of distribution functions can be found in the more advanced textbooks on mathematical statistics, e.g. Kendall - Stuart (1969), chapt. 11. In the same book we also find the distribution of the $r-th$ order statistic, (formula 11.34). Further derivations for order statistics are found in Mann et al (1974), chapt. 3.5 and 3.8, and Rohatgi (1976) chapt´s 4.5 and 13.6.

(1) The parent distribution of x_i is $f(x) = \frac{1}{\sqrt{2\,\pi}}\; e^{-x^2/2}$

(2) The distribution function of $y_i = x_i^2$ is

$$f(y) = \frac{1}{\sqrt{2\,\pi\,y}}\; e^{-y/_2}\;, \qquad y \geq 0$$

$$0 \qquad\qquad , \qquad y < 0$$

and the sum z of two y_i , $z = y_1 + y_2$ (point error) is distributed

$$f(z) = \frac{1}{2}\; e^{-z/_2}\,, \qquad z \geq 0$$

$$0 \qquad , \qquad z < 0\;.$$

(3) The distribution function of the $r - th$ order statistic $Z_{(r)}$ is

$$F\big(Z_{(r)}\big) = \frac{n!}{(r-1)!\;\;(n-r)!}\;\left[\int_0^{Z_r} f(Z)\,dz\right]^{r-1}\;\left[1 - \int_0^{Z_r} f(Z)\,dz\right]^{n-r}\cdot f$$

(4) The joint distribution of the r smallest Z_i is

$$f_{min} = f\big(Z_{(1)} \ldots Z_{(r)}\big) = \frac{n!}{(n-r)!}\;\left[1 - \int_0^{Z} f(Z)\,dz\right]^{n-r}\;\prod_{j=1}^{r} f(Z_i)\;.$$

(5) The joint distribution of the b largest Z_i is

$$f_{max} = f\big(R_{(n-b)} \ldots Z_{(n)}\big) = \frac{r(n)}{r(n+1)}\;\left[\int_0^{Z_{(n-b)}} f(Z)\,dz\right]^{n-b}$$

$$\int_{y_{(n-b)}}^{y_{(n-b+1)}} f(Z)\,dz\;\cdot\;\int_{y_{(n-b+1)}}^{y_{(n-b+2)}} f(Z)\,dz\;\ldots\;\int_{Z_{(n-1)}}^{Z_{(n)}} f(Z)\,dz\;\cdot b\;\cdot f(Z)\;.$$

(6) The distribution of $F_{(b,r)}^{max} = u$ can then be found from

$$H(u) = F_1(uv)\ f_2(v)\, dv$$

where

$$F_1 = \int_0^{\sum_{n-b}^{n} y_{(i)}} f_{max}\ dy\ , \quad f_2 = f_{min}$$

$$u = \sum_{n-b}^{n} y_{(i)} \Big/ \sum_{1}^{r} y_i \qquad v = \sum_{1}^{r} y_{(i)} \ .$$

In (3) we have squares of integrals, and this leads to very complicated calculations. The calculations are, however, relatively simple if the parent frequency function is of the exponential type, viz.

$$f(x) = \frac{1}{a}\ e^{-\frac{x}{a}} \ .$$

Thus, if we define the squared standardized point residual for the observed image point as

$$z = \frac{v_x^2}{\sigma_O^2\ q_{v_x v_x}} + \frac{v_y^2}{\sigma_O^2\ q_{v_y v_y}} \ ,$$

this is a sum of two random variables being $N(0,1)$ distributed, and this sum is distributed according to χ^2 with two degrees of freedom,

$$f(z) = \frac{1}{2}\ e^{-z/2} \qquad z \geq 0$$

$$F(z) = 1 - e^{-z/2} \qquad z \geq 0 \ ,$$

which is an exponential distribution.

Thus, working with the squared standardized residual would open the possibility to derive the exact distribution functions for those test statistics that we need for testing hypotheses on blunders and outliers.

Then, there is the problem of "Studentization", which arises when we do not know σ^2 , but have to estimate the variance from observations. This has been discussed by Pope (1975). (See also Anscombe (1960)). In this case we use the same observations to determine both V and σ^2 (internal Studentization), then nominator and denominator were mutually dependent, and we arrive at a so-called τ (tau)-distribution for $\frac{V}{s_O \sqrt{q_{VV}}}$. From this we have to derive the distribution of our variate z.

In (6) we also have to consider the linear dependence of the two quadratic forms originating from the same adjustment. We know that, if nominator and denominator are based on two independent samples from the same distribution, the F- ration will be distributed as Snedocor´s F with b and $n-p-b$ degrees of freedom. But, first, the nominator contains the b largest values of the sample, and secondly, the independence of nominator and denominator requires that

$$(I - A_2\, Q\, A_2^T\,)\,(I - A\, Q\, A^T\,)\,(I - A_1\, Q\, A_1^T\,) = 0 \quad .$$

(Graybill (1961), Theorem 4.21).

The matrices $(I - A_2\, Q\, A_2^T\,)$ and $(I - A_1\, Q\, A_1^T\,)$ are two hyperdiagonal submatrices of $(I - A\, Q\, A^T\,)$,

$$\begin{bmatrix} I - A_1\, Q\, A_1^T & -A_1\, Q\, A_2^T \\ -A_2\, Q\, A_1^T & I - A_2\, Q\, A_2^T \end{bmatrix} .$$

The quadratic forms are calculated over all observations, and thus we have to multiply the three matrices

$$\begin{bmatrix} I - A_1\, Q\, A_1^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I - A_1\, Q\, A_1^T & -A_1\, Q\, A_2^T \\ -A_2\, Q\, A_1^T & I - A_2\, Q\, A_2^T \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & I - A_2\, Q\, A_2^T \end{bmatrix} ,$$

which obviously is equal to zero only if

$$(I - A_1\, Q\, A_1^T\,)\,(-A_1\, Q\, A_2^T\,)\,(I - A_2\, Q\, A_2^T\,) = 0 \quad .$$

This is the case if and only if $A_1\, Q\, A_2^T = 0$ which happens when the design matrix A is such that the observations of group 1 and 2 determine two different groups A and B of unknowns, i.e. we could have adjusted the two groups separately instead.

Since the exact distribution function seems to be very difficult to derive by algebraic means, there remains the possibility to calculate the distribution by means of computer simulation. Some hints can be found in Mann et al (1974), chapt. 7.2. The distribution function has to be simulated for different sizes of group 1 and group 2, and for each combination of sizes some 10 000 samples have to be simulated. The interesting percentage values (e.g. 5%, 1%, 0.1%) should be printed in tables. There is a particular problem to be solved, namely the simulation of the linear dependence between nominator and denominator through off-diagonal elements of the Q_{vv} matrix.

Further, simulations of the non-central distribution are then needed for the calculation of the power of the test. Thus, computer simulation is not a trivial task either.

4.5 BLUNDER LOCALIZATION

Let us now return to the unsolved problem of dividing the observations into two groups. Let us try the following procedure.

(0) Consider all observations in group 1. Set $b_O < r$ as the maximum number of blunders to be tested. Make an initial adjustment. Go to (2).

(1) Make a sequential adjustment of all observations in group 1.

(2) Compute the standardized residuals of group 1

$$w_i = v_i / s_O \sqrt{q_{v_i v_i}} \quad .$$

(3) Select the observation in group 1 with max $|w_i|$ and refer it to group 2. Check if there is another residual

v_j having $q_{v_i v_j} / \sqrt{q_{v_i v_i} \cdot q_{v_j v_j}} = \pm 1$, if so, they are 100 % correlated and blunder location is not possible. The blunder can be either of the observations. The inverse of the submatrix corresponding to these residuals in Q_{vv} does not exist. If this happens or if the number of blunders have reached b_O the procedure is <u>stopped</u>. Otherwise proceed to (4).

(4) Test the variance ratio between group 2 and 1. If the hypothesis $H_O : Var\ 1 = Var\ 2$ is accepted it might be so that there are more blunders that influence $Var\ 1$ so that the test is biased and therefore go to (3) and select the next absolutely largest $|w_i|$.

(5) If H_O is rejected the observations in group 2 are regarded to contain blunders. Estimate them :

$$V = (I - A_2\, Q\, A_2^T)^{-1}\, \hat{V}_2 = (I + A_2\, Q_1\, A_2^T)\, \hat{V}_2 \quad .$$

Try to explain the blunder and correct the observation accordingly. Go to (6). If it is not possible to explain and correct the blunder, refer the observation to group 3 which is outside the procedure. Go to (2).

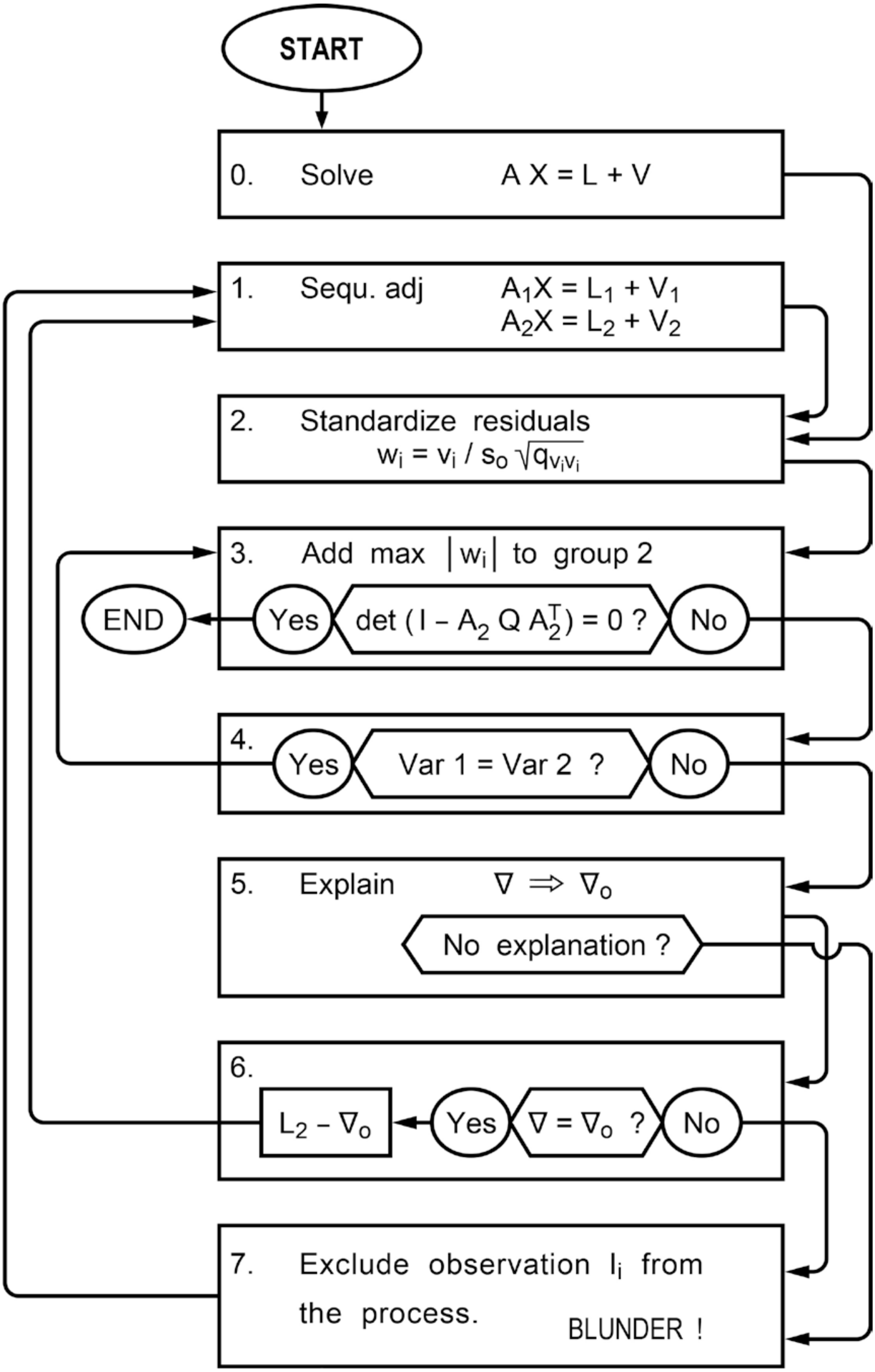
(6) Test the new discrepancies after correction of the blunders to see if the observation can now be recalled to group 1. If so go to (1).

(7) If the correction was not successful, refer the observation to group 3. Go to (1).

Another method for grouping the observations in blunder-free and mistakes is the following.

(1) Make an initial adjustment, all observations regarded as belonging to group 1, set $F_O = 0$, $j = 0$.

Graph of the procedure



(2) Compute the standardized residuals w_i of group 1, set $j = j + 1$.

(3) Refer the absolutely largest $|w_i|$ to group 2.

(4) Compute $\bar{X}_1,\ Q_1, \overline{V}_1, \overline{V}_2,\ \bar{s}_O^2$ by sequential adjustment.

(5) Calculate $F_j = \dfrac{\overline{V}_2^T\ (I - A_2\ Q\ A_2^T)^{-1}\ \overline{V}_2}{\overline{V}_1^T\ (I - A_1\ Q_1\ A_1^T)^{-1}\ \overline{V}_1}$

(6) If $F_i \geq F_{(j,r)p\%}^{max}$ then go to (2).

(7) If $F_j < F_{(j,r)p\%}^{max}$ then recall the last (the $j : th$) observation to group 1, and the observations in group 2 are now regarded as blunders.

(8) Stop blunder localization.

These methods are suggested by intuition and they have to be tested in theoretical and empirical studies. The effectiveness of the localization can preferably be studied by computer simulation.

5 STOCHASTIC PROPERTIES OF IMAGE COORDINATES

5.1 TRUE ERRORS AND RESIDUALS

In most adjustment it is assumed that the observations are normally distributed with mean zero and a common variance σ^2 , and that the observations are independent:

$$A\,X = L + e \;, \qquad E(e) = 0 \;,$$

$$E\left(e_i e_j^T\right) = \sigma^2\, I$$

The equations are given the same weight $P = I^{-1} = I$.
Not very seldom, the equations are given different weights P, based on the assumption that the observations are independent and have different variances σ_i^2 :

$$E\left(e_i e_j^T\right) = \sigma^2\, D \,, \qquad P = D^{-1} \quad (D:$$

It is very uncommon that the observations in the collinearity model (chap. 2) are given the stochastic properties saying that the observations are correlated (or even dependent) and have different variances

$$E\left(e_i e_j^T\right) = \sigma^2\, D^{1/2}\, C\, D^{1/2} = Cov \,,$$

$$P = \frac{1}{\sigma^2}\, Cov^{-1} = D^{1/2}\, C^{-1}\, D^{-1/2} \,.$$

C is a correlation matrix with $c_{ii} = 1$ in the main diagonal, and the off-diagonal elements are $-1 \le c_{ij} \le 1$.

$D^{1/2}$ is a diagonal matrix with elements equal to the square root of the elements in D.

$D^{-1/2}$ is the inverse of $D^{1/2}$.

The information in the weight matrix P is known a priori, i.e. before the adjustment; a priori weights of the observations are derived from the a priori variances and covariances. The individual true errors e of the observations L, however, are not known generally, and the adjustment is based on the equations

$$A\,X = L + V \,.$$

The properties of V sometimes have been studied in order to check the assumption on the stochastic properties of e.

Very often, there have been produced histograms showing the distribution of v_i . (see e.g. Hallert et al (1964), (1967)). The χ^2 -test has been applied to show the normal distribution. Vector diagrams have been plotted to show dependence or independence of the residuals. Sometimes a dependence has been described as function of the distance between pairs of points in the image, the model or the block; Torlegård (1967), Kupfer (1973), Ackermann (1978). Correlations between observations in different photos having the same location in the image have been studied by Ackermann (1978). Bachmann, Hawawini (1978) have studied the correlation between observations as a function of time. Hallert (1967), Morén (1967) and Brown (1969) have studied the variation of the magnitude of the residuals as a function of the radial distance from the principal point.

In almost all of these investigations the residuals are used directly as they appear. The residuals V have been used as observations on the true errors e. This is an approximation that can be justified sometimes, but far from always. We know that the residuals are functions of the observations and the design matrix A

$$V = (A^O - I)\,L \ .$$

If we happened to know the true error e, they will be mixed in the residuals through the relation

$$V_e = (A^O - I)\,e \ ,$$

but then the adjustment is an unnecessary operation, because the unknowns can directly be determined from the consistent system of equations $A\,X = L + e$ with $n = p$ equations, the only condition being rank $(A) = p$.
However, the dimensions of $(A^O - I)$ are (n,n) , and the rank $(A^O - I) = r = n - p$, so it is, in principle, impossible to find the true errors from the residuals.
But what we can do is to derive the covariance matrix for the residuals, and use the standardized residuals w_i for our further studies.

$$Cov\,V = \sigma^2\,(I - A^O) = \sigma^2\,Q_{vv}$$

$$w_i = v_i/\sigma\,\sqrt{q_{v_i v_i}} \ .$$

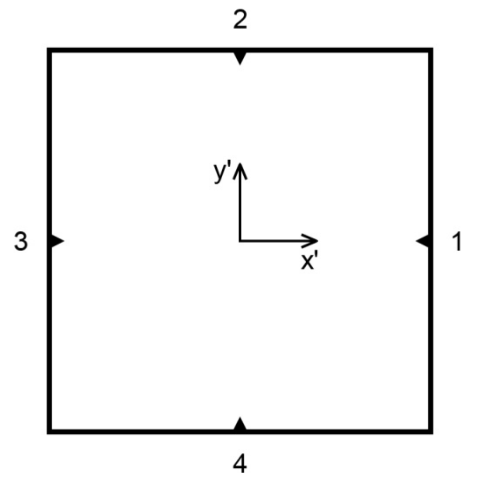
This, anyhow, is better than using the residuals directly, especially if the $q_{v_i v_i}$ vary considerably. If all residuals have the same precision (i.e. the same $q_{v_i v_i}$), the standardization is not very important for tests on distribution and correlations.

The variation of the $q_{v_i v_i}$ terms depends very much on the homogeneity and symmetry of the geometry behind the observations. As an example, we can use the affine coordinate transformation on fiducial marks.

Ex 5.1

The four mid-side marks common in Zeiss cameras yield

$$I - A^O = \frac{1}{4}\begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$



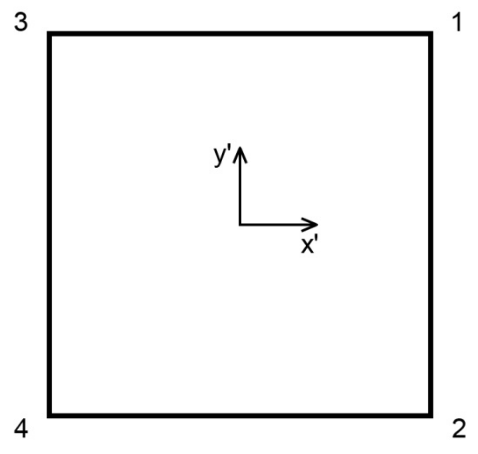
where the $q_{v_i v_i}$ all are equal. The relative is

1/4 = (4 - 3) / 4 = 0.25 .

Ex 5.2

The four corner marks (Jena UMK, Wild RC-series) yield

$$I - A^O = \frac{1}{4}\begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$



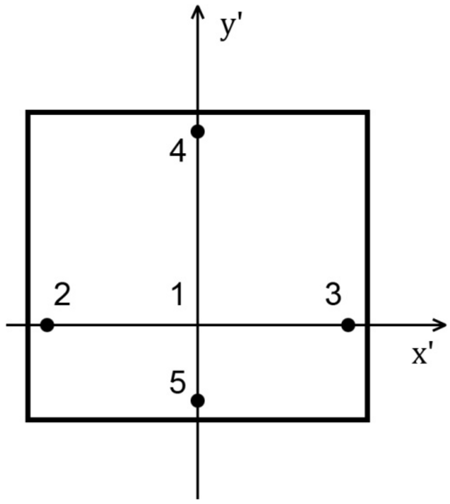
where all the $q_{v_i v_i}$ are equal, too. The relative redundancy is again

1/4 = (4 - 3) / 4 = 0.25 .

Ex 5.3

By contrast we have the unsymmetric geometry of the fiducials in the Wild P31 and P32 cameras, where we get

$$I - A^O = \frac{1}{1306} \begin{bmatrix} +1032 & -274 & -274 & -154 & -330 \\ & +379 & +379 & -154 & -330 \\ & & +379 & -154 & -330 \\ & & & +147 & +315 \\ & & & & +675 \\ \text{symmetric} & & & & \end{bmatrix}$$



The relative redundancy is 2612 / 1306 / 5 = (5 - 3) / 5 = 0.4 .

The minimum local relative redundancy is 147 / 1306 = 0.11 ,

the maximum is 1032 / 1306 = 0.79 .

In this case it is very important to standardize the residuals before comparison or blunder localization, as the ratio between the largest and the smallest $q_{v_i v_i}$ is as large as 1032 / 147 = 7.0 .

The main diagonal is [0.79 0.29 0.29 0.11 0.50] .

The correlations between the residuals are given by the matrix

$$\begin{bmatrix} 1 & -.44 & -.44 & -.40 & -.40 \\ & 1 & 1 & -.65 & -.65 \\ & & 1 & -.65 & -.65 \\ & & & 1 & 1 \\ & & & & 1 \\ \text{symmetric} & & & & \end{bmatrix}$$

and from this we can see that points *2* and *3* compensate each other totally, as do points *4* and *5*. Thus, we only have reliable control for blunders in point *1*, which has no total correlation with any other point.

Ex 5.4

For the Hasselblad camera with the 25 - point-reseau we get

$(I - A^O) \cdot 50 =$

$$
= \left[\begin{array}{ccccc ccccc ccccc ccccc ccccc}
40 & -8 & -6 & -4 & -2 & -8 & -6 & -4 & -2 & 0 & -6 & -4 & -2 & 0 & 2 & -4 & -2 & 0 & 2 & 4 & -2 & 0 & 2 & 4 & 6 \\
 & 43 & -6 & -5 & -4 & -6 & -5 & -4 & -3 & -2 & -4 & -3 & -2 & -1 & 0 & -2 & -1 & 0 & 1 & 2 & 0 & 1 & 2 & 3 & \\
 & & 44 & -6 & -6 & -4 & -4 & -4 & -4 & -4 & -2 & -2 & -2 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & & \\
 & & & 43 & -8 & -2 & -3 & -4 & -5 & -6 & 0 & -1 & -2 & -3 & -4 & 2 & 1 & 0 & -1 & -2 & 4 & 3 & & & \\
 & & & & 40 & 0 & -2 & -4 & -6 & -8 & 2 & 0 & -2 & -4 & -6 & 4 & 2 & 0 & -2 & -4 & 6 & & & & \\
 & & & & & 43 & -5 & -3 & -1 & -1 & -6 & -4 & -2 & 0 & 2 & -5 & -3 & -1 & 1 & 3 & & & & & \\
 & & & & & & 46 & -3 & -2 & -1 & -4 & -3 & -2 & -1 & 0 & -3 & -2 & 1 & 0 & & & & & & \\
 & & & & & & & 47 & -3 & -3 & -2 & -2 & -2 & -2 & -2 & -1 & -1 & -1 & & & & & & & \\
 & & & & & & & & 46 & -5 & 0 & -1 & -2 & -3 & -4 & 1 & 0 & & & & & & & & \\
 & & & & & & & & & 43 & 2 & 0 & -2 & -4 & -6 & 3 & & & & & & & & & \\
 & & & & & & & & & & 44 & -4 & -2 & -0 & 2 & & & & & & & & & & \\
 & & & & & & & & & & & 47 & -2 & -1 & & & & & & & & & & & \\
 & & & & & & & & & & & & 48 & & & & & & & & & & & & \\
 & & \mathit{symmetric} & & & & & & & & & & \mathit{symmetric} & & & & & & & & & & \mathit{symmetric} & &
\end{array} \right]
$$

y
$\uparrow$

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

$\rightarrow x$

Here, the variation between the largest and the smallest $q_{v_i v_i}$ is 48 / 40 = 1.2 .
The redundancy is 44/50 = (25 - 3) / 25 = 0.88 .
Minimum local redundancy is 0.80 (corner point) and maximum local redundancy is 0.96 (center point).

As the residuals in this case have approximately the same standard error it does not seem to be very important to standardize the residuals before comparison, analysis, or error detection. In the case of block adjustment of aerial triangulation the standard errors of the residuals are rather similar in magnitude, too. This has been discussed by Grün (1979b), who suggests a simplified method for data-snooping. By taking into consideration the variation of the standard errors of the residuals he can avoid the laborious calculation of the matrix $I - A^O$.

For close range applications of photogrammetry, the geometry, as a rule, is such that the standard errors of the residuals vary considerably, and thus the standardization of the residuals is necessary. Furthermore, various applications have very different geometries, and both these circumstances indicate the importance of deriving matrix $I - A^O$ in order to have an effective method when analyzing residuals.

Now, if we study the correlations between residuals after an adjustment, we still cannot be sure that we know what we are doing. The correlations between the residuals theoretically can have three different causes:

(1) A priori correlation between the true errors through the matrix C.

(2) Blunders (∇) that have not been detected, localized and deleted from the observations in the adjustment cause spurious correlation. Such blunders are spread around in the residuals by the operator

$$V_b = (A^O - I)\,\nabla \ ,$$

and if the off-diagonal elements are $\neq 0$, which is the case when we have redundant observations in the adjustment to determine unknowns X through indirect observations L , then we get spurious correlation.

(3) Systematic errors of the mathematical model (lacking fidelity) such that $E\ (e) \neq 0$ (or expressed as $A\,X \neq E\ (L)$, which is the same). The effect of the systematic errors in the observations are also spread around in the same way as are blunders.

5.2 EMPIRICAL STUDIES

Now, what type of experiment could possibly answer the question on the stochastic nature of the true errors e_i in the observations l_i . Well, the causes for correlation quoted under (2) and (3) above have to be eliminated. Let us assume that we have been successful in the elimination of blunders using the methods described in chap. 4.5, and by a careful procedure, occupational skill, and other means available to avoid blunders. Then, we have to eliminate the effect of the systematic errors of the model, which traditionally can be done by repetition of the experiment under the same exterior conditions, so that the variation in the observations are not influenced by changes of the systematic errors. This, of course is impossible for aerial photography and block triangulations, but for close range photogrammetry in laboratory environment we can repeat our experiment after a short time interval, and under very similar exterior environmental conditions. Let us assume one experiment including photography, photographic processing, measurement and adjustment with n observations l_i from which we obtain the n residuals v_i , where $i = 1 \dots n$. The outcome of the experiment can be regarded as one observation point in n-dimensional observation space. This experiment is now repeated m times, where m is large number, say 100 or 200. This gives us m sets of observations l_i and m sets of residuals v_i^k where $k = 1 \dots m$; or m points in n - dimensional observation space. Thus, we have one sample of size m on the variables λ_i , the coordinate axes in the n - dimensional observation space, and from this sample we are able to estimate the means

$$\bar{\mu}_i = \sum_{k=1}^{m} l_i^k \qquad \text{and the variances} \qquad \sigma_i^2 = \sum_{k=1}^{m} \left(l_i^k - \bar{\mu}_i\right)^2 / (m-1) \ .$$

In this way we obtain n means $\bar{\mu}_i$ and variances $\bar{\sigma}_i^2$. Then, we can test the hypothesis that they all have the same mean $\mu_0 = 0$ and the same variance σ_0^2 . Furthermore, we can also estimate the correlation between the observations by using the estimator r_{ij} , giving the estimate $\bar{r}_{ij}$:

$$\bar{r}_{ij} = \sum_{k=1}^{m} \left(l_i^k - \bar{\mu}_i\right)\left(l_j^k - \bar{\mu}_j\right) / \sqrt{\bar{\sigma}_i^2 \cdot \bar{\sigma}_j^2} \ .$$

Then the significance of $\bar{r}_{ij}$ for all pairs i, j of points can be tested. It may even be possible to suggest a function for $\bar{r}_{ij}$ and $\bar{\sigma}_i^2$ depending on the position in the photo, for $\bar{r}_{ij}$ depending on distance between points i, j in the photo, time interval between the measurements of l_i and l_j , or something else. To find such a function it is necessary to use a rather large number n^k of observations l_i in each experiment k. Similar experiments have been suggested by Pope (1975) in order to study the stochastic properties of observations.

What has been done up to now, are studies with just one experiment k with a large number of observations l_i , thus $m = 1$ and $n = 100 \dots 10{,}000$. If conclusions from these studies can be made concerning the properties of true errors, this is very much dependent on whether or not systematic errors of the mathematical model have been corrected, and whether or not all blunders have been eliminated.

Brown (1969) has shown in such a study that there are, for a certain aerial camera, functions for σ^2 with the position in the photo as argument. He derived two functions, one for the radial and another for the tangential error of image coordinates with the radius from the principal point as independent variable. Studies by Hallert (1967) and Morén (1967) suggest just one function for the standard error of the image coordinates as a function of radius. Brown´s approach seems to be very reasonable as the physical reality and other knowledge also suggest different functions in radial and tangential directions. The resolution and MTF of lenses are separated in this way, and the random variation of the flatness of the image surface carried by the emulsion has an influence only in the radial direction. But, on the other hand, there are also physical matters that suggest the variation to be described in cartesian coordinate variables rather than polar, and such a matter is the random variation of image position depending on the photographic processing. Brown (1969) has shown the fact that independent polar variances causes the cartesian image coordinates to be correlated.

The error propagation is obtained from

$$\begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix} = \frac{1}{r^2} \begin{bmatrix} y & -x \\ x & y \end{bmatrix} \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_t^2 \end{bmatrix} \begin{bmatrix} y & x \\ -x & y \end{bmatrix}$$

which gives the correlation coefficient

$$\rho_{xy} = (\sigma_r^2 - \sigma_t^2) / (\sigma_r^2 + \sigma_t^2) \ .$$

This correlation increases if there are other physical causes that make the x and y coordinates on the same point to be correlated. The covariances of the contributing components will be added. It should be noted that these studies emphasize the variances of image coordinates of single points. Brown has not given any information on the covariances between different points. It is of course possible that the radial and tangential error show correlation between different points and that such correlation can be dependent on the distance between the points and their radii from the principal point.

Ackermann (1978) has shown that the correlation between different points can be considerably reduced after introduction of additional parameters in the mathematical model. This is an evidence for the presence of systematic error in the model. Thus, his study is rather a motivation for the use of additional parameters in the adjustment, or a more sophisticated coordinate refinement procedure prior to the adjustment, than it is a determination of the stochastic behavior of the true image errors.

5.3 CARTESIAN OR SPHERICAL AND CYLINDRICAL COORDINATES

Analytical photogrammetry is almost entirely based on cartesian coordinate systems. Such a system is also the most convenient one as comparators and plotters provide rectangular coordinates. The photos are flat, and as such easily measured with an x-y- digitizer. But there are also instruments to measure polar coordinates available for photogrammetry, e.g. the DBA "bug" comparator, and the now obsolete picture theodolites. The latter instrument really measures the angles in the bundle of rays that is reconstructed from the photo, and thus these instruments are based on a design that is closer to the nature of photogrammetric recordings, namely the bundles of rays and the angles between the rays in the bundle.

It is of course no principal difficulty to convert all the cartesian-based formulae for analytical photogrammetry into spherical or cylindrical form. The question is what we would gain by doing so. Following the evaluation procedure in photogrammetry we find:

–– The mensuration if images is made in comparators that gives x, y. Conversion to angles is necessary. No gain.

–– In both cases, relative orientation is based on series expansions, and the same amount of computation is needed. No gain.

–– Absolute orientation seems to be more easily performed on cartesian model coordinates since the absolute coordinates most often are given in such a system. Here it seems to be a drawback to work with other than cartesian model coordinates.

–– Block triangulation based on bundle angles can have the advantage that available adjustment techniques for geodetic space net works can offer some advantages, especially for simultaneous adjustment of geodetic and photogrammetric observations. This might be an advantage, but modern aerial triangulation methods and adjustments are now very efficient and it must be doubted if the angular approach can offer a better method in general.

–– Theory of errors might be the best field for a gain to be expected, e.g. for the understanding and correction of systematic and random errors that preferably are divided into radial and tangential components (see, e.g., Brown (1969)). For studies of error propagation in a stereomodel it seems to be easier to describe the orientation of error ellipsoids in cylindrical rather than rectangular coordinates because of the fact that point intersections lie in epipolar planes. This makes the error components more independent of each other.

6 THE MULTI-CONCEPT IN CLOSE RANGE PHOTOGRAMMETRY

6.1 HIGH ACCURACY PROCEDURES

The analytical methods provide new possibilities in order to increase the accuracy in photogrammetry. Accuracy here comprises precision, fidelity and reliability. The price for an increased accuracy has to be paid with better instruments, careful project planning, more observations (i.e. larger measuring times), and rigorous adjustment. The multi-photogrammetry concept is more related to planning, observation and adjustment than to instruments. The multi-concept is the idea of check and control in all phases, repetition and replication, and redundancy in the determination of every unknown variable. The means to reach the utmost accuracy in close range photogrammetry (and it may be applied to photogrammetry in general in most cases) can be summarized as follows

– High quality pictures for the measurements. The better one can identify object features to be determined, the better will be the accuracy. The importance of photographic quality is not always recognized by photogrammetrists who mostly emphasize geometric problems. Not only high geometric quality of image coordinates obtained through sophisticated camera calibration, but also photographic experience is important (emulsions, filters, illumination, exposure, processing etc.).

– High precision comparators for image coordinate readings. Calibration of the measuring instrument reveals systematic errors, that have to be corrected for.

– Multi-readings of all points on each plate in order to detect blunders in numbering, identification and recording of coordinates. Averaging reduces the variance due to reading the images.

– Multi-fiducials in order to give redundancy in the transformation of coordinates from the comparator system to the camera system, and to control the departures of film deformation from the commonly applied conformal or affine fiducial transformation assumptions.

– Multi-targets to signalize object points in the photograph. The regularity in the pattern of such targets reduces the risk of measuring other images than the signalized points. Local irregularities of the imaging system (atmosphere, lens, emulsion, comparator, operator, etc.) will be averaged out.

– Multi-frames on each station can be arranged in several ways, e.g.

  1. repeated photography with the same orientation

  2. repeated photography with rotation around the camera axis

  3. repeated photography with camera axis directions changed, yielding partly overlapping bundles from the same station. Imperfections of the imaging system will be averaged out and the precision of the resulting bundle will be increased.

– Multi-stations for point intersections in object space are necessary to provide a tool for blunder detection in the final adjustment. The stations should be positioned as to give optimal intersection angles at the object points, and each point should be determined from at least four stations in order to give redundancy also after deletion of one ray due to a blunder in the observations.

– Multi-control means that the transformation into the object space coordinate system has to be based on redundant information not only in the form of given coordinates of control points, but also in the form of geometric object space conditions (lines, planes, angles, distances, parallelism, etc.) and direct observations on the orientation elements.

- Multi-purpose program with rigorous adjustment is based best on the bundle approach. For close range applications the number of bundles and the number of object points are often limited such that least squares adjustment can contain all unknowns in a direct solution, also providing the error propagation to unknowns and functions of observations and unknowns. The adjustment process must take into consideration all sorts of observations, and therefore, a priori information on the weights of the observations have to be known and considered. Additional parameters are necessary to design a mathematical model that has high fidelity to the real geometry of the images. Rigorous adjustment based on a general approach does not impose any restrictions on the choice of cameras, location of photo stations, camera orientations, type of object space control, etc. The term "multi-purpose program" also covers the possibility to use the program both for the calibration of cameras and for the determination of object geometry.

6.2 MULTI - READINGS

Hottier (1976) has made a very comprehensive study of the gain in accuracy that can be obtained by

a) repeating the settings,

b) using multi-targets, and

c) talking the average of more picture frames on each station.

He found that the gain in accuracy by repetition of the settings is at most 30% (decrease of RMS in check points) for up to 7 settings on single targets in single frames. He also states that the effect of repeated settings is smaller than the effect of multi-targets and multi-frames, and for an optimal gain of accuracy in relation to the measuring work, he recommends one setting, two targets, two frames, or one setting, one target, three frames.

With computer-assisted data capture there is the possibility to control the setting precision interactively. The requirements on the precision can be formulated in different ways such as maximum range, maximum standard deviation of the mean, rejection of out-liers after adjustment, etc., see e.g. Dorrer (1977).

Hottier worked under "controlled" conditions with a test field, and it can be assumed that blunders were immediately detected, localized and eliminated. For practical close range photogrammetric projects, however, the procedure must be such that blunders are detected, localized and eliminated without a priori knowledge on the object.

Repeated readings of image points is a well established method to detect and localize blunders. Mistakes in points identification and numbering is often detected immediately. The order of reading the points when repeating is commonly reverse to the order of the first set. If the maximum range criterion is applied, the points with large differences will be measured a third time to see which of the readings on the points is being likely to be an out-lier. If this third reading also seems to be an out-lier the readings are repeated until any two of them fall within the predetermined tolerances. These two are then regarded as representative for the point. Intuitively we can say that, if the tolerance is too wide in relation to the setting precision, out-liers will contaminate the result; and if it is too narrow, the measuring work will be enlarged without yielding any better results, because we are deleting good observations until it happens that any two of them are close enough to each other.

If the tolerance is extremely small, it has the effect that a lot of good observations are thrown away before two observations fall within the tolerance interval. Thus, it is rather important to have tolerance limits that are reasonable. The following example will demonstrate the effect of different tolerance ranges. Suppose the true error of our observations is normally distributed according to $N\left(0, \sigma/\sqrt{2}\right)$. The differences between repeated observations are then distributed according to $N\,(0, \sigma)$. Using the acceptance interval $\pm 3\,\sigma$ for the differences leads to rejecting 0.3% of the good observations. The acceptance interval of $\pm 1\,\sigma$ leads to a rejection rate of 31.7%. The power of these is shown in Fig. 6.1 where we find that a power of 80% leads to acceptance of erroneous observations if the blunder is smaller than $2.9\,\sigma$ and $2.3\,\sigma$, respectively. The standard deviation of the mean will be smaller than $\sigma/2$, because we have truncated the distribution of the differences at $\pm 3\,\sigma$ and $\pm 1\,\sigma$, respectively.

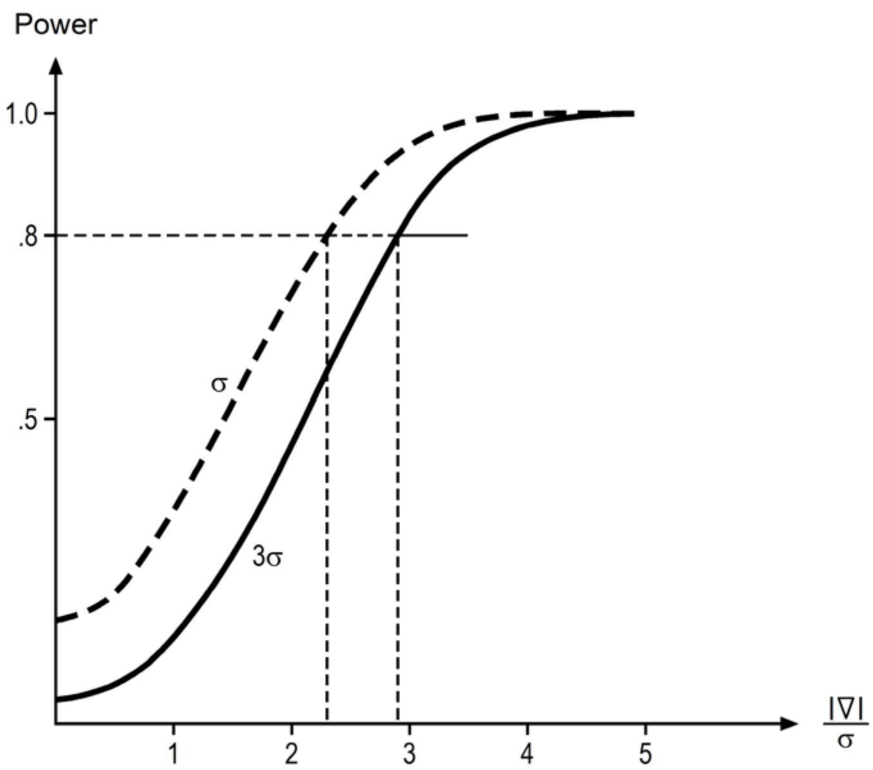


Fig. 6.1

Power functions for testing outliers

Ex 6.1

Twenty points have been observed twice each. The true errors of the first setting are unknown to the observer, but he has differences between the double readings. Points having differences exceeding the tolerances are repeated until two observations are accepted. The values in the following table 6.1 represent a simulated sample with single observations $N(0, 0.71)$ and the differences $N(0, 1)$. The sample means and the sample variances are given at the end of the table 6.1.

Table 6.1:

True errors of simulated sample and successive differences applying the $1\,\sigma$ - rule.

Point no	True error 1st setting		Differences applying the $1\,\sigma$ - rule								
			2nd	1st	3rd	1st	4th	1st	Further		Rem
	e_x	e_y	d_x	d_y	d_x	d_y	d_x	d_y	d_x	d_y	
1	-0.952	0.884	0.464	0.060							
2	0.136	-0.141	0.137	-2.526	-0.555	-0.513					
3	-0.850	-0.204	2.455	-0.531	0.046	-0.525					
4	0.279	1.280	-0.323	-0.194							
5	-0.739	0.974	-0.068	0.543							
6	0.596	0.413	0.290	-1.558	0.321	0.595					
7	0.666	0.860	-0.288	0.187							
8	0.739	0.518	1.298	-1.190	2.945	0.881	-1.005	-0.044	0.712	0.203	6th
9	0.022	0.284	0.241	0.022							
10	0.546	0.160	-0.957	0.525							
11	0.445	0.265	1.486	1.022	1.974	-0.934	0.007	-0.162			
12	-0.380	-1.372	-0.354	-0.472							
13	0.553	0.175	-0.634	1.279	-0.258	1.579			0.376	0.300	3rd
14	0.042	-0.347	0.697	3.521	0.412	0.161					
15	0.353	0.470	0.926	0.571							
16	-0.298	-0.095	1.375	-1.851	0.439	-1.885			-0.926	-0.034	3rd
17	1.206	-0.103	0.785	0.194							
18	0.823	-0.352	-0.963	1.192	-0.035	0.371					
19	0.625	0.323	-0.853	-0.501							
20	-0.211	0.752	-1.865	-0.278	1.179	-1.501	0.769	-0.136			
sample mean	0.085	0.237	0.192	0.001							
st. dev.	0.782	0.595	1.030	1.298							
rms	0.766	0.623	1.023	1.265							

Applying the $3\,\sigma$ - rule on d_x and d_y, observation no. 14 would be repeated, giving 3rd - 1st $d_x = 1.394$ and $d_y = 0.906$, and thus
3rd - 2nd $d_x = 0.697$ and $d_y = 2.615$.

We obviously delete setting no. 2 on point 14.

Now, we estimate the precision of the setting from the differences that we have after applying the 1σ - rule and the 3σ - rule. But as we here have simulated data we know the true errors and then we can calculate the mean, standard deviation and root mean square value of the true errors after taking the average of the accepted two observations per point. The decrease of the root mean square (RMS) error after averaging is the measure of the gain in accuracy that we obtain by only applying the acceptance tolerance for repeated settings.

<u>Table 6.2:</u>

Precision estimated from differences between double readings.

The precision refers to the difference which is $N(0,1)$

	$\infty\sigma$		3σ		1σ	
	x	y	x	y	x	y
Mean	*0.192*	*0.001*	*0.227*	*-0.130*	*-0.024*	*0.074*
St. dev.	*1.030*	*1.298*	*1.060*	*1.028*	*0.560*	*0.370*
Rms	*1.023*	*1.265*	*1.058*	*1.0-0*	*0.546*	*0.369*

<u>Table 6.3:</u>

Accuracy from true errors of the average of two settings.

The accuracy refers to the average which is $N(0,0.5)$

	$\infty\sigma$		3σ		1σ	
	x	y	x	y	x	y
Mean	*0.276*	*0.264*	*0.326*	*0.239*	*0.235*	*0.295*
St. dev.	*0.701*	*0.786*	*0.744*	*0.755*	*0.637*	*0.801*
Rms	*0.737*	*0.811*	*0.795*	*0.773*	*0.664*	*0.834*

Thus, in this sample the observable improvement in x goes from 1.023 to 1.058, resp. 0.546, but it corresponds to the real improvement from 0.737 to 0.795, resp. 0.664, and in y the observable improvement goes from 1.265 to 1.010, resp. 0.369, but it corresponds to the real improvement from 0.811 to 0.773, resp. 0.834. The conclusion must be that smaller tolerances do not yield any better results in relation to the true values of the observed quantities. A tolerance of 5σ may be as good as one of 1σ but without unnecessary work by avoiding rejection of good observations. In order to decide in which of the two repeated observations the blunder occurs, a third, or more, reading is as a rule done, until two readings are close enough. But the difficulty can also be overcome by using the original observations in an adjustment where observations in other points help to localize the blunder.

<u>Ex 6.2</u>

Four fiducial marks are measured twice. In one of the fiducial marks the double readings differ considerably.

If the averages of the double measurements are treated as observations in an affine coordinate transformation we obtain the following matrices

$$A = \begin{bmatrix} 1 & a & b \\ 1 & a & -b \\ 1 & -a & b \\ 1 & -a & -b \end{bmatrix}$$

$$N = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4a^2 & 0 \\ 0 & 0 & 4b^2 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4a^2 & 0 \\ 0 & 0 & 1/4b^2 \end{bmatrix}$$

$$Q_{vv} = I - A^O = \frac{1}{4}\begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

From the last matrix it is obvious that an error in any of the observations will be distributed with equal magnitudes in all residuals. The trace is 1, the relative redundancy $1/4 = 0.25$, all diagonal elements 0.25. The algebraic correlation between the residuals is $+1$ or -1 in all combinations and thus it is not possible to localize the blunder. If we eliminate one of the observations, the matrix $I - A^O$ will have only zero elements; there is no redundancy left.

The inverse $(I - A_2\, Q\, A^T)^{-1}$ does not exist for any two observations.

But, on the other hand, if we introduce the original eight observations in the adjustment of the affine transformation we get the following:

$$A = \begin{bmatrix} 1 & a & b \\ 1 & a & b \\ 1 & a & -b \\ 1 & a & -b \\ 1 & -a & b \\ 1 & -a & b \\ 1 & -a & -b \\ 1 & -a & -b \end{bmatrix}$$

$$N = 8 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & b^2 \end{bmatrix}$$

$$Q = \frac{1}{8} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & {}^{1}\!/_{a^2} & 0 \\ 0 & 0 & {}^{1}\!/_{b^2} \end{bmatrix}$$

$$I - A^O = \frac{1}{8} \cdot \begin{bmatrix} 5 & -3 & -1 & -1 & -1 & -1 & 1 & 1 \\ & 5 & -1 & -1 & -1 & -1 & 1 & 1 \\ & & 5 & -3 & 3 & 3 & -1 & -1 \\ & & & 5 & 1 & 1 & -1 & -1 \\ & & & & 5 & -3 & -1 & -1 \\ & & & & & 5 & -1 & -1 \\ & & & & & & 5 & -3 \\ & & & & & & & 5 \\ symmetric & & & & & & & \end{bmatrix}$$

The trace is 5 , relative redundancy $5/8 = 0.625$, the diagonal elements are all 0.625 , the correlation between the double readings are $r_d = 0.6$, while the correlation to the other readings is $r_v = \pm\, 0.2$. Now, it is easy to localize the blunders (after standardization by dividing the residuals by s_O and the square root of the corresponding diagonal element of $I - A^O$), as the major part of the blunder (0.625) stay at the position of its origin, and only a smaller part $(0.375$, resp. $0.125)$ is transferred to the other residuals. Let us say that we have a blunder in the last observation, and after deletion we get

$$Q_1 = \frac{1}{40} \cdot \begin{bmatrix} 6 & -1/a & -1/b \\ & 6/a^2 & 1/ab \\ & & 6/b^2 \\ symmetric & & \end{bmatrix}$$

$$I - A^O = \frac{1}{40} \cdot \begin{bmatrix} 24 & -16 & -4 & -4 & -4 & -4 & 8 \\ & 24 & -4 & -4 & -4 & -4 & 8 \\ & & 24 & -16 & 4 & 4 & -8 \\ & & & 24 & 4 & 4 & -8 \\ & & & & 24 & -16 & -8 \\ & & & & & 24 & -8 \\ & & & & & & 16 \\ symmetric & & & & & & \end{bmatrix}$$

The trace is $160/40 = 4$, the relative redundancy is $4/7 = 0.571$, the residuals of the double observations have $q_{vv} = 24/40 = 0.60$, while the single one has $q_{vv} = 16/40 = 0.4$, all correlation coefficients are absolutely smaller than 1 (0.67 , 0.41 and 0.17) , which makes it possible to localize a second blunder in the observations, even when it occurs in the last point.

The conclusion that can be drawn from this example is the following: In order to avoid rejection of good observations and unnecessary repetition of observations the original measurements rather than their averages should be used in the next adjustment step of the procedure. The localization of blunders is then done easier because the design matrix A is changed. This, however, requires in practice that the computation can be done as soon as there are enough repetitions of observations. It is not necessary to measure all points twice before adjustment; the testing can start as soon as a solution is possible, and then succeeding observations are added to the adjustment by sequential techniques referred to in chapter 4.1. Analytical plotters and comparators with on-line computational facilities offer the hardware needed for such procedures.

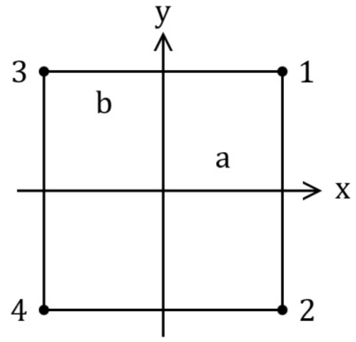
6.3 MULTI - FIDUCIALS

In most cameras there are four fiducial marks. Using a conformal fiducial transformation of coordinates from the comparator system to the camera system we have four unknowns, eight observations and four redundant observations giving a relative redundancy of 0.5 , which is just on the limit of acceptability when thinking of blunders. But what sort of correlations exist between the residuals after the adjustment? Can we localize and eliminate blunders, and still have redundancy? The answer can be found in the matrix $I - A^O$.

Ex 6.3

For the case of a conformal transformation on four corner fiducials we have

$$A = \begin{bmatrix} 1 & 0 & a & -b \\ 0 & 1 & b & a \\ 1 & 0 & a & b \\ 0 & 1 & -b & a \\ 1 & 0 & -a & -b \\ 0 & 1 & b & -a \\ 1 & 0 & -a & b \\ 0 & 1 & -b & -a \end{bmatrix}$$



$$
I - A^O = \frac{1}{4\,r^2} \cdot \begin{bmatrix} 2r^2 & 0 & -2a^2 & +2ab & -2b^2 & -2ab & 0 & 0 \\ & 2r^2 & -2ab & -2a^2 & -2ab & -2b^2 & 0 & 0 \\ & & 2r^2 & 0 & 0 & 0 & -2b^2 & +2ab \\ & & & 2r^2 & 0 & 0 & -2ab & -2b^2 \\ & & & & 2r^2 & 0 & -2a^2 & -2ab \\ & & & & & 2r^2 & +2ab & -2a^2 \\ & & & & & & 2r^2 & 0 \\ symmetric & & & & & & & 2r^2 \end{bmatrix}
$$

where $r^2 = a^2 + b^2$.

The trace is 4 , relative redundancy $4/8$, all $q_{vv} = 0.5$ and the correlation coefficients 0 , a^2/r^2 , b^2/r^2 or $\pm\, ab/r^2$ which, for
$a = b = r/\sqrt{2}$ give $\rho = 0$ or $\rho = \pm\, 0.5$.
Blunder localization is feasible, but does there remain redundancy after elimination ?
Using the formula given in chapter 4.1 we find

$$
(I - A_2\, Q\, A_2^T)^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}
$$

$$
A_1\, Q\, A_2^T = \frac{1}{4r^2} \cdot \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -2b^2 & -2ab \\ -2ab & -2b^2 \\ -2a^2 & -2ab \\ 2ab & -2a^2 \end{bmatrix}
$$

$$A_1\, Q\, A_2^T\, (I - A_2\, Q\, A_2^T)^{-1}\, A_2\, Q\, A_1^T =$$

$$= \frac{8}{16\, r^4} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & r^2 b^2 & 0 & 0 & abr^2 \\ & & & r^2 b^2 & abr^2 & 0 \\ & & & & r^2 a^2 & 0 \\ symmetric & & & & & r^2 a^2 \end{bmatrix}$$

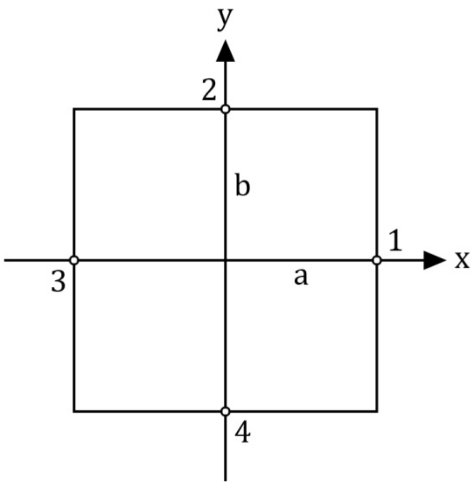
$$Q_1 = \frac{1}{2r^2} \cdot \begin{bmatrix} r^2 & 0 & -a^2 & ab & -b^2 & -ab \\ & r^2 & -ab & a^2 & ab & -b^2 \\ & & a^2 & 0 & 0 & ab \\ & & & a^2 & -ab & 0 \\ & & & & b^2 & 0 \\ symmetric & & & & & b^2 \end{bmatrix}$$

The trace is 2, relative redundancy $2/6$, the q_{vv} are 0.5, $a^2/2r^2$ or $b^2/2r^2$ (for $a = b = r/\sqrt{2}$ the latter are 0.25), the correlation coefficients are 0, ± 1, $\pm\, a/r$ or $\pm\, b/r$ (for $a = b = r/\sqrt{2}$ the latter two are $\pm\, 0.71$), and we still have redundancy, but not enough for blunder localization in points 2 and 3, because here the correlation is ± 1 between v_{x2} and v_{y3}, resp. v_{y2} and v_{x3}.

Ex 6.4

For the case of conformal transformation on four midside fiducials we obtain analogously

$$A = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & 0 & a \\ 1 & 0 & 0 & -b \\ 0 & 1 & b & 0 \\ 1 & 0 & -a & 0 \\ 0 & 1 & 0 & -a \\ 1 & 0 & 0 & b \\ 0 & 1 & -b & 0 \end{bmatrix}$$



$I - A^O =$

$$= \begin{bmatrix} r^2+2b^2 & 0 & -r^2 & -2ab-r^2 & +2a^2 & 0 & r^2 & -2ab \\ & r^2+2b & 2ab & -r^2 & 0 & -r^2+2a^2 & -2ab & 0 \\ & & r^2+2a^2 & 0 & -r^2 & -2ab & -r^2+2b^2 & 0 \\ & & & r^2+2a^2 & 2ab & -r^2 & 0 & -r^2+2b^2 \\ & & & & r^2+2b^2 & 0 & -r^2 & -2ab \\ & & & & & r^2+2b^2 & 2ab & -r^2 \\ & & & & & & r^2+2a^2 & 0 \\ symmetric & & & & & & & r^2+2a^2 \end{bmatrix} \cdot \frac{1}{4r^2}$$

The trace is $=4$, relative redundancy 0.5 , $q_{v_x v_x} = \frac{1}{4} + \frac{a^2}{2r^2}$ and $q_{v_y v_y} = \frac{1}{4} + \frac{b^2}{2r^2}$, both equal 1/2 for $a = b = r/\sqrt{2}$.

The correlation coefficients are 0 or $\pm 1/2$ for $a{=}b$, and blunder localization is possible in any point; but after elimination of one point further localization of blunders is not possible, as in the previous example.

It is very common to use an affine transformation for the fiducial marks, and here two more parameters are introduced as unknowns, the design matrix A is correspondingly changed and the redundancy is decreased to such an extent that blunder localization is impossible on four fiducial marks with just one observation. Here, more fiducial marks are necessary, as e.g. in the Wild P31 and P32 with 5 or 17, or in the Hasselblad MK 70 with 25 fiducial marks. The Q_{vv} -matrices for affine transformations have been presented and discussed in chapter 5.1 and 6.2.

In reseau cameras and in some other types, the fiducial marks have the shape of a cross. In this case one can preferably take four (or two times four) measurements on the arms of the crosses and compute the intersection rather than repeat the settings in the centre, in order to increase the precision. An example of a measuring instruction for cross-shaped fiducial is given in appendix 1.

6.4 MULTI - TARGETS

The points to be measured are very often targeted before photography. In aerial triangulation pre-targeting of all points (known, unknown and tie) has proved to be an efficient method to increase the accuracy, thereby avoiding errors in artificial point transfer and bad point definition when using natural transfer points. Multi-targeting has been used in aerial (e.g. Kupfer (1973) and Hvidegaard (1976)) as well as in close range photogrammetry (Hottier (1976)). Multi-targets can be designed in several ways depending on the relevant parameters in the specification. Such parameters are, e.g., the size and shape of the measuring mark in the comparator, the average image scale, the variation of the image scale within and between the pictures, the angle between the target plane and the image plane (convergency angle), method of attaching the targets to the points on the object (gluing, self-adhesive, painting, nailing) number of targets per signalized point, environmental conditions, resolution viewing magnification, etc. Some examples of multi-targets are shown in figs. 6.2 - 6.5.

The sevenfold target has the advantage that one can choose the targets such that the average of the observations coincides with the position of the central one for any number of targets from 1 to 7. For 2, 3 or 4 targets there are several combinations and the operator can choose the one that has the best pictorial quality. The cross-like target is suited for oblique photography with varying image scale. The five-circle target is good for varying image scale, but it is limited to 2 or 3 targets per point. Hottier (1976) found that the maximum gain of accuracy, by increasing the number of targets per object point, is about 40%, which is attained for 4 - 5 targets. He recommends as a practical optimum one setting, one target, and three frames, or one setting, two targets and two frames. If only one frame is available it seems to be a good compromise to have two settings on two targets or one setting on three or four targets as judged from Hottier´s results.

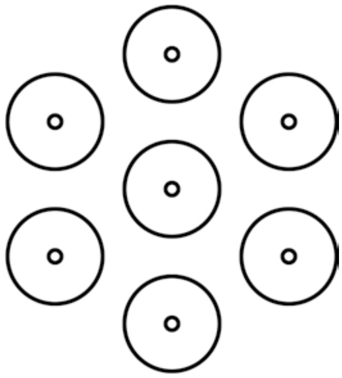


Fig. 6.2
The sevenfold target,
suited for any number
of targets from 1 to 7

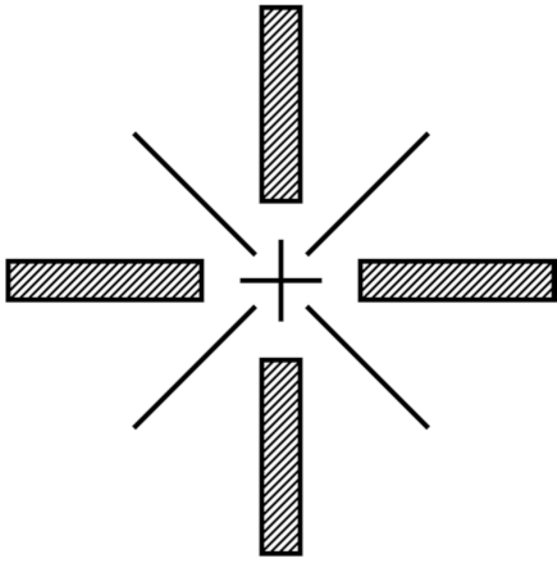


Fig. 6.3
The star-like target,
suited for oblique photos
in varying scales

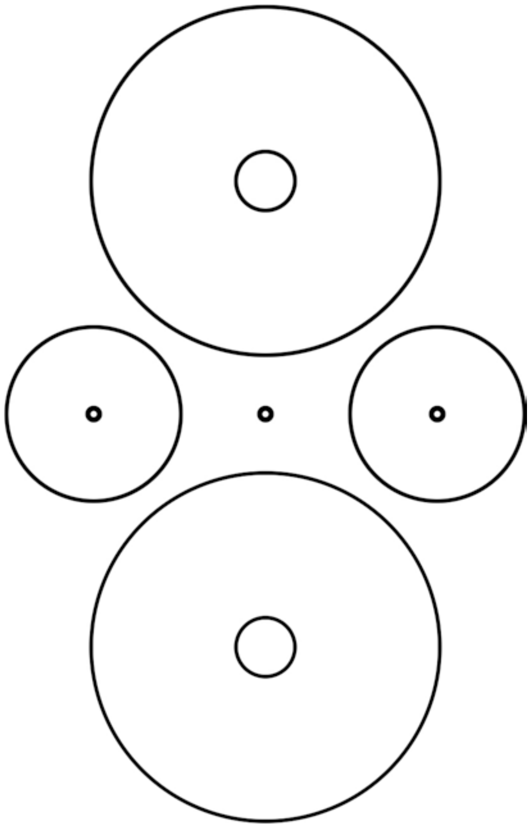


Fig. 6.4
The five-circle target,
suited for varying image
scales

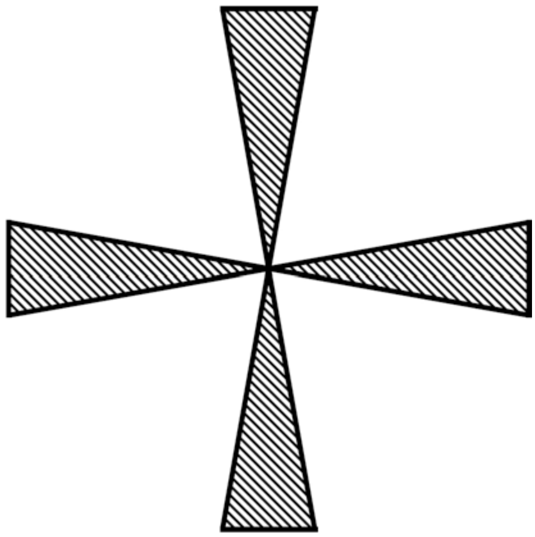


Fig. 6.5
The cross-like target,
suited for oblique photos
in varying scales

6.5 MULTI - FRAMES

A most efficient means to increase the precision of the reconstructed bundle of rays is to take more pictures on each station and use their average in the following steps. Hottier (1976) has studied the effect of this technique for a close range normal case stereomodel, and he found that the maximum gain in precision was 40% for up to 5 frames per station (measured with one setting on single targets). For practical purposes recommends one setting per target, two (three) targets per point and two (three) frames per station, which yields 40% (50%) gain of precision.

For the duplicated frames Hottier used the same exterior orientation. The effect of the random errors is reduced through averaging, but systematic and constant errors remain the same. Such systematic errors of our mathematical model of the imaging geometry can, e.g., be caused by imperfections of the lens, principal point error, bending of plates, film flattering and film deformation. These errors can be described in terms of radial and tangential components in the image plane. The tangential component, and that part of the radial component that is not rotationally symmetric can be determined (or averaged out) by rotating the camera around the camera axis between the pictures. The rotationally symmetric part of the radial component is not determinable. Here a quadratic image format is ideal, since coverage will be identical in the four cardinal positions. Taking the average of the bundles of rays from frames with κ - rotations 0, $\pi/2$, $\pi/2$, $3\pi/2$ corresponds in some way to the station adjustment in geodetic triangulation.

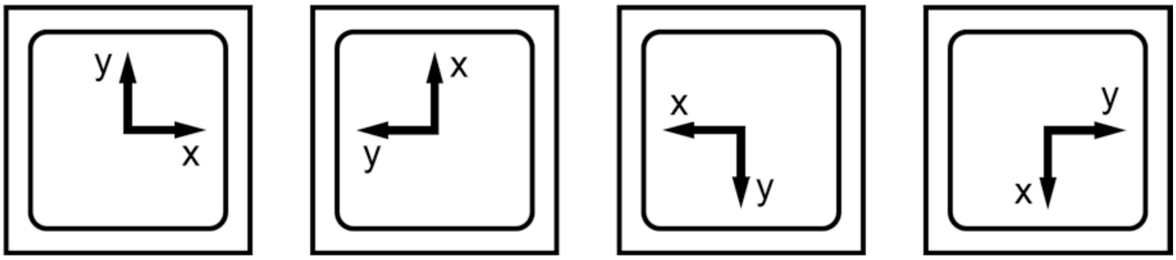


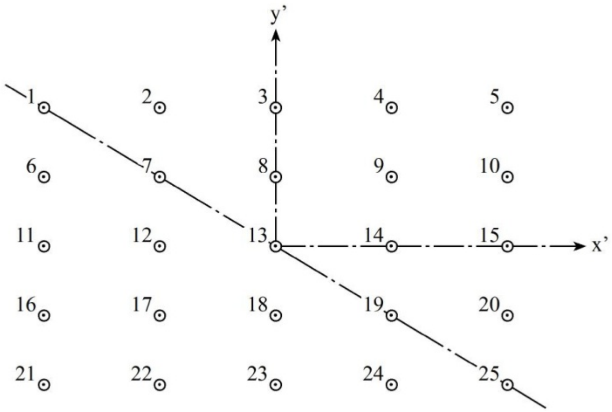
Fig. 6.6
Multi-frames on the same station with the camera axes in the same direction but with different κ - rotations make it possible to eliminate tangential image errors and the irregular part of the radial image errors.

Each frame is measured and the observations are transformed to the camera system. After rotating the camera systems to one and the same reference position, the sets of image coordinates can be compared. Small discrepancies will always occur due to setting precision and error propagation from the fiducial transformation, systematic and random errors (in the camera-film performance, changes of the exterior orientation other than just the κ - rotation, and blunders. Therefore, for comparison of different sets of coordinates a differential perspective transformation is appropriate. If tangential distortion is considerable the observation equations have to be amended with terms covering this effect.

Table No 6.4

	1		2		3		4		5		7		8		9		10		13		14		15		19		20		25	
	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y
1 x	.76	.09	-.14	.09	-.06	.09	-.00	.08	.03	.08	-.09	-.01	-.04	-.00	-.00	.00	.00	.01	-.02	-.05	-.01	-.03	-.02	-.01	-.01	-.01	-.05	.02	-.08	.09
y		.76	-.02	-.17	-.09	-.10	-.11	-.04	-.08	.03	-.01	-.09	-.07	-.05	-.08	-.00	-.04	.04	-.05	-.02	-.04	.00	.01	.02	-.01	-.01	.05	-.01	.09	.02
2 x			.88	.01	-.09	.04	-.05	.08	-.00	.11	-.09	-.03	-.07	-.00	-.04	.02	-.00	.05	-.05	-.02	-.04	-.00	-.01	.02	-.03	-.01	-.01	.01	-.01	.02
y				.86	-.04	-.10	-.08	-.07	-.08	.04	.01	-.07	-.03	-.05	-.05	-.03	-.05	-.00	-.02	-.02	-.03	-.01	-.02	.00	-.01	-.01	.02	-.01	.05	-.05
3 x					.90	0.	-.09	.04	-.06	.09	-.07	-.03	-.08	0.	-.07	-.03	-.04	.06	-.06	0.	-.05	.02	-.02	.04	-.03	.01	.00	.02	.02	-.01
y						.90	-.04	-.10	-.09	-.10	.03	-.05	0.	-.05	-.03	-.05	-.06	-.05	0.	-.02	-.02	-.02	-.04	-.02	-.01	-.01	-.02	-.01	.01	-.02
4 x							.88	-.01	-.14	.02	-.04	-.02	-.07	.00	-.09	.03	-.09	-.03	-.05	.02	-.06	.04	-.05	.06	-.03	-.04	-.00	.05	.04	.01
y								.86	-.09	-.17	.05	-.03	.03	-.05	-.01	-.07	-.10	-.09	.02	-.02	-.01	.01	-.06	-.04	-.01	-.01	-.05	-.00	-.04	.00
5 x									.76	-.09	-.00	-.00	-.04	.00	-.09	.04	-.17	.02	-.02	.05	-.05	.07	-.10	.09	.00	.08	-.04	.11	.03	.08
y										.76	.08	-.00	.07	-.05	.01	-.07	-.09	-.09	.05	-.03	-.00	-.04	-.09	-.05	-.00	-.00	-.08	-.00	-.08	.03
7 x											-.93	-.01	-.06	.00	-.04	.02	-.01	.04	-.05	-.01	-.04	-.00	-.01	.01	-.02	-.01	-.01	-.01	-.01	-.01
y												.93	-.02	-.06	-.02	-.04	-.01	-.03	-.01	-.05	-.01	-.04	-.01	-.03	-.01	-.03	-.01	-.03	-.01	-.01
8 x													.93	0.	-.06	.02	-.03	.04	-.06	0.	-.05	.01	-.02	.02	-.04	-.00	-.01	-.00	.00	-.03
y														.94	-.00	-.06	-.01	-.06	0.	-.05	-.01	-.05	-.02	-.05	-.02	-.04	-.03	-.03	-.04	-.01
9 x															.93	.01	-.07	.03	-.05	.01	-.06	.02	-.05	.03	-.04	.02	-.03	.03	-.00	.00
y																.93	-.01	-.09	.01	-.05	-.00	-.06	-.03	-.07	-.02	-.04	-.05	-.04	-.08	-.00
10 x																	-.86	-.01	-.02	.02	-.05	.03	-.10	.04	-.03	.05	-.07	.08	-.04	.08
y																		.88	.02	-.05	.00	-.07	-.04	-.09	-.03	-.04	-.08	-.05	-.11	-.00
13 x																			.94	0.	-.05	0.	-.02	0.	-.05	-.01	-.02	-.02	-.02	-.05
y																				.94	0.	-.06	0.	-.06	-.01	-.05	-.02	.05	-.05	-.02
14 x																					.94	0.	-.05	0.	-.06	.00	-.05	-.00	-.04	-.00
y																						.93	0.	-.08	-.02	-.06	-.03	-.07	-.07	-.04
15 x																							.90	0.	-.05	.03	-.10	.04	-.10	.09
y																								.90	-.03	-.07	-.04	-.08	-.09	-.06
19 x																									-.93	-.01	-.07	-.03	-.09	-.04
y																										.93	.01	-.09	-.04	.09
20 x																											.86	.01	-.17	-.09
y																												.88	-.02	-.14
25 x																													.76	.09
y																														.76

The matrix is symmetric.
Further the point symmetry of the grid yields a grid symmetry, e.g. $q_{x9,x2} = q_{x17,y6}$



After the perspective transformation has been done we can study the residuals in order to detect, localize and eliminate occurring blunders, and for that purpose we need the Q_{vv} - matrix, which as usual is $(I - A^O)$.

Assume that we have pictures taken with a camera having a format slightly larger than 2/3 x 2/3 of the principle distance c (e.g. Hasselblad MK 70 with Biogon 60) and that the perspective transformation is based on 25 well distributed image points located in grid intersections with the x' and y' coordinates equal to 0 , $\pm\ c/6$, $\pm\ c/3$. The Q_{vv} - matrix has 50 x 50 elements, but due to the point symmetry there is a good symmetry in the matrix as well and it is not necessary to calculate more than 1/8 of the elements. The diagonal elements are shown in Table 6.4. The maximum absolute value of the correlation coefficients between the residuals is 0.210, and it occur between v_x of a corner point and v_x of its closet neighbour having the same x' - coordinate (and similarly for v_y and y'). It can be noted that the maximum value of $q_{v_i v_i}$ does not belong to the central point. The corner points take the smallest values as expected. The trace of the matrix is 44, which is the number of degrees of freedom in the adjustment. The mean value of the diagonal elements is 0.880 , and the range thus 0.762-0.944 . In this case it is easy to localize and eliminate blunders because the correlation coefficients are small, the inverse exist, and the diagonal elements are well over 0.5 .

In order to determine or eliminate the effect of the rotationally symmetric part of the radial component, the frames have to be taken with the camera axis pointing in different directions. This has been applied by Borchers (1965) in structural deformation measurements. He determined the bending of the plates from the radial components and the structural deformation from the tangential components of the image displacement vector. Taking nine frames as indicated in Fig. 6.7 we can determine both tangential and radial error components. For very high accuracy requirements, e.g. camera calibration, the "nine-frame-pattern" can be repeated for the four cardinal κ - rotations giving in total 36 frames of the same bundle of rays. This certainly is too much to be practical, and the optimum combination might be found by an investigation devoted to that particular problem. A combination of 4 κ - rotated central pictures, two ω - rotated and two φ - rotated would possibly yield an optimum. To facilitate a convenient photography one has to design and build a camera support with gimbals axes going through the exterior projection centre, so that the camera position is unchanged when the directions are varied. For ultra precise work the eccentricities might have to be considered in the computations. The systematic components can be formulated as additional parameters in the mathematical model as it is done in aerial block adjustment.

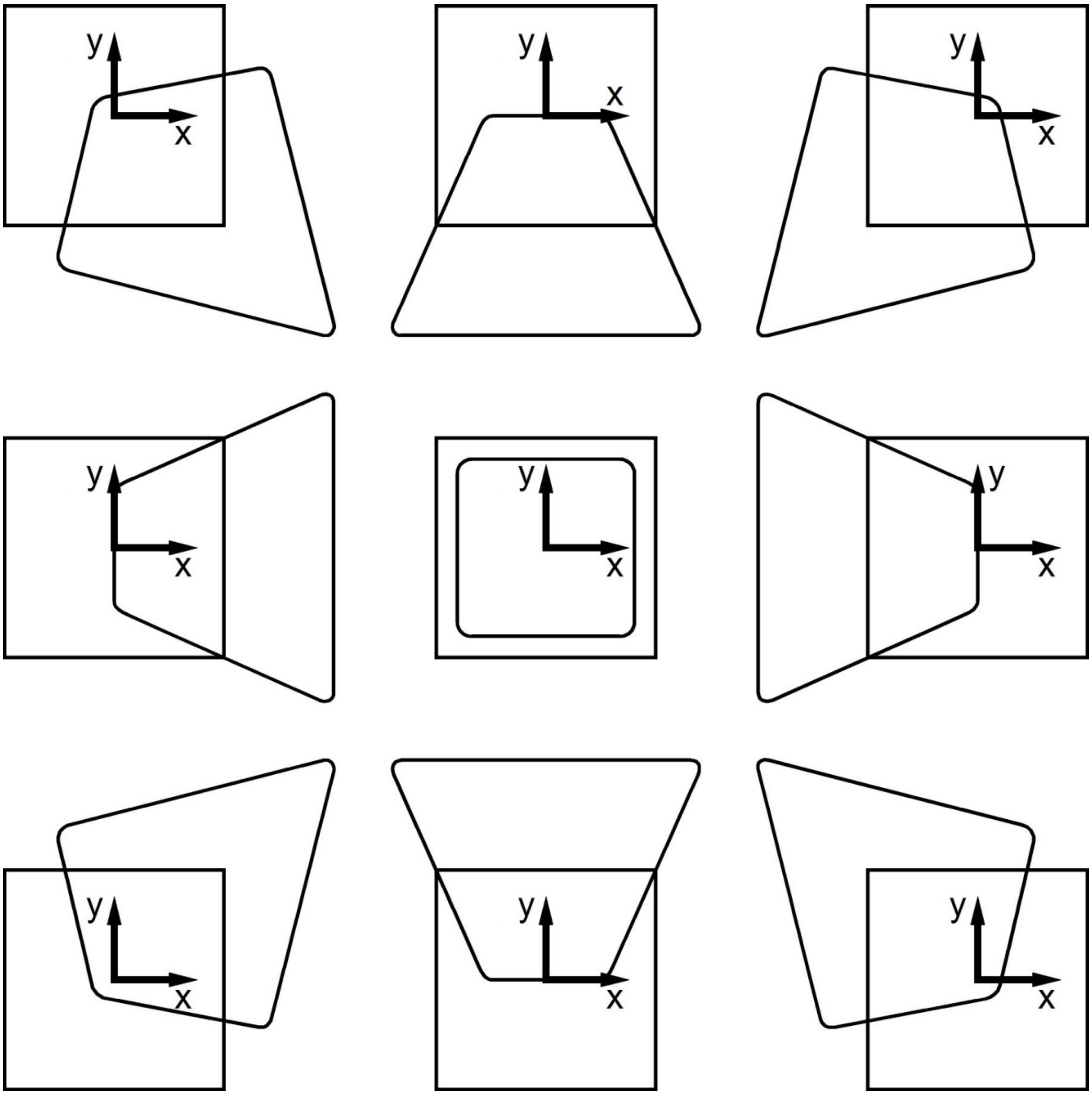


Fig. 6.7
Nine-frame experimental design with different directions of the camera axes that makes it possible to eliminate radial image errors.

But there is a difference between aerial and close range photogrammetry as to the exterior orientation of the frames. In aerial block adjustment there are a large number of frames all having different exterior orientation and typically 60 % by 20 % or 60 % by 60 % overlap. In close range we can have groups of frames with the same exterior orientation except for some or all three directions of the camera axis. This can certainly provide advantages compared to the aerial case. The possibilities have not yet been studied for close range applications. There are some studies on the effect of multiple strips with different flight directions in aerial photography for the determination of additional parameters, e.g. Thomas (1977).

6.6 MULTI - STATIONS

The concept of multi-stations is well known in photogrammetry through bundle block adjustment and a series of analytical solutions in close range photogrammetric measurement problems. The ordinary case in aerial block adjustment with vertical photography over flat terrain from the same altitude with one and the same 15 x 23 x 23 camera in parallel strips standardized overlap and control points according to well known thumb-rule patterns has to be generalized in close range photogrammetry with respect to camera orientation (interior and exterior), object geometry, overlaps, control points, object geometry conditions, targeting, etc. It is typical for high precision close range photogrammetry to try planning the photography in such a way that the ray intersections in the reconstructed model will be as good as possible, preferably under right angles. This leads to convergent photography with nearly 100 % overlap, and to get a homogeneous precision one often uses long focal lengths if the space around the object is large enough, in order to give approximately the same image scale over the entire object. Under these circumstances it is not very convenient to measure the pictures in pairs in a stereo comparator and treat the data in units of stereomodels. The appropriate approach obviously is the bundle adjustment, and thus the pictures preferably are measured one by one, and the image coordinates are the observations to be adjusted. Most of the theoretical and practical investigations into the accuracy of analytical close range photogrammetry have been related to the precision of the object coordinates. Some investigations treat the formulation of the proper mathematical model for the imaging geometry. Very few reports are given on studies of the reliability of the experimental design in close range photogrammetry. One recent report is given by Grün (1978b). He applies the "data-snooping" according to Baarda, defines measures for precision and reliability of the experimental design, and he simulates examples with close range bundle block having 2 - 4 frames. The precision indicator PI defined by Grün is the mean value of the root of the diagonal elements of the inverse of the normal equation matrix Q_{xx} , thus $PI = tr\left(\sqrt{Q_{xx}}\right) / p$. The reliability indicator RI is defined as the mean of the $q_{v_i v_i}$ values, thus $RI = tr\ (Q_{vv})/r$. (I think that a more proper definition of the precision indicator would be $PI = tr\ (Q_{xx})/p$. Grün´s results show that good precision is obtained from good intersection angles of the rays in the object points, i.e. good base-to distance ratios. This holds also for pairs of pictures. Good reliability on the other hand is obtained if four pictures, three of which do not lie in the same straight line, form a block to determine the object points with rays of which not more than two are in the same plane. This holds also for small

base-to-distance ratios. For such a photoquadruple the reliability indicator is $RI = 0.62$. In the case of a stereo-pair the $q_{v_i v_i}$ - values will be zero for the coordinate that is orientated in the epipolar plane direction (zero variance problem). This, of course, is well known; it is just the vertical coordinate that is controlled in the stereo-pair case (vertical parallax). For three pictures with camera axes in the sample plane the situation is slightly better, but not good enough, as the reliability is much better for y' than x'. Grün suggests that the three camera stations be chosen such that the intersection rays in the object points form three planes which do not coincide. In this way a more uniform reliability is expected in the different coordinate directions. The importance of multiple-ray intersection for the reliability has been further studied by Grün (1979b), but for aerial blocks. It appears from that study that, at least four intersecting rays are necessary for a good reliability, and that the rays must form at least four different planes intersecting but not coinciding in the object point. The $q_{v_i v_i}$ - values are for these points on the average 0.50 . (Grün uses in his paper $\sqrt{q_{vv}}$, thus obtaining 0.70 as an average). The results presented by Grün (1978b) and (1979b) indicate that the following thumb-rule might be valid:
In order to obtain good reliability in photogrammetric block adjustment, it should be such a geometry that the object points are determined from at least four rays that pairwise, form epipolar planes that do not coincide; and to obtain good precision these epipolar planes should intersect at right angles. The geometry is demonstrated in Fig. 6.8. The validity of this thumb-rule should be investigated and studied in theory and practice.

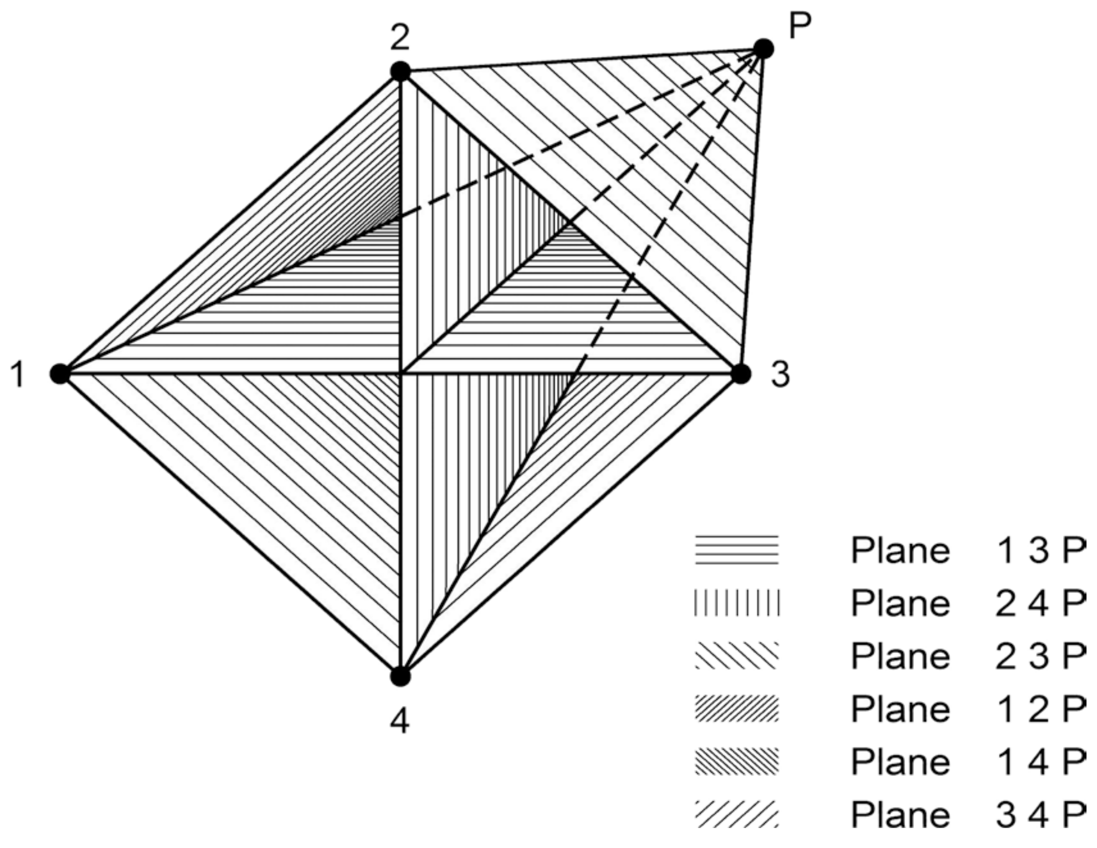


Fig. 6.8 Epipolar planes form four pictures 1, 2, 3 and 4 of one point P.

6.7 MULTI - CONTROL

In most cases control is given as coordinates of full (XYZ) , planimetric (XY) or height (Z) control points. In some aerial block adjustment programs there is a possibility to use the information that all points on the shoreline of a lake have the same but unknown height. In some cases auxiliary data from statoscope, APR and the like can be used in the adjustment. Similar information originating from both object and photography can efficiently be used in close range photogrammetry.

In analogue methods approximate values are used in the orientation process, e.g. in aerial photogrammetry, where vertical photography is assumed, the operator can use this information to orient the model even if the control is insufficient, say, the height control is mainly located in a line. Then the operator uses the approximation to the vertical to orient the model in height so that at least planimetry can be plotted and perhaps also topographic form-lines if they can serve the purpose for which the map is intended. If there is insufficient control in the numerical orientation, this leads to a singular matrix of observation equation coefficients. A general method to overcome this problem is to add a number of equations which correspond to direct observations on the unknowns in the form of approximate values. These equations are given weights that take into account the uncertainty of the approximations. Direct observations on the exterior orientation parameters can also be introduced in the system of observation equations. This is often the case for levelled metric cameras and photo-theodolites. For stereometric cameras with fixed base the distance between the two projection centres can be introduced as a ficticious observation with weight corresponding to the precision of the base. In the same way ficticious observations on the interior orientation elements can be introduced, and corrections will be computed to these values in the adjustment. See e.g. Wrobel-Kruck (1978) and Hell (1979).

In this way a singular system of equations will always be avoided. As approximations have to be calculated anyhow, or given in input, for the linearization, they all are available, and in the described way the same algorithm can be used for all cases, also such where control is missing. The effect will also show up in the errors of the unknowns and functions of them, in such a way that the elements in the normal equation inverse $Q = (A^T\,P\,A)^{-1}$ will be large for the weakly controlled unknowns. For uncontrolled unknowns x_i the $Q_{x_i x_i}$ corresponds to the weight of the approximations $P_{i\,i} = s_0^2/s_i^2$ given a priori.

Utilization of vertical and horizontal planes, straight lines, angles, etc., for the relative and absolute orientation in analogue methods of close range photogrammetry has also to be formulated in a mathematical way and must be included in the adjustment (Kager-Kraus (1976), and Hell (1979)). Doing so, there will sometimes be introduced new unknown parameters in the adjustment, e.g. position and attitude of an arbitrary plane in the object space. These condition equations are given weights that correspond to the precision with which the condition can describe the corresponding matter in reality. It happens very often that these conditions are non-linear, and thus they have to be linearized before they are entered into the equation system.

6.8 MULTI - PURPOSE PROGRAM

To summarize, we now have a system of equations that has the following structure:

$A\,Y + B\,X \quad + G\,Z$	$= L_1\,, P_b$	–	image coordinates
$C\,X$	$= L_2\,, P_g$	–	given object control coordinates
$D\,Y$	$= L_3\,, P_y$	–	conditions and observations on exterior orientation elements
$I\,Y$	$= L_4\,, P_{ny}$	–	approximations on the exterior orientation elements
$I\,X$	$= L_5\,, P_{nx}$	–	d : o on object points
$I\,U$	$= L_5\,, P_{nu}$	–	d : o on geometric condition parameters
$I\,Z$	$= L_6\,, P_{nz}$	–	d : o on additional parameters
$E\,X + F\,U$	$= L_7\,, P_e$	–	geometric conditions in object space

where the

$A \;\; B \;\; C \;\; D \;\; E \;\; F \;\; G$ are coefficient matrices

Y vector of corrections to approximations of exterior orientation elements $(X_O \;\; Y_O \;\; Z_O \;\; \omega \;\; \varphi \;\; \kappa)_i$
six elements for each photo i

X vector of corrections to approximations of object space coordinates $(X \;\; Y \;\; Z)_\kappa$
3 unknowns for each point κ

Z vector of corrections to approximations of interior orientation parameters and additional parameters
$\left(x'_O \;\; y'_O \;\; c \;\; a_3 \;\; a_5 \;\; \ldots \;\; p_1 \;\; p_2 \;\; \ldots \;\; a_{ij} \;\; b_{ij}\right)_l$
a set of selected parameters for each camera l used

U vector of corrections to approximations of unknown parameters in geometric object space conditions

I unit matrix

$L_1 \ldots\; L_7$ right hand side term in the linearized observation equation (often called discrepancies)

$P_b \ldots\; P_e$ weight matrices for the observation equations.

The equations of type 2 and 5 are of the same kind, the only difference being that the weights of the given control points are much higher than those of the unknown points for which we have just approximations. The same holds for equations of type 3 and 4.

The model is very general, it can be used for calibration purposes, for point determination, for resection, etc. For camera calibration the interior orientation elements Z are unknowns. If the calibration is based on testfield then X is known with high accuracy.

$$
\begin{aligned}
A\,Y + B\,X + G\,Z \;&=\; L_1\,, P_b \\
C\,X \;&=\; L_2\,, P_g \\
I\,Y \;&=\; L_4\,, P_{ny} \\
+\,I\,Z \;&=\; L_6\,, P_{nz}
\end{aligned}
$$

If the given points are regarded as free from errors we have $P_g = \infty$.
Then, $X = C^{-1}\,L_2$. Introducing this in the first set of equations we get

$$
\begin{aligned}
A\,Y + G\,Z &= L_1 - B\,C^{-1}\,L_2\,, P_b \\
I\,Y &= L_4 \qquad\qquad, P_{ny} \\
I\,Z &= L_6 \qquad\qquad, P_{nz}
\end{aligned}
$$

As here in this case all coordinates of all points are known we have $C = I$.
L_1 are measured image coordinates and $B\,C^{-1}\,L_2 = B\,L_2$ are given object coordinates $B\,L_2$ using the approximate values on Y and Z. Often P_{ny} and P_{nz} are put to zero, so that we only have equations of the first type. The number of photos can be limited when test-field are used but they must have a point distribution in space such that $(A\,;G)$ is non-singular.

If self-calibration is used, all or most of the elements in X are unknowns but more photos of the same object with varying exterior orientation Y are introduced in the adjustment compared to the test-field case. A large number of equations for geometric object space conditions can be introduced, which is the case for e.g. the plumbline method.

Testfield calibrations and self-calibrations give a set of constants for the interior orientation Z and the intention is to use them in future projects as given values in functions that correct the image coordinates for the systematic errors before adjustment. The type 1 equation then is written in the form

$$A\,Y + B\,X = L_1 - \,G\,Z \;.$$

It is important to note here that the set of constants in Z should be the same as in the calibration. The calibration should also be designed such that the estimate of Z is not correlated so that of Y after the adjustment of the calibration observations. If a subject Z_1 of the calibration parameters Z will be used in later projects, this subset Z_1 should be chosen such that it is uncorrelated to the remaining set Z_2 . The normal equation in calibration takes the form

$$N_{11}\,Y + N_{12}\,Z = H_1$$

$$N_{21}\,Y + N_{22}\,Z = H_2 \quad .$$

Partitioning the calibration variables in two groups, Z_1 and Z_2 , the inverse N^{-1} can be written in the form

<table>
<tr><td>Q_{YY}</td><td colspan="2">Q_{YZ}</td></tr>
<tr><td rowspan="2">Q_{ZY}</td><td>$Q_{Z_1Z_1}$</td><td>$Q_{Z_1Z_2}$</td></tr>
<tr><td>$Q_{Z_2Z_1}$</td><td>$Q_{Z_2Z_2}$</td></tr>
</table>

We now require the calibration adjustment to yield

$$Q_{YZ} \quad = 0$$

$$Q_{Z_1Z_2} = 0 \ .$$

The reach this the whole calibration procedure has to be studied, because Q depends on the design of the experiment.

Wrobel (1978) has drawn attention to the fact that the determined calibration constants should be, if possible, uncorrelated to exterior element and between themselves in order to be easily used later on.

On-the-job calibration means that all types of unknowns are included in the adjustment, exterior elements, interior elements, point coordinates and geometric condition parameters. The same pictures are, at the same time, used for the calibration and for the project. Grün has discussed the problem of over-parameterization (Grün (1978a) and (1978b)) which has the effect that the standard errors of the unknowns and the covariance between them increase although the standard error of unit weight decreases. A possible solution to this is to design the additional parameters to be orthogonal to the other unknowns, and then combine this with statistical tests on their significance. The general bivariate polynomial approach is thus abandoned. For aerial triangulation Grün (1979a) has found that additional parameters i.e. on-the-job-calibration, are somewhat superior to testfield calibration methods. In many close range applications it may be difficult to get a good design of the project for on-the-job calibration, in other cases it is easier. Our opinion principle is to expect higher accuracy if more observations are included in the process, which is the case for projects based on testfield calibration. On the other hand, on-the-job calibration determines the interior orientation of exactly the same photos as for the project, which eliminates the effect of variation of systematic errors between calibration and project.

The multi-purpose program concept contains so many different features that great flexibility is needed for the practical use of the program for calculations. The possibility of interactive work to edit input and output seems to be an advantage that is worthwhile to be tried, in order to facilitate the effective use of the computational tools. These tools are partly the software and programs written by the photogrammetrists, partly the hardware such as computer peripherals, output devices, interactive numerical and graphical terminals, input from image coordinate measuring instruments, and the like. As the design and solutions in analytical close range photogrammetry vary considerably from one project to the other, the interactive approach to the adjustment calculation must be the best way to follow.

7 CONCLUSIONS

Classical theory of errors of measurements divide errors in three groups: random, systematic and gross errors (blunders). The combined effect of these types of errors gives the accuracy. The random errors are supposed to be normally distributed and usually also independent, the systematic errors follow some known rules and the observations can be corrected for their effect, the gross errors are blunders or mistakes made by observers or malfunctioning equipment.

The random errors and their effect are connected to the concept of precision. The systematic errors are related to the mathematical model for the adjustment of the photogrammetric observations. The better the model fits to the reality, the better is the model fidelity. The gross errors and blunders are related to the concept of reliability of the adjustment. The accuracy thus comprises three parts: precision, model fidelity and reliability.

All three parts of accuracy are improved by redundant observations. Redundancy is needed in each step of the photogrammetric procedure and in the determination of the final result. Some main steps of the procedure where improvement through redundancy can be obtained are:

- photography : the importance of good photographic quality is not always recognized
- comparators : correction of systematic errors determined by calibration
- multi-readings : repeated settings on each image point
- multi-fiducials : more than four fiducial marks are of value
- multi-targets : local irregularities of the imaging and measuring system (atmosphere, lens, emulsion, comparator, operator, etc.) is averaged out by more fold targets on each object point

- multi-frames : more exposures on each station give an averaged bundle of rays, and the camera can be rotated ω, ϕ and κ between the exposures so as to estimate or eliminate systematic errors of the bundle of rays

- multi-stations : more than the necessary two photography stations for a stereopair yield very effective means to improve accuracy by having each object point determined by three, four and more rays intersecting from various directions in space

- multi-control : absolute orientation and control of model deformation is obtained not only by given co-ordinates of control points but also by geometric object space conditions such as lines, planes, distances, etc.

- multi-purpose program : a rigorous computer program is needed to handle all information, it has to be flexible for various purposes and mathematical models in photogrammetry, it should preferably be based on interactive checks on intermediate results.

The multi-concept will be very expensive to introduce in all steps. It does not seem to be necessary to do so in order to improve precision, fidelity and reliability. Hottier (1976) has given recommendation for combinations of numbers of settings, targets and frames for photogrammetric intersection from two stations to improve precision. Grün (1978) has shown the effect of base-distance-ratio and intersection from more than two stations on precision and reliability. Calibration of cameras and comparators and additional parameters in adjustments have improved the model fidelity. Provided there is a general computer program available for the adjustment it seems to be reasonable to assume that intersection of object points from four different photography stations yields a good precision and reliability. The improvement of the model fidelity can be achieved by precalibration or multi-frames on each station combined with on-the-job calibration. Research, development and practical experience are needed to find the relation between time consumption, cost, and accuracy improvement so as to find the best design of photogrammetric solution of a particular measuring task.

REFERENCES

Ackermann, F., Schilcher, M. (1978): Auto- and Cross-Correlation of Image Coordinates. Nachrichten aus dem Karten- und Vermessungswesen, Reihe II, Nr. 36.

Anscombe, F. J. (1960): Rejection of Outliers. Technometrics, Vol. 2, No. 2, May 1960, pp. 123-147.

Baarda, W. (1965): Precision Accuracy and Reliability of Observations. Computing Centre of the Delft Geodetic Inst., 1965.

Baarda, W. (1967): Statistical Concepts in Geodesy. Netherlands Geodetic Commission, Vol. 2, No. 4, 1967.

Baarda, W. (1968): A Testing Procedure for Use in Geodetic Networks. Netherlands Geodetic Commission, Vol. 2, No. 5, 1968.

Baarda, W. (1976): Reliability and precision of networks. Computing Center of the Delft Geodetic Inst., 1976.

Baarda, W. (1977): Measures for the accuracy of geodetic networks. Paper presented to the IAG international symposium on optimization of design and computation and control of networks, Sopron, Hungary, July 3-9, 1977.

Bachmann, W. K., Hawawini, Y. J. (1978): Détermination expérimentale de la précision de l´orientation relative au sens large dans un appareil de restitution analogique. École Polytechnique Fédérale de Lausanne. Lausanne, 1978.

Bain, L. J. (1978): Statistical Analysis of Reliability and Life-Testing Models. Dekker, New York and Basel, 1978.

Bjerhammar, A. (1973): Theory of errors and generalized matrix inverses. Elsevier, Amsterdam, London, New York, 1973.

Borchers, P. (1965): The Photogrammetric Study of Structural Movements in Architecture. ISP Archives, Vol. XV, part 6, Lisbon, 1965.

Bouloucos, T., et al. (1979): The reliability of the observations of points in planimetric independent model blocks. ITC Journal 1979-2, pp. 226-248.

Brown, D. C. (1969): Advanced methods for the calibration of metric cameras. Paper for the 1969 Symposium on computational photogrammetry at Syracuse Univ., 1969.

Dorrer, U. (1977): Software Aspects in Desk-Top Computer-Assisted Stereoplotting. Photogrammetria, 33 (1977), pp. 1-18.

Ebner, H. (1976): Self Calibrating Block Adjustment. Bildmessung und Luftbildwesen, No. 44, pp. 128-139.

Förstner, W. (1976): Statistical Test Methods for Blunder Detection in Planimetric Block Adjustment. ISP Archives Helsinki 1976. Paper presented to Comm. III of ISP.

Förstner, W. (1978): Die Suche nach groben Fehlern in photogrammetrischen Lageblöcken. DGK, Reihe C, Nr. 240, München, 1978.

Förstner, W. (1979): On internal and external reliability of photogrammetric coordinates. Proc. American Society of Photogrammetry, Washington D.C., 1979, pp. 294-310.

Graybill. A. F. (1961): An Introduction to Linear Statistical Models. Volume I. McGraw-Hill, New York, Toronto, London, 1961.

Grün, A. (1978a): Progress in Photogrammetric Point Determination by Compensation of Systematic Errors and Detection of Gross Errors.
Nachrichten aus dem Karten- und Vermessungswesen,
Reihe II: Übersetzungen, Heft Nr. 36.
Frankfurt a M., pp. 113-140, 1978.

Grün, A. (1978b): Accuracy and statistics in close range photogrammetry. ISP Archives, Vol. XXII, part V, Proceedings Photogrammetry in Industry. Royal Inst. of Technology, Stockholm, 1978.

Grün, A. (1979a): Self-Calibration versus Testfield Calibration. Paper presented to the meeting if ISP Working Group III/3 in Aalborg, Denmark, May 17-18, 1979.

Grün, A. (1979b): Gross errors detection in bundle adjustment. Paper presented at Aerial triangulation symposium in Brisbane, Australia, 1979.

Hallert, B., et al. (1964): Fundamental Problems in Photogrammetry. ISP Working Group Commission II. ISP Archives Lisbon, 1964.

Hallert, B., et al. (1967): Quality problems in photogrammetry. ISP Archives Comm. II Working group on fundamental problems. Stockholm, 1967.

Hein, G., Lenzel, K. (1979): Zur Genauigkeit und Wirtschaftlichkeit verschiedener Interpolations- und Prädiktionsmethoden. Zeitschrift für Vermessungswesen, Heft 11, Nov. 1979, pp. 492-505.

Heindl, G., Reinhart, E. (1979): Detecting outliers with linear programming by introducing tolerances. Paper presented at the XVII-IUGG general assembly, Canberra, Australia, Dec. 1979.

Hell, G. (1979): Terrestrische Bildtriangulation mit Berücksichtigung zusätzlicher Beobachtungen. DGK, Reihe C, Nr. 252, München, 1979.

Hottier, P. (1976): Accuracy of Close Range Analytical Restitutions: Practical Experiments and Prediction. Photogrammetric Engineering and Remote Sensing, 1976, pp. 345-375.

Hvidegaard, J. (1976): Densification of trigonometric nets. ISP Archives Helsinki 1976. Presented paper Commission III.

Jacobsen, K. (1980): Versuch zur Optimierung der mit Bündelblockausgleichungen zu erreichenden Genauigkeit. Inst. f. Photogrammetrie, Techn. Univ. Hannover.

Kager, H., Kraus, K. (1976): Gemeinsame Ausgleichung photogrammetrischer, geodätischer und fiktiver Beobachtungen. Geowissenschaftliche Mitteilungen 8, 1976, pp. 113-133. Technische Universität Wien.

Kendall, M. G., Stuart, A. (1969): The Advanced Theory of Statistics. Griffin, London, 1969.

Kupfer, G. (1973): Image Geometry as Obtained from Rheidt Test Aera Photography. OEEPE Official Publication No. 8, Inst. f. Angewandte Geodäsie, Frankfurt a. M., 1973, pp. 255-281.

Mann, N. R., Schafer, R. E., Singpurwalla, N. D. (1974): Methods for Statistical Analysis of Reliability and Life Data. Wiley, New York, London, Sydney, Toronto, 1974.

Mikhail, E. M. (1976): Observations and least squares. IEP-A Dun-Donnelley Publisher, New York, 1976.

Molenaar, M. (1976): Error detection in planimetric block adjustment. ITC-Journal, 1976, No. 2, pp. 341-356.

Molenaar, M. (1978): Preadjustment error detection in independent model blocks. ITC-Journal, 1978-3, pp. 514-541.

Morén, A. (1967): Geometrical Quality of Aerial Photographs. Royal Inst. of Technology, Stockholm, 1967.

Pelzer, H. (1979): Zur Beurteilung der Zuverlässigkeit geodätischer Messungen. Festschrift Karl Gerke. Geodätische Schriftenreihe der TU Braunschweig, Nr. 1, 1979, pp. 187-205.

Pope, A. J. (1975): The statistics of residuals and the detection of outliers. Paper presented to the IUGG & IAG XVI General Assembly in Grenoble, France, 1975.

Rauhala, U. (1974): Array Algebra with Applications in Photogrammetry and Geodesy. Thesis, Stockholm, 1974, and Fotogrammetriska Meddelanden, Vol. VI, No. 6, Royal Inst. of Technology, Stockholm, 1974.

Rohatgi, V. K. (1976): An Introduction to Probability Theory and Mathematical Statistics. Wiley, New York, London, Sydney, Toronto, 1976.

Shut, G. H. (1976): Review of interpolation methods for digital terrain models. ISP Archives, Comm. III. Invited paper, 1976.

Stefanovic, P. (1978a): Blunders and least squares. ITC-Journal 1978-1, pp. 122-157.

Stefanovic, P. (1978b): Analytical photogrammetry: computational efficiency and blunder detection. Paper presented to the ISP Comm. III symposium in Moscow, 1978. ISP Archives 1978.

Thomas, J. (1977): Variation der Bildfluganordnung. Ihre Auswirkung auf die Bündelausgleichung bei Vorliegen zufälliger und systematischer Bildfehler. DGK, Reihe C, Nr. 233, München, 1977.

Torlegård, K. (1967): On the Determination of Interior Orientation of close-up Cameras under Operational Conditions using Threedimensional Test Objects. Thesis, Royal Inst. of Technology, Stockholm, 1967.

Wrobel, B., Kruck, E. (1978): Passpunktbestimmungen und Fassaden durch Bündelblockausgleichung mit Bildern von Stereomeß-kammern. Paper presented to the CIPA symposium in Sibenik, Yugoslavia, 1978.

Wrobel, B. (1978a): Kammerkalibrierung mit verknüpften Bildverbänden bei nicht formatfüllendem Testfeld. ISP Archives, Vol. XXII, Part V, Proceedings Photogrammetry for Industry. Royal Inst. of Technology, Stockholm, 1978.

Appendix

PROGRAM MULTI - PHOTOGRAMMETRY

INSTRUCTIONS FOR INPUT OF COMPARATOR MEASUREMENTS

Each record starts with an identifier defining the type of record. The identifier is a two digit number [00 , 99] . Measurements from mono- and stereocomparators are accepted. Readings in mm-units. The measuring file starts with a title record (id = 10) and ends with a stoprecord (id = 99). The file contains measurements on an arbitrary number of photos (or pair of photos in case of a stereocomparator). A series of measurements on a photo (pair of photos) begins with id = 12 and ends with id = 98. The same photo (pair of photos) can be measured several times with changed positions in the comparator. Such a replication begins id = 12 and ends id = 98. With each series the observations on the image points may be repeated once, i.e. double measurement on the same point is allowed between id = 12 and id = 98. Such double readings are identified with the point number. There can be single and double readings on the same photo. For each photo the readings are of four kinds : fiducial marks (id = 20-29), control points (id = 30-39, 60-69), auxiliary points (id = 40-49, 70-79) and new points (id = 50-59, 80-89). id = X0 : no double readings. id = X1 double readings. If the fiducial marks are crosses, they can be measured on the four bars instead of centrally. id = 24 means four single readings on the bars, id = 28 means four double readings on the bars. For object points with multi-targets the second digit of the id-number indicates the number of targets measured for that point. If id ≥ 62 there are double readings on multi-targets. Point numbers are positive and have at most 6 digits (000 001 , 999 999).

Specification for the identifiers.

id = 10 "text string with max 72 characters"

id = 11 comp. no.
Identification no for the comparator. Has to be the same as in the comparator calibration file.

id = 12 p1, k1 (p2, k2)
p1 is an identification number for the photo,
k1 is the camera number (When a stereocomparator is used, p2 is the photo on the parallax carriage taken with camera no k2.).

id = 20-
-id = 89 pt, x, y, (px, py)
pt : point no (000 001 - 999 999)
x : x coordinate in mm
y : y coordinate in mm
px : x-parallax, in case of stereocomparator
py : y-parallax, in case of stereocomparator

id = 20 single reading on fiducial mark

id = 21 double reading on fiducial mark

id = 24 single reading on a bar of fiducial mark

id = 28 double reading on a bar of fiducial mark

id = 30 single reading on control point, single target

id = 31 double reading on control point, single target

id = 32 single reading on control point, 2-fold target

id = 62 double reading on control point, 2-fold target

id = 33 single reading on control point, 3-fold target

id = 63 double reading on control point, 3-fold target

id = 34 single reading on control point, 4-fold target

id = 64 double reading on control point, 4-fold target

id = 35 single reading on control point, 5-fold target

id = 65 double reading on control point, 5-fold target

id = 36 single reading on control point, 6-fold target

id = 66 double reading on control point, 6-fold target

id = 37 single reading on control point, 7-fold target

id = 67 double reading on control point, 7-fold target

id = 4X single readings on auxiliary points

id = 7X double readings on auxiliary points

The second digit X has the same meaning as for id = 3X and id = 6X series above. The auxiliary points are observed extra to give a strong connection between the photos. A pair of photos has a common object space. Auxiliary points can be taken in such positions as to

a) cover the common image area

b) cover the object space volume.

id = 5X single readings on new points to be determined

id = 8X double readings on new points to be determined

The second digit X has the same meaning as for id = 3X and id = 6X series above.

id = 98 pt, x, y (, px, py)
End of photos (Dummy recording of pt, x, y)

id = 99 pt, x, y (, px, py)
End of measurement file (Dummy, recording of pt, x, y)

Note 1:

id = 24 Readings on bars of the fiducial mark is for a single four bar recording done such that the two first readings are taken on opposite bars and the two last on those bars perpendicular to the first ones. The point no is that of the fiducial mark for all 4 settings.

Note 2:

id = 28 For double readings on the bars the pattern above is repeated later on, preferably at the end of the photo readings. The point no is that of the fiducial mark for all 8 settings.

Note 3:

id = 24 For cameras without fiducial marks the side lines (the edges) of the image format can be used. The four bar readings on each fiducial mark and here instead four readings on the two sides forming the image corner. The location of readings 1 - 4 should be the same as when calibrating the camera to avoid bias. The point no is the same for all 4 settings, i.e. that of the fiducial mark, here = image corner.

Note 4:

id = 28 Double readings on side lines are possible. The point no is the same for all 8 settings, i.e. that of the fiducial mark, here = image corner.

Note 5:

Photos are measured in a right handed xy system with the photos in positions as diapositives. Parallax readings are defined from the comparator file.