

Novel Analytical Calculation Method for the Non-Linear Ψ -i-Characteristic of Switched-Reluctance-Machines in the Aligned Rotor Position

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Abstract - Knowing the non-linear Ψ -i-characteristic is crucial for the design of switched reluctance machines, especially for the aligned position. So far, analytical calculations are based on complex models of the magnetic circuit or on functions needing a fitting procedure (using measured or FEM-calculated data). In this paper, a method is presented that requires only very few input data. In addition, these input data can be deduced easily from the geometry of the machine. Comparisons with measured data show quite a good correlation, qualifying this method to be used in the design procedure of new switched reluctance machines.

- The approach in [4] is a physically motivated model with analytical equations. The given results are good, but the model itself is very complex with numerous input parameters.
- The model in [5] is based on the calculation of magnetic resistances and therefore it is less complicated. Nevertheless, even for this model numerous input parameters are required to calculate the magnetic resistances.

I. INTRODUCTION

The non-linear Ψ -i-characteristic (flux linkage versus phase current) is essential, if the performance of switched reluctance machines has to be calculated. In general there are three methods to get this characteristic:

- Measurements give the correct results for a special machine, but this can not be used for the development of new machine designs.
- FEM calculations are suitable for the design of new machine topologies, but there is the drawback of very time-consuming procedures.
- Analytical calculations are fast, but it is complicated to model the highly non-linear Ψ -i-characteristic.

Concerning the analytical calculation there are some proposals from different authors, which are shortly commented in the following:

- In [1] a polynomial approximation is given, but measurements or FEM calculations are required for the fitting procedure.
- An exponential approximation is given in [2], but again measurements or FEM calculations are required for the fitting procedure.
- A series method is given in [3]; but also this method requires measurements or FEM calculations for the fitting procedure.

In this paper, an analytical approach for calculating the non-linear Ψ -i-characteristic will be discussed, with the following special advantages:

- There are only few input parameters to calculate this characteristic.
- These input parameters easily can be deduced from the machine geometry.

II. THE ALIGNED POSITION OF SWITCHED RELUCTANCE MACHINES

A. General remarks

The non-linear Ψ -i-characteristic of switched reluctance machines in the aligned position has the following features:

- For low phase currents the machine is unsaturated. The small air-gap between stator tooth and rotor tooth determines the Ψ -i-characteristic, which is linear:

$$\Psi(i) = A \cdot i. \quad (1)$$

- For high phase currents the machine is strongly saturated. As in this situation the iron parts behave like air again the Ψ -i-characteristic is linear, but with different

slope than before and with an offset:

$$\Psi(i) = B \cdot i + C. \quad (2)$$

- The part in between is the non-linear transition between these two linear branches. This non-linear part is of special interest, because it crucially determines the machine performance. Up to now this non-linear transition curve can only be determined so far that it is always below both linear branches.
- All parameters A, B, and C can be extracted from the machine geometry. However, this procedure will be published later. In this paper we concentrate on the extraction of the parameters from measured data.

The following figure shows these attributes together with a characteristic current I_{sat} (indicating the end of the first linear branch):

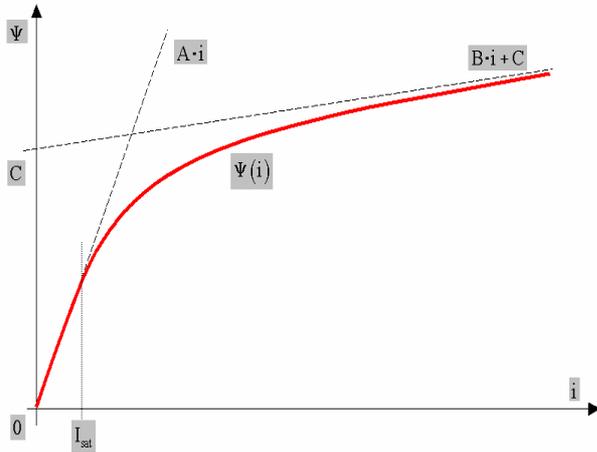


Fig. 1: Non-linear Ψ - i -characteristic of a switched reluctance machine in aligned position.

B. Mathematical description

For the mathematical description of the non-linear Ψ - i -characteristic the following equation will be used:

$$\Psi(i) = \begin{cases} A \cdot i & \text{if } i \leq I_{\text{sat}} \\ (B \cdot i + C) \cdot \left(1 - E \cdot e^{-\frac{i}{I_{\text{sat}}}}\right) & \text{if } i > I_{\text{sat}} \end{cases} \quad (3)$$

This equation is physically motivated, because:

- For low currents ($i \rightarrow 0$), we get: $\Psi(i) = A \cdot i$. This corresponds to the requirements.

- For very high currents ($i \rightarrow \infty$), we get: $\Psi(i) = B \cdot i + C$. This again corresponds to the requirements.

What we have now is a description of the non-linear Ψ - i -characteristic with three known parameters (A, B, C; known from the geometry of the machine) and two unknown variables (I_{sat} and E). These two unknowns can now be calculated from two additional constraints:

- The absolute value of both branches of (3) must be the same at $i = I_{\text{sat}}$ (because the flux value must be continuous).
- The slope of both branches of (3) must be the same at $i = I_{\text{sat}}$ (because the inductivity value must be continuous).

From these constraints we get:

$$A \cdot I_{\text{sat}} = (B \cdot I_{\text{sat}} + C) \cdot (1 - E \cdot e^{-1}) \quad (4)$$

and

$$A = \left. \frac{d}{di} \Psi(i) \right|_{i > I_{\text{sat}}, i \rightarrow I_{\text{sat}}} \quad (5)$$

With

$$\begin{aligned} \left. \frac{d}{di} \Psi(i) \right|_{i > I_{\text{sat}}} &= \\ &= \frac{d}{di} \left[(B \cdot i + C) \cdot \left(1 - E \cdot e^{-\frac{i}{I_{\text{sat}}}}\right) \right] \\ &= \frac{d}{di} \left[B \cdot i + C - B \cdot i \cdot E \cdot e^{-\frac{i}{I_{\text{sat}}}} - C \cdot E \cdot e^{-\frac{i}{I_{\text{sat}}}} \right] \\ &= B + 0 - B \cdot E \cdot \left[e^{-\frac{i}{I_{\text{sat}}}} + i \cdot \left(-\frac{1}{I_{\text{sat}}}\right) \cdot e^{-\frac{i}{I_{\text{sat}}}} \right] \\ &\quad - C \cdot E \cdot \left(-\frac{1}{I_{\text{sat}}}\right) \cdot e^{-\frac{i}{I_{\text{sat}}}} \\ &= B + \left[\frac{C \cdot E}{I_{\text{sat}}} - B \cdot E \cdot \left(1 - \frac{i}{I_{\text{sat}}}\right) \right] \cdot e^{-\frac{i}{I_{\text{sat}}}} \end{aligned}$$

we get from (5):

$$A = B + \frac{C \cdot E}{I_{\text{sat}}} \cdot e^{-1} \quad (6)$$

C. Solution for the unknowns

With (4) and (6) we have two equations for the two unknowns I_{sat} and E . Solving (6) for the unknown I_{sat} we get:

$$I_{\text{sat}} = \frac{C \cdot E}{(A - B) \cdot e} \quad (7)$$

Introducing (7) into (4) gives with some fundamental transformations:

$$E^2 + \frac{2 \cdot (A - B) \cdot e}{B} \cdot E - \frac{(A - B) \cdot e^2}{B} = 0 \quad (8)$$

From the set-up, (3), we know that $E > 0$ is true. Together with

$$A > B \Rightarrow \frac{2 \cdot (A - B) \cdot e}{B} > 0$$

it can be deduced that from the two solutions of the quadratic equation (8) only that solution containing the positive sign is relevant. Therefore, we get:

$$E = -\frac{(A - B) \cdot e}{B} + \sqrt{\frac{(A - B)^2 \cdot e^2}{B^2} + \frac{(A - B) \cdot e^2}{B}}$$

$$= \frac{(A - B) \cdot e}{B} \left[-1 + \sqrt{1 + \frac{B}{A - B}} \right] \quad (9)$$

Introducing (9) into (7) gives:

$$I_{\text{sat}} = \frac{C}{B} \left[-1 + \sqrt{1 + \frac{B}{A - B}} \right] \quad (10)$$

Equations (9) and (10) are the solutions for the two unknowns. With this, the non-linear Ψ -i-characteristic for the aligned position is determined completely.

III. COMPARISON WITH MEASUREMENTS

The method described above was checked against measurements for a certain reluctance machine. From the measurements, the following data could be extracted:

$$A = 1.01 \cdot 10^{-3} \frac{\text{Vs}}{\text{A}}, \quad B = 0.037 \cdot 10^{-3} \frac{\text{Vs}}{\text{A}},$$

$$C = 0.017 \text{Vs}$$

Using (9) and (10) we get:

$$E = 1.346, \quad I_{\text{sat}} = 8.713 \text{A}.$$

With these data the following comparison can be drawn: the measured Ψ -i-characteristic is shown in red, the calculated one in blue:

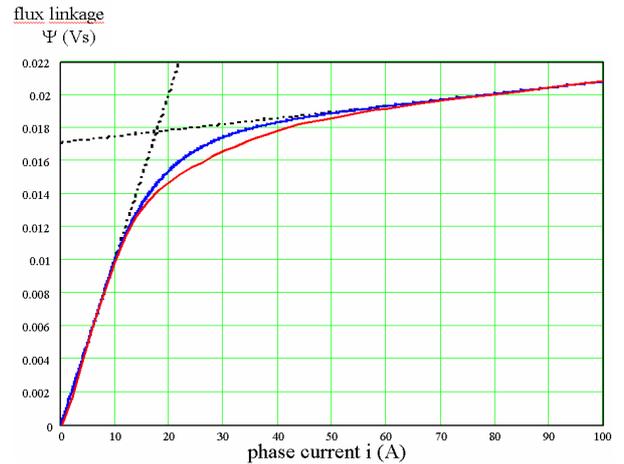


Fig. 2: Non-linear Ψ -i-characteristic of a switched reluctance machine in aligned position; red: measurement; blue: calculation.

In Fig.2 the calculation data deviate with a maximum of 6% from the measurement data, which is very good for analytical calculation while in Fig. 3 there is a deviation up to 25%.

Another important characteristic is the inductivity

versus phase current. With $L = \frac{\partial \Psi}{\partial i}$ these

characteristics (measurement and calculation) can be deduced from the above Ψ -i-characteristics.

The results are shown in the following Fig. 3.

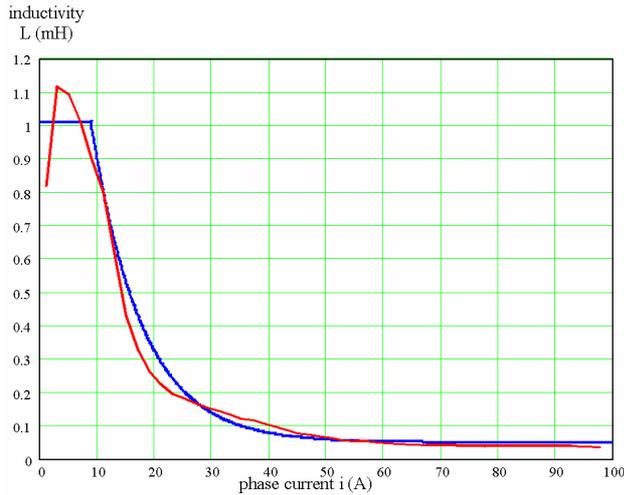


Fig. 3: Inductivity versus phase current characteristic of a switched reluctance machine in aligned position; red: measurement; blue: calculation.

IV. CONCLUSION

The presented method of calculating the non-linear ψ - i -characteristic for the aligned rotor position is based on the knowledge of the linear branches of the ψ - i -characteristic. These parameters can be deduced from the geometry of the machine. Consequently, complex models or functions needing fitting procedures are avoided. Comparisons with measured data show quite a good correlation, qualifying this method to be used in the design procedure of new switched reluctance machines.

The calculation of the non-linear ψ - i -characteristic for arbitrary rotor positions will be published in a subsequent paper [6].

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