# Serial-parallel Graphs

```
grammar SerialParallel
   nonterminal SP(2), S(0);
   terminal
                a(2);
   start
                S;
   S()
          ::= SP(x,y)
                                [ init
                                           ]
                                           ]
   SP(x,y) ::= a(x,y)
                                [ edge
            | SP(x,y) SP(x,y) [ parallel ]
            | SP(x,u) SP(u,y) [ serial
                                           ]
end
```

**Note:** When describing conflicts,  $\circ$  represents a node that has not yet been processed whereas  $\bullet$  represents a node that has been processed already, but that is not the value of any current parameter.

Note: States  $q_0, q_2, q_3, q_4, q_5, q_6$  have conflicts

State  $q_0(a, b)$ 

S()	$ ightarrow$ . SP $(oldsymbol{a},oldsymbol{b})$
$SP({m a},{m b})$	$ ightarrow$ . SP $(oldsymbol{a},oldsymbol{b})$ SP $(oldsymbol{a},oldsymbol{b})$
$SP({m a},{m b})$	$ ightarrow$ . SP $(oldsymbol{a}, n_1)$ SP $(n_1, oldsymbol{b})$
$SP(\boldsymbol{a}, \boldsymbol{b})$	$ ightarrow$ . a $(oldsymbol{a},oldsymbol{b})$
$SP(\boldsymbol{a}, n_2)$	$\rightarrow$ . SP $(a, n_3)$ SP $(n_3, n_2)$
$SP(\boldsymbol{a}, n_4)$	$ ightarrow \mathbf{SP}(\boldsymbol{a},n_4)\mathbf{SP}(\boldsymbol{a},n_4)$
$SP(\boldsymbol{a}, n_5)$	$ ightarrow$ . a $(oldsymbol{a},n_5)$

$$\begin{array}{c} \frac{\mathsf{SP}(n_0, n_1)}{n_0 = \boldsymbol{a}, n_1 \uparrow} \rightarrow q_2(n_0, n_1, \boldsymbol{b}) \\ \hline \mathbf{SP}(n_0, n_1) \\ \hline n_0 = \boldsymbol{a}, n_1 = \boldsymbol{b} \rightarrow q_3(n_0, n_1) \\ \hline \mathbf{a}(n_0, n_1) \\ \hline n_0 = \boldsymbol{a}, n_1 \uparrow \qquad q_1(n_0, n_1) \\ \hline \mathbf{a}(n_0, n_1) \\ \hline n_0 = \boldsymbol{a}, n_1 = \boldsymbol{b} \rightarrow q_1(n_0, n_1) \end{array}$$

### **Conflicts:**

• shift  $a(a, \circ)$ , shift a(a, b)

State  $q_1(a, b)$  $[SP(a, b) \rightarrow a(a, b) \cdot [edge] ]$ 

State  $q_2(a, b, c)$ 

$SP({m a},n_1)  o SP({m a},{m b})$ . $SP({m b},n_1)$
$SP(oldsymbol{a},oldsymbol{b})  o SP(oldsymbol{a},oldsymbol{b})$ . $SP(oldsymbol{a},oldsymbol{b})$
$SP(oldsymbol{a},oldsymbol{c})  o SP(oldsymbol{a},oldsymbol{b})$ . $SP(oldsymbol{b},oldsymbol{c})$
$SP({m a},{m b})  o .SP({m a},{m b})  SP({m a},{m b})$
$SP(\boldsymbol{a},\boldsymbol{b})   o  sP(\boldsymbol{a},n_2)SP(n_2,\boldsymbol{b})$
$SP(oldsymbol{a},oldsymbol{b})  o$ . a $(oldsymbol{a},oldsymbol{b})$
$SP(\boldsymbol{a}, n_3) \rightarrow SP(\boldsymbol{a}, n_4)  SP(n_4, n_3)$
$SP(\boldsymbol{a}, n_5) \rightarrow SP(\boldsymbol{a}, n_5) SP(\boldsymbol{a}, n_5)$
$SP({m a},n_6)  o \mathtt{la}({m a},n_6)$
$SP(\boldsymbol{b}, n_7) \rightarrow SP(\boldsymbol{b}, n_8) SP(n_8, n_7)$
$SP(\boldsymbol{b}, n_9) \rightarrow SP(\boldsymbol{b}, n_9) SP(\boldsymbol{b}, n_9)$
$SP(oldsymbol{b}, n_{10})  ightarrow  extbf{a}$ ( $oldsymbol{b}, n_{10}$ )
$SP(\boldsymbol{b}, \boldsymbol{c}) \longrightarrow SP(\boldsymbol{b}, n_{11}) SP(n_{11}, \boldsymbol{c})$
$SP(\boldsymbol{b}, \boldsymbol{c}) \rightarrow SP(\boldsymbol{b}, \boldsymbol{c})  SP(\boldsymbol{b}, \boldsymbol{c})$
$SP(\boldsymbol{b},\boldsymbol{c}) \rightarrow a(\boldsymbol{b},\boldsymbol{c})$

$$\begin{array}{c} \frac{\mathsf{SP}(n_0, n_1)}{n_0 = \boldsymbol{a}, n_1 \uparrow} & q_2(n_0, n_1, \boldsymbol{b}) \\ \hline \mathbf{SP}(n_0, n_1) & \\ n_0 = \boldsymbol{a}, n_1 = \boldsymbol{b} & q_4(n_0, n_1) \\ \hline \mathbf{SP}(n_0, n_1) & \\ n_0 = \boldsymbol{b}, n_1 \uparrow & q_5(n_0, n_1, \boldsymbol{c}, \boldsymbol{a}) \\ \hline \mathbf{SP}(n_0, n_1) & \\ n_0 = \boldsymbol{b}, n_1 = \boldsymbol{c} & q_6(n_0, n_1, \boldsymbol{a}) \\ \hline \mathbf{a}(n_0, n_1) & \\ n_0 = \boldsymbol{a}, n_1 \uparrow & q_1(n_0, n_1) \\ \hline \mathbf{a}(n_0, n_1) & \\ n_0 = \boldsymbol{b}, n_1 = \boldsymbol{b} & q_1(n_0, n_1) \\ \hline \mathbf{a}(n_0, n_1) & \\ n_0 = \boldsymbol{b}, n_1 \uparrow & q_1(n_0, n_1) \\ \hline \mathbf{a}(n_0, n_1) & \\ n_0 = \boldsymbol{b}, n_1 \uparrow & q_1(n_0, n_1) \\ \hline \mathbf{a}(n_0, n_1) & \\ n_0 = \boldsymbol{b}, n_1 = \boldsymbol{c} & q_1(n_0, n_1) \end{array}$$

#### **Conflicts:**

• shift a(a, b), shift  $a(b, \circ)$ , shift a(b, c), shift  $a(a, \circ)$ 

State  $q_3(a, b)$ 

$SP(\boldsymbol{a}, \boldsymbol{b})$	$ ightarrow {\sf SP}({m a},{m b})$ . ${\sf SP}({m a},{m b})$	
S()	$ ightarrow SP(oldsymbol{a},oldsymbol{b})$ .	[init]
$SP(\boldsymbol{a}, \boldsymbol{b})$	$ ightarrow \mathbf{sP}(oldsymbol{a},oldsymbol{b}) SP(oldsymbol{a},oldsymbol{b})$	
$SP(\boldsymbol{a},\boldsymbol{b})$	$ ightarrow$ . SP $(oldsymbol{a}, n_1)$ SP $(n_1, oldsymbol{b})$	
$SP({m a},{m b})$	$ ightarrow$ . a $(oldsymbol{a},oldsymbol{b})$	
$SP(\boldsymbol{a},n_2)$	$\rightarrow$ . SP $(a, n_3)$ SP $(n_3, n_2)$	
$SP(\boldsymbol{a},n_4)$	$\rightarrow$ . SP $(\boldsymbol{a}, n_4)$ SP $(\boldsymbol{a}, n_4)$	
$SP(\boldsymbol{a}, n_5)$	$ ightarrow$ . a $(oldsymbol{a},n_5)$	

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 \begin{array}{c} \frac{\mathsf{SP}(n_0, n_1)}{n_0 = \boldsymbol{a}, n_1 \uparrow} & q_2(n_0, n_1, \boldsymbol{b}) \\ \hline \mathsf{SP}(n_0, n_1) & & \\ n_0 = \boldsymbol{a}, n_1 = \boldsymbol{b} & q_4(n_0, n_1) \\ \hline \boldsymbol{a}(n_0, n_1) & & \\ n_0 = \boldsymbol{a}, n_1 \uparrow & & q_1(n_0, n_1) \\ \hline \boldsymbol{a}(n_0, n_1) & & & \\ n_0 = \boldsymbol{a}, n_1 = \boldsymbol{b} & & q_1(n_0, n_1) \end{array}
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### **Conflicts:**

• shift  $a(a, \circ)$ , shift a(a, b)

 $\begin{array}{l} \mbox{State } q_4(a,b) \\ \hline SP(a,b) \rightarrow SP(a,b) \cdot SP(a,b) \\ SP(a,b) \rightarrow SP(a,b) SP(a,b) \cdot [parallel] \\ \hline SP(a,b) \rightarrow \cdot SP(a,b) SP(a,b) \\ SP(a,b) \rightarrow \cdot SP(a,n_1) SP(n_1,b) \\ SP(a,b) \rightarrow \cdot a(a,b) \\ SP(a,n_2) \rightarrow \cdot SP(a,n_3) SP(n_3,n_2) \\ SP(a,n_4) \rightarrow \cdot SP(a,n_4) SP(a,n_4) \\ SP(a,n_5) \rightarrow \cdot a(a,n_5) \end{array}$ 

$$\begin{array}{c} \underbrace{\mathsf{SP}(n_0, n_1)}_{n_0 = \mathbf{a}, n_1 \uparrow} & q_2(n_0, n_1, \mathbf{b}) \\ \hline \underbrace{\mathsf{SP}(n_0, n_1)}_{n_0 = \mathbf{a}, n_1 = \mathbf{b}} & q_4(n_0, n_1) \\ \hline \underbrace{\mathsf{a}(n_0, n_1)}_{n_0 = \mathbf{a}, n_1 \uparrow} & q_1(n_0, n_1) \\ \hline \underbrace{\mathsf{a}(n_0, n_1)}_{n_0 = \mathbf{a}, n_1 = \mathbf{b}} & q_1(n_0, n_1) \end{array}$$

## Conflicts:

• reduce *parallel*, shift a(a, b), shift  $a(a, \circ)$ 

State  $q_5(a, b, c, d)$ 

rial]

$$\begin{array}{c} \frac{\mathsf{SP}(n_0, n_1)}{n_0 = \mathbf{a}, n_1 \uparrow} & q_2(n_0, n_1, \mathbf{b}) \\ \hline \mathbf{SP}(n_0, n_1) & q_4(n_0, n_1) \\ \hline \mathbf{SP}(n_0, n_1) & q_5(n_0, n_1, \mathbf{c}, \mathbf{a}) \\ \hline \mathbf{SP}(n_0, n_1) & q_5(n_0, n_1, \mathbf{c}, \mathbf{a}) \\ \hline \mathbf{SP}(n_0, n_1) & q_6(n_0, n_1, \mathbf{a}) \\ \hline \mathbf{n}_0 = \mathbf{b}, n_1 = \mathbf{c} & q_6(n_0, n_1, \mathbf{a}) \\ \hline \mathbf{a}(n_0, n_1) & q_1(n_0, n_1) \\$$

#### **Conflicts:**

• shift  $a(a, \circ)$ , reduce *serial*, shift  $a(b, \circ)$ , shift a(a, b), shift a(b, c)

State  $q_6(a, b, c)$ 

$SP(oldsymbol{a},oldsymbol{b})  o SP(oldsymbol{a},oldsymbol{b})$ . $SP(oldsymbol{a},oldsymbol{b})$	
$SP(oldsymbol{c},oldsymbol{b})  o SP(oldsymbol{c},oldsymbol{a})  SP(oldsymbol{a},oldsymbol{b})$ .	[serial]
$SP({m a},{m b})  ext{ }  o \mathtt{sP}({m a},{m b})  SP({m a},{m b})$	
$SP({m a},{m b})  ext{ }  o {m s}SP({m a},n_1)SP(n_1,{m b})$	
$SP(oldsymbol{a},oldsymbol{b})  ext{ }  o  extsf{a}(oldsymbol{a},oldsymbol{b})$	
$SP(\boldsymbol{a},n_2)  ightarrow SP(\boldsymbol{a},n_3)  SP(n_3,n_2)$	
$SP({m a},n_4)  ightarrow sP({m a},n_4)  SP({m a},n_4)$	
$SP({m a},n_5)  o$ . a $({m a},n_5)$	

$$\begin{array}{c} \underbrace{\mathsf{SP}(n_0,n_1)}{n_0 = \boldsymbol{a}, n_1 \uparrow} & q_2(n_0,n_1, \boldsymbol{b}) \\ \hline & \underbrace{\mathsf{SP}(n_0,n_1)}{n_0 = \boldsymbol{a}, n_1 = \boldsymbol{b}} & q_4(n_0,n_1) \\ \hline & \underbrace{\mathsf{a}(n_0,n_1)}{n_0 = \boldsymbol{a}, n_1 \uparrow} & q_1(n_0,n_1) \\ \hline & \underbrace{\mathsf{a}(n_0,n_1)}{n_0 = \boldsymbol{a}, n_1 = \boldsymbol{b}} & q_1(n_0,n_1) \end{array}$$

#### **Conflicts:**

• shift a(a, b), shift  $a(a, \circ)$