FEM in der Antriebstechnik

- ANSYS -

Dr.-Ing. Gurakuq Dajaku

FEAAM GmbH

Institute for Electrical Drives and Actuators

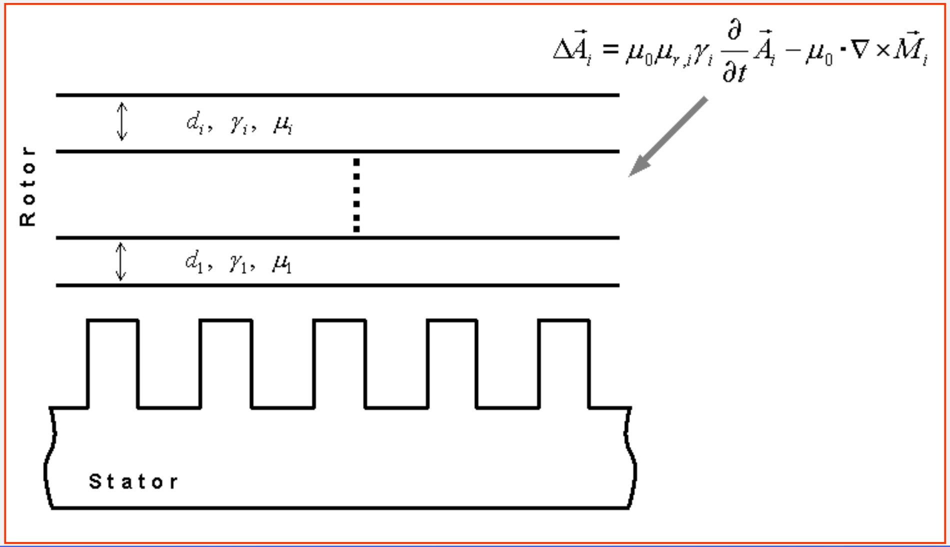
University of Federal Defense Munich

1. Analysis of Electric Machines

- The working principles of electrical machines can be described as the interaction of electromagnetic fields,
- An electromagnetic phenomenon can be expresses in mathematical form by Maxwell's electromagnetic equations,
- These equations can be solved by various methods: **analytical** and **numerical methods**,
- The analytical methods are considered to be inadequate when the problem involves magnetic saturation and complex geometry,
- The numerical methods are better equipped to handle such difficult problems. Among the various numerical methods, the finite element method (FEM) is widely used because of its flexibility and reliability,
- Of course, also other calculation methods exist for solving the electromagnetic problems, such as the Magnetic Equivalent Circuit (MEC) method.

2. Analytical Method using Maxwell's Equations

- Using analytical methods the solution of electromagnetic field inside the electric machines is based on the solution of Maxwell's differential equations for the magnetic and electric regions in the electric machine, which may be represented in integral form,
- The electromagnetic field equations for individual regions are formulated in terms of the magnetic vector potential or scalar potential,
- The method of separating variables is used to find the solution for the vector potential,
- For the complex geometries and if the non-linearity of materials has to be considered, the mathematical models become too complex and the solution of the mathematical equations is too difficult.



3. Magnetic Equivalent Circuits (MEC)

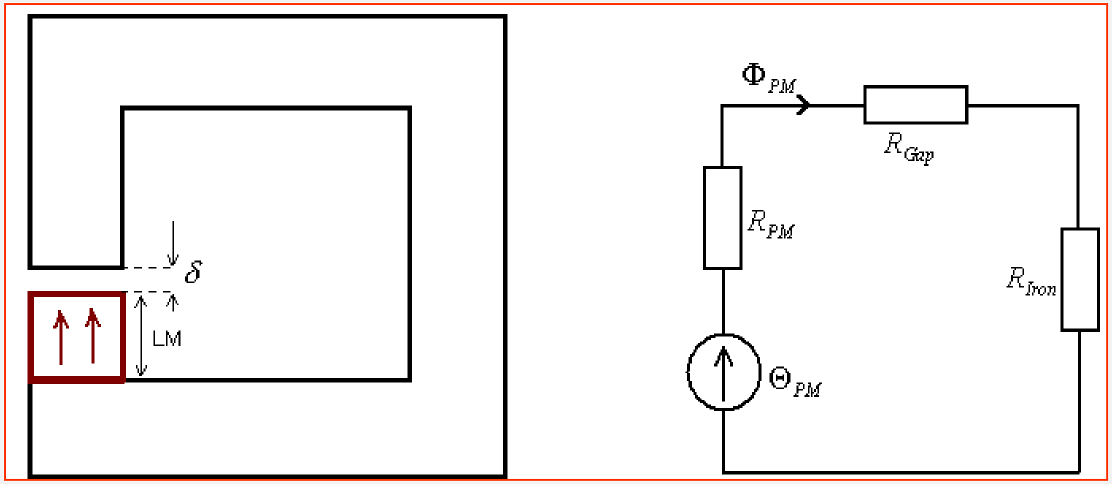
- Magnetic Equivalent Circuits (MEC) is a technique for modeling electromagnetic machines that can support both steady-state and dynamic simulations,
- MEC networks consist of reluctances, magnetomotive force (MMF) sources, and magnetic flux sources, and are an analogue to resistive electric networks,
- Reluctances represent flux tubes in the geometry of the modeled device whose values depend on the geometry, and the electromagnetic properties of the materials,
- Magnetic reluctance:

$$R_m = \frac{l}{\mu_0 \cdot \mu_r \cdot A}$$

- Magnetic flux:

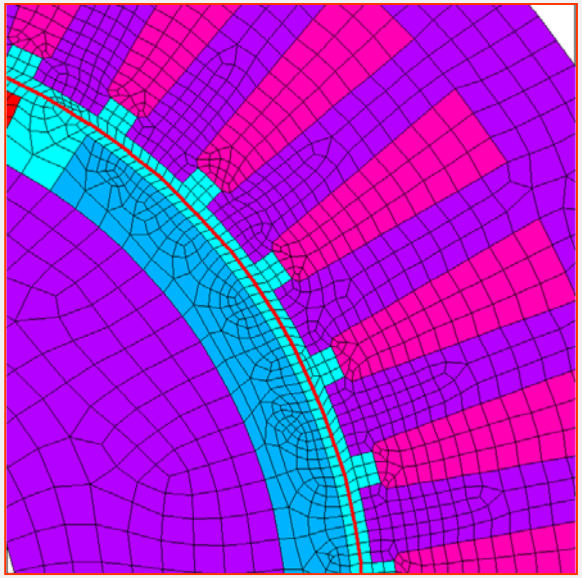
$$\Phi = \frac{\Theta}{R_m} = \Lambda \cdot \Theta$$

$$\Theta = N \cdot i, \qquad \text{- MMF source}$$



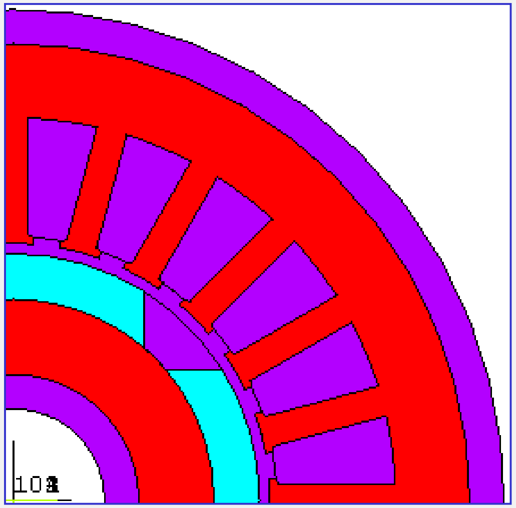
4. Finite Element Method (FEM)

- The finite-element (FE) method allows precise determination of machine parameters through the magnetic field solutions as it takes into account the actual distribution of windings, details of geometry, and non-linearity of magnetic materials,
- Using the finite element method, the whole analysis domain is divided into elementary sub-domains, which are called finite elements, and the field equations are applied to each of them,
- The FE methods are essentially based on the determination of the distribution of the electric and magnetic fields in the structures under study, based on the solution of **Maxwell's equations**,
- Primary simulation results are values for the magnetic vector potential, and flux, flux-density, MMF, eddy currents, stored energy, forces, torques, etc. may be calculated as secondary, post-processing results.

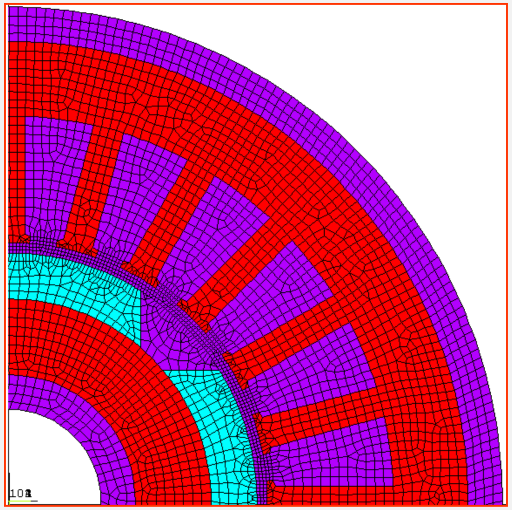


4. Finite Element Method (FEM)

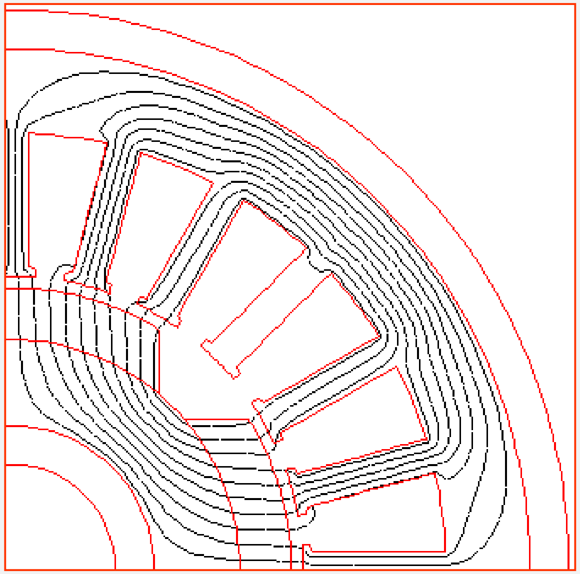
Geometry



FE mesh



FE results



5. Maxwell's Equations

- The governing laws of electromagnetic field problems can be expressed with well known Maxwell's equation in differential form. These are given as,

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \vec{B} = 0$$

- and equations for material,

$$\vec{B} = \mu \cdot \vec{H} + \mu_0 \cdot \vec{M}$$

$$\vec{J} = \gamma \cdot \vec{E}$$

- where, $\vec{H}$ is magnetic field intensity, $\vec{B}$ is magnetic flux density, $\vec{E}$ is electric field intensity, $\vec{J}$ is electric current density, $\vec{M}$ is residual magnetization , $\mu = \mu_0 \cdot \mu_r$ is permeability, and $\nabla = \vec{i}_x \frac{\partial}{\partial x} + \vec{i}_y \frac{\partial}{\partial y} + \vec{i}_z \frac{\partial}{\partial z}$
- Each of the above electromagnetic quantities can be a function of three space coordinates ***x, y, z***.

5. Maxwell’s Equations

- The flux density can be expressed in term of magnetic vector potential A as,

$$\vec{B} = \nabla \times \vec{A}$$

- Using the above equations leads to the second order differential equation for the vector potential:

$$\Delta\vec{A} = \mu_0 \mu_r \gamma \frac{\partial}{\partial t}\vec{A} - \mu_0 \cdot \nabla \times \vec{M}$$

where, $\Delta = \vec{i}_x \frac{\partial^2}{\partial x^2} + \vec{i}_y \frac{\partial^2}{\partial y^2} + \vec{i}_z \frac{\partial^2}{\partial z^2}$

- The three components of B in Cartesian coordinate system are:

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}; \qquad B_y = \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}; \qquad B_z = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

6. Electromagnetic Computation

- Electromagnetic Torque:

$$T_e = \frac{L}{\mu_0} \int_0^{2\pi} r^2 B_r B_\theta d\theta$$

- Electromagnetic Forces:

$$F_{rad} = \frac{1}{2\mu_0} \int_S \left[B_r^2(\theta_s, t) - B_\theta^2(\theta_s, t) \right] dS$$

$$F_{\tan} = \frac{1}{\mu_0} \int_S \left[B_r^2(\theta_s, t) - B_\theta^2(\theta_s, t) \right] dS$$

- Flux-linkage:

$$\phi = \int_S \vec{B} \cdot d\vec{s}$$

$$rot\left(\vec{A}\right) = \vec{B}, \qquad \Rightarrow \phi = \oint \vec{A} \cdot d\vec{l}$$

- Magnetic Energy and Co-energy:

$$W_{\mathrm{m}} = \int_V \int_0^B H dB d\mathrm{v}$$

$$W_{\mathrm{co\_eng}} = \int_V \left(B \cdot H \right) d\mathrm{v} - W_{\mathrm{m}}$$

- Eddy current losses:

$$P_{edd,mag} = \frac{1}{T} \int_0^T \int_{\mathrm{V}} \sigma \left(\frac{\partial A_z}{\partial t} \right)^2 \cdot d\mathrm{v} \cdot dt$$